MT1001 20/21 S1

Sunday, 24 January 2021 10:45 PM

1. (a)

For MKT 1:

$MR = 56 - 8x$

$MC = 8$

For MKT 2:

$MR = 83 - 10x$

$MC = 8$

(ii) For MKT 1:

$f(x) = 56x - 4x^2 - 8x - 120$

When $MR = MC$, i.e. $x = \frac{56-8}{8} = 6$

$f(x)_{max} = f(6) = 24$

For MKT 2:

$f(x) = 83x - 5x^2 - 8x - 120$

When $MR = MC$, i.e. $x = \frac{83-8}{10} = 7.5$

$f(x)_{max} = f(7.5) = 161.25$

(iii) That is $P_1 = P_2 \Rightarrow 5x_2 - 4x_1 = 27$

$MR_1 + MR_2 = 139 - 8x_1 - 10x_2$

$MC_1 + MC_2 = 16$

$\begin{cases} MR_1 + MR_2 = MC_1 + MC_2 \\ 5x_2 - 4x_1 = 27 \end{cases}$ Solved for $\begin{cases} x_1 = 4.3125 \\ x_2 = 8.85 \end{cases}$

Thus,

$f(x_1)_{max} = f(4.3125) = 12.60$

$f(x_2)_{max} = f(8.85) = 152.14$

(b) $p = 52 - 0.025q$

$\frac{dp}{dq} = -0.025$

$\varepsilon = \frac{1}{0.025}\left(\frac{52 - 0.025q}{q}\right) = \frac{2080}{q} - 1$

(i) $\varepsilon > 1$

That is $\frac{2080}{q} > 2$, $0 < q < 1040$, $p > 26$

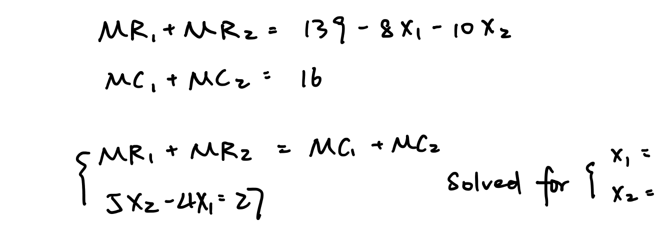
(ii) $\varepsilon < 1$

That is $\frac{2080}{q} < 2$, $1040 < q$, $0 < p < 26$

(iii) It is inelastic. Thus, $\varepsilon < 1 \Rightarrow 0 < p < 26$

(iv) It is elastic. Thus, $\varepsilon > 1 \Rightarrow p > 26$

2. (a)



After 3 years, the balance will be

$\$400{,}000 \times (1 + \frac{4\%}{2})^6 = \$450{,}465$

Thus,

$\frac{40{,}000}{\frac{4\%}{2}}\left(1 - \frac{1}{(1+\frac{4\%}{2})^t}\right) \cdot (1 + \frac{4\%}{2}) = 450{,}465$

Solved for $t = 12 = 6$ years

That is, 9 years are needed.

(b) After 3 years, let's set the last payment Y

$450{,}465 = \frac{40{,}000}{0.02} \times \left(1 - \frac{1}{1.02^{11}}\right) \times 1.02 + \frac{Y}{1.02^{12}}$

Solved for $Y = \$51{,}161.6 \times 1.02^{12}$

$Y = \$64885.28$

3. (a) Let $a = \sqrt{x^2+1}$. $da = \frac{x}{\sqrt{x^2+1}}dx$

Thus, $\int \frac{1}{x} \cdot \frac{1}{x} \cdot da = \int \frac{1}{a^2-1} da$

$= \int \frac{1}{a+1} \cdot \frac{1}{a-1} da$

$= \frac{1}{2}\int \frac{1}{a-1} - \frac{1}{a+1} da$

$= \frac{1}{2}[\ln(a-1) - \ln(a+1)] + C$

(b) $\int_1^\infty \frac{\ln x}{\sqrt{x^3}} dx = \int_1^\infty \frac{1}{x} \cdot \frac{\ln x}{\sqrt{x}} dx$

$= \int_{\ln 1}^{\ln \infty} \frac{1}{\sqrt{x}} \cdot d(\ln x)$. Let $a = \ln x$

$= \int_0^\infty e^{-\frac{a}{2}} da$

$= (-2) \cdot e^{-\frac{a}{2}} \Big|_0^\infty$

$= (-2) \cdot (0 - 1) = 2$

(c) $P_0 = 50 = 50e^{(\frac{q_0^2}{36} - 1)}$ we have $\frac{q_0^2}{36} - 1 = 0$

Thus $q_0 = 6$.

$PS = \int_0^6 50 - 50e^{(\frac{q^2}{36}-1)} dq$

$= 300 - 50\int_0^6 e^{(\frac{q^2}{36}-1)} dq$

Using Simpson's rule with $n = 6$:

$f(x) = e^{(\frac{x^2}{36}-1)}$

$\Delta x = \frac{6}{n} = 1$

$\int_0^6 e^{\frac{q^2}{36}-1} dq \approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + \cdots + 4f(x_5) + f(x_6)]$

where $x_0 = 0$, $x_6 = 6$

≈ 0.2878

4. (a) $f_x(x, y) = e^{(1-x^2-y^2)} \times [2x - 2x \cdot (x^2 + 2y^2)]$

$= e^{(1-x^2-y^2)} \times 2x \times (1 - x^2 - 2y^2)$

$\Rightarrow$ ① $x = 0$ ② $x^2 + 2y^2 = 1$

$f_y(x, y) = e^{(1-x^2-y^2)} \times [4y - 2y \cdot (x^2 + 2y^2)]$

$= e^{(1-x^2-y^2)} \times 2y \times (2 - x^2 - 2y^2)$

$\Rightarrow$ ① $y = 0$ ② $x^2 + 2y^2 = 2$

A (0, 0)

B (0, 1)

C (0, -1)

D (1, 0)

E (-1, 0)

$f_x = e^{1-x^2} \cdot 2x \cdot (1 - x^2)$ ↓

$f_y = e^{1-y^2} \cdot 2y \cdot (2 - 2y^2)$

$f_{xx} = e^{1-x^2} \cdot [2 - 6x^2 - 2x \cdot (2x - 2x^3)]$

$= e^{1-x^2} \cdot (2 - 10x^2 + 4x^4) < 0$

when $x = \pm 1$

$f_{yy} = e^{1-y^2} \cdot [4 - 12y^2 - 2y \cdot (4y - 4y^3)]$

$= e^{1-y^2} \cdot 4 \cdot (1 - 5y^2 + 2y^3)$

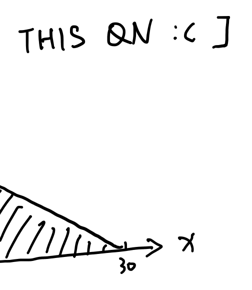
$\begin{cases} f_{yy} > 0 & \text{when } y = 0 \\ f_{yy} < 0 & \text{when } y = \pm 1 \end{cases}$

Thus, point A is a relative minimum.

Whereas B.C.D.E are relative maximum.

[SORRY, I'M NOT SURE ABOUT THIS QN :( ]

(b) $C_A = 3$, $C_B = 5$ $\Rightarrow 3x + 5y = 90$



mean $= \frac{\int_y \int_x Q(x,y)\,dx\,dy}{\int_y \int_x 1\,dx\,dy}$

$= \frac{\int_y \left[\frac{x^3}{3} + y^2x + 5ye^{\frac{x}{5}}\right]_0^{30} dy}{\frac{1}{2} \times 18 \times 30}$

$= \frac{\int_y 9000 + 30y^2 + 5ye^6 + 5y\,dy}{270}$

$= \frac{1}{270} \times \left[9000y + 10y^3 + \frac{5}{2}e^6y^2 + \frac{5}{2}y^2\right]_0^{18}$

$= \frac{1}{270} \times (162k + 58320 + 327587)$

≈ 2029

ALL THE BEST ! :)