MT1001 19/20 S1

Wednesday, 13 January 2021 8:57 PM

1. (a) $N'(x) = 32x - 4x^2$

$N''(x) = 32 - 8x = 0 \Rightarrow$ Diminishing Point $x = 4$

$N(4) = 180 + 16 \times 4^2 - \frac{4}{3} \times 4^3 = 521.33 \Rightarrow (4, 521.33)$

At $(4, 521.33)$, $N'(4) = N'(x)_{max} = 64$ when the ad. cost is \$4,000 .

(b) Revenue $f(p) = p \cdot 7\ln(\frac{10}{p})$

$f'(p) = 7\ln(\frac{10}{p}) + \frac{7}{10}p^2 \cdot (-\frac{10}{p^2}) = 7\ln\frac{10}{p} - 7 = 0$

$\Rightarrow p = \frac{10}{e} \approx 3.68$ thus $q = 7\ln e = 7$

$\Rightarrow$ At a price of \$3680 , revenue is at max. & $p = 7000$

price elasticity $= q' \cdot \frac{p}{q} = 7[\ln(\frac{10}{p}) - 1] \times \frac{p}{q}$

We have $\frac{10}{p} = e^{\frac{q}{7}}$, thus $p = \frac{10}{e^{\frac{q}{7}}} = 10e^{-\frac{q}{7}}$

$\Rightarrow 7[\ln e^{-\frac{q}{7}} - 1] \times 10\frac{e^{-\frac{q}{7}}}{q} = \frac{(-q-7) \times 10}{q \cdot e^{\frac{q}{7}}}$

(c) revenue $f(x) = [(140 - x) \times 4 + 500] \times x$

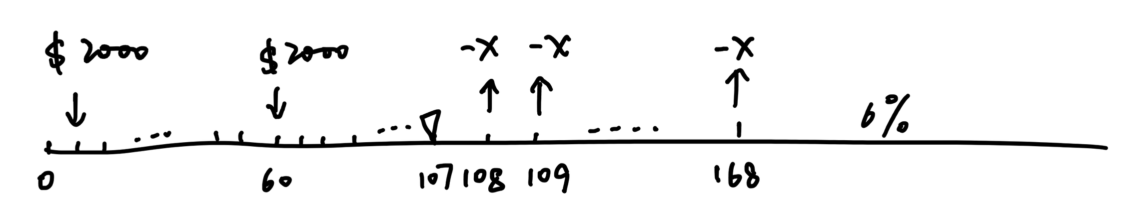
$= 1060x - 4x^2$

$f'(x) = 1060 - 8x \Rightarrow x = \$132.5$

profit at max. $= [(140 - 132.5) \times 4 + 500] \times (132.5 - 11)$

$= \$64395$

2. (a)



In the 107-th month:

$FV = \frac{2000}{\frac{6\%}{12}}[(1 + \frac{6\%}{12})^{60} - 1] \cdot (1 + \frac{6\%}{12})^{107-60} = \$176402$

Thus, $PV = \frac{x}{\frac{6\%}{12}}[1 - \frac{1}{(1+\frac{6\%}{12})^{168-108+1}}] = \$176402$

Solved for $x = \$3362.39$

(b) $FV = 1500000 = \frac{x}{4.5\%}[(1 + 4.5\%)^4 - 1]$

Solved for x : 350615

Year	Balance'	Deposit	Balance''
1	0	350615	350615
2	366393	350615	717008
3	749273	350615	1099888
4	1149383	350617	1500000

3. (a) $\int \frac{1}{x^{\frac{3}{2}} + x} dx = \int \frac{1}{x} \cdot \frac{1}{\sqrt{x} + 1} dx$

$a = \sqrt{x}$. $dx = 2a \cdot da \Rightarrow \int \frac{1}{a^2} \cdot \frac{1}{a+1} \cdot 2a \cdot da$

$= 2\int \frac{1}{a} \cdot \frac{1}{a+1} da$

$\frac{A}{a} + \frac{B}{a+1} = \frac{(A+B)a + A}{a(a+1)} \Rightarrow \begin{cases} A = 1 \\ B = -1 \end{cases}$

$= 2\int \frac{1}{a} - \frac{1}{a+1} da$

$= 2(\ln a - \ln(a+1))$

(b) $\frac{d}{dx}\int_0^{x^3-1} f(t)\,dt = 1$

Thus $f(x) = x + M$

$f(7) = 7 + M$

$\Rightarrow \int_0^{342} x + M\,dx = 7 = [\frac{x^2}{2} + Mx]_0^{342} = 58482 + 342M$

Solved for $M = -171$

Thus $f(7) = 7 - 171 = -164$ & $f'(7) = 1$

(c) $\int_0^1 x - f(x)\,dx = \int_0^1 x - x^2e^{x^2-x} dx$

$= \frac{1}{2} - \underline{\int_0^1 x^2e^{x^2-x} dx}$

$= \frac{1}{2} - 0.2878 \approx 0.2122$

Gini Index $= \frac{0.2122}{0.5} \approx 0.4244$

Formula $= \frac{\frac{1}{2} - \int_0^1 x^2e^{x^2-x}dx}{\frac{1}{2}}$

$F(x) = x^2e^{x^2-x}$

$F'(x) = [2x + x^2(2x-1)]e^{x^2-x}$

By Simpson's Rule, $\Delta x = \frac{1-0}{6} = \frac{1}{6}$

integral $\approx \frac{\Delta x}{3} \cdot [F(x_0) + 4F(x_1) + \dots + 4F(x_5) + F(x_6)]$

where $x_0 = 0$ and $x_6 = 1$

we have integral ≈ 0.2878

4. (a) $\frac{dD}{dx} = \frac{dD}{du} \cdot \frac{du}{dx} + \frac{dD}{dv} \cdot \frac{dv}{dx}$

$= 4(2u - \sqrt{v}) \cdot y - \frac{1}{y} \cdot \frac{2}{2\sqrt{v}}(2u - \sqrt{v})$

$= 8xy^2 - 4y\sqrt{\frac{x}{y}} - \frac{1}{y \cdot \sqrt{\frac{x}{y}}} \cdot (2xy - \sqrt{\frac{x}{y}})$

$= 8xy^2 - 6\sqrt{xy} + \frac{1}{y}$

$\frac{dD}{dy} = \frac{dD}{du} \cdot \frac{du}{dy} + \frac{dD}{dv} \cdot \frac{dv}{dy}$

$= 4(2u - \sqrt{v}) \cdot x - x \cdot \frac{1}{\sqrt{v}}(2u - \sqrt{v})$

$= 8x^2y - 4x\sqrt{\frac{x}{y}} - \frac{2x^2y}{\sqrt{\frac{x}{y}}} - x$

(b) x - \$70

y - \$60

$\Rightarrow 15x^{0.7}y^{0.3} = 3000 \Rightarrow x^{0.7}y^{0.3} = 200 \Rightarrow y = (\frac{200}{x^{0.7}})^{\frac{10}{3}}$

Thus, $f = 70x + 60y$

$= 70x + 60 \cdot 200^{\frac{10}{3}} \cdot x^{-\frac{7}{3}}$

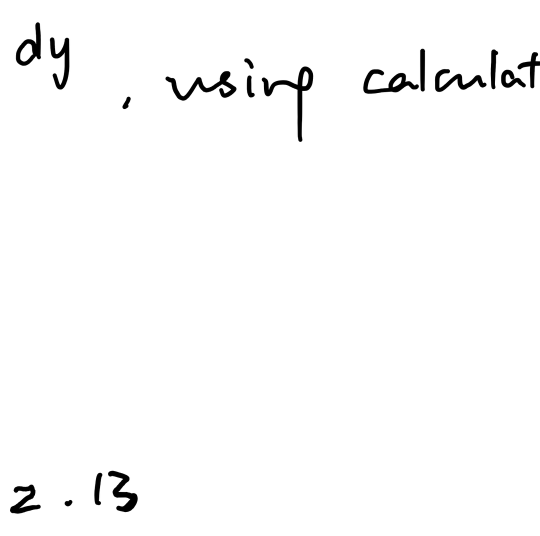
$f'(x) = 70 + 2807057029 \cdot x^{-\frac{10}{3}} \cdot (-\frac{7}{3}) = 0$

Solved for $x = 246.2289$

Thus $f = 70 \times 246.2289 + 60 \times 200^{\frac{10}{3}} \times 246.2289^{-\frac{7}{3}}$

$= \$26722.97$

(c) Notice that $z > 0$



volume $= \int_0^1\int_0^1 z\,dx\,dy$

$= \int_0^1 [\frac{x^3}{3} + y^2 \cdot x + \int_0^1 e^{(1-x)^2}]_0^1 dy$, using calculators

$= \int_0^1 \frac{1}{3} + y^2 + 1.46\,dy$

$\approx 1.796 + \frac{y^3}{3}\Big|_0^1 = 2.13$

All the best ;)