MT1001 2017-2018

1. (a) supply = demand

$$1.8+0.3x=\frac{4.5}{x}+2.5$$

$$0.3x^2-0.7x-4.5=0$$

$$x_1=4.411 \quad x_2=-2.078 \text{ (not suitable)}$$

$$1.8+0.3\times4.411=3.1233$$

$$\therefore (4.411,\ 3.123)$$

(b) after tax $p=(1+10\%)[1.1+0.6\,(2.7)^{0.2x}]$

$$p=1.21+0.66\,(2.7)^{0.2x}$$

$$1.21+0.66\,(2.7)^{0.2x}=\frac{4.5}{x}+2.5$$

$$\frac{4.5}{x}-0.66\,(2.7)^{0.2x}+1.29=0$$

$$f(x)=\frac{4.5}{x}-0.66\,(2.7)^{0.2x}+1.29$$

$$f'(x)=\frac{-4.5}{x^2}-0.132\,(2.7)^{0.2x-1}$$

$$f(5.5)=0.1401>0 \qquad f(6)=-0.1336<0$$

$$x_0=6$$

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=5.53287$$

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}=5.94936$$

$$x_3=x_2-\frac{f(x_2)}{f'(x_2)}=5.94936$$

(c) $E(p)=\frac{p\,f'(p)}{f(p)}=\frac{5\times(-0.18)}{3.4}=-0.26<0$ $\quad |-0.26|<1 \quad \therefore$ inelastic

$$f'(p)=\frac{-4.5}{x^2} \quad f'(5)=-0.18 \quad f(5)=3.4$$

2.(a) (i) MP= Marginal Revenue- Marginal Cost

① $MP = p_1x_1 - (6x_1+90)$

$= (45-3x_1)x_1 - (6x_1+90)$

$= -3x_1^2 + 39x_1 - 90$

② $MP = p_2x_2 - (6x_2+90)$

$= (64-4x_2)x_2 - 6x_2 - 90$

$= -4x_2^2 + 58x_2 - 90$

(ii) ① $MP' = -6x_1+39=0 \quad x_1 = \frac{39}{6}$

② $MP' = -8x_2+58=0 \quad x_2 = \frac{29}{4}$

Total profit: $p_1x_1 + p_2x_2 - 6(x_1+x_2) - 90$

$= (45 - 3\times\frac{39}{6})\times\frac{39}{6} + (64-4\times\frac{29}{4})\times\frac{29}{4} - 6\times(\frac{39}{6}+\frac{29}{4}) - 90$

$= 267$

(b) $100{,}000 = R\times\dfrac{[1+\frac{5\%}{12}]^4-1}{\frac{5\%}{12}}$

$R = 24844.291$

3.(a) $\int\frac{x+1}{x^2+xe^x_n}dx$

$= \int\frac{x+1}{x(x+e^x_n)}dx$

$= \int\frac{1+\frac{1}{x}}{x+e^x_n}dx$ Set $u = x+e^x_n$ $\frac{du}{dx} = 1+\frac{1}{x}$

$= \int\frac{du}{u}$

$= \ln^{|u|} + C$

$= e_n^{|x+e^x_n|} + C$

(b) $f(x) = \int_0^1 |t(t-x)|\,dt$

$= \int_0^1 |t^2 - tx|\,dt$

Set $y = t^2 - tx$ $\quad y_{min} = y(\frac{t}{2}) = \frac{t^2}{2} \geq 0 \quad \therefore y > 0$

$f(x) = \int_0^1 (t^2 - tx)\,dt$

$= \left[\frac{t^3}{3} - \frac{x}{2}t^2\right]_0^1$

$= \frac{1}{3} - \frac{x}{2} \qquad 0 \leq x \leq 1$

$f'(x) = -\frac{1}{2} < 0 \quad \therefore f_{min}(x) = f(1) = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$

(c) (i) $FV = \int_0^T f(t) e^{r(T-t)}\,dt$

$= e^{rT} \int_0^T f(t) e^{-rt}\,dt$

$= e^{0.5} \int_0^5 t \cdot e^{-0.1t}\,dt$

(ii) $\Delta x = \frac{5-0}{8} = \frac{5}{8} \qquad x_0 = 0,\ x_1 = \frac{5}{8},\ x_2 = \frac{5}{4},\ x_3 = \frac{15}{8},\ x_4 = \frac{5}{2},\ x_5 = \frac{25}{8},\ x_6 = \frac{15}{4},$

$x_7 = \frac{35}{8},\ x_8 = 5$

$e^{0.5}\int_0^5 t \cdot e^{-0.1t}\,dt = \frac{5}{24}\left[f(0) + 4f(\frac{5}{8}) + 2f(\frac{5}{4}) + 4f(\frac{15}{8}) + 2f(\frac{5}{2}) + 4f(\frac{25}{8}) + \right.$

$\left. 2f(\frac{15}{4}) + 4f(\frac{35}{8}) + f(5)\right]$

$= \frac{5}{24} \times \left[0 + \frac{5}{2} + \frac{5}{2} + \frac{15}{2} + 5 + \frac{25}{2} + \frac{15}{2} + \frac{35}{2} + 5\right]$

$= \frac{5}{24} \times 60$

$= 12.5$

4.(a) $D = (u+2v)^{\frac{1}{2}} \qquad \frac{\partial D}{\partial x} = \frac{\partial D}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial D}{\partial v} \cdot \frac{\partial v}{\partial x}$

$= \frac{y}{2\sqrt{u+2v}} + \frac{\ln y + ye^x}{\sqrt{u+2v}}$

$= \frac{y}{2\sqrt{xy + \frac{1}{y} + 2x\ln y + 2ye^x}} + \frac{\ln y + ye^x}{\sqrt{xy + \frac{1}{y} + 2x\ln y + 2ye^x}}$

$\frac{\partial D}{\partial y} = \frac{\partial D}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial D}{\partial v} \cdot \frac{\partial v}{\partial y}$

$= \frac{y(x - \frac{1}{y^2})}{2\sqrt{xy + \frac{1}{y} + 2x\ln y + 2ye^x}} + \frac{\frac{x}{y} + e^x}{\sqrt{xy + \frac{1}{y} + 2x\ln y + 2ye^x}}$

(b) $g(x,y) = 0.3x + 0.6y - 12000 = 0$

$F(x,y,\lambda) = 16x^{0.25}y^{0.75} + \lambda(0.3x + 0.6y - 12000)$

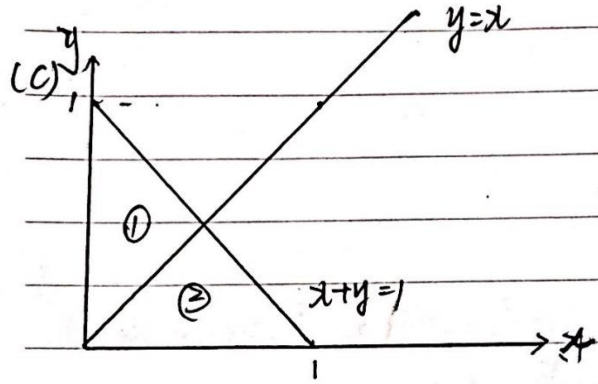
$= 16x^{0.25}y^{0.75} + 0.3\lambda x + 0.6\lambda y - 12000\lambda$

$$\begin{cases} F_x = 4x^{-0.75}y^{0.75} + 0.3\lambda = 0 \\ F_y = 12x^{0.25}y^{-0.25} + 0.6\lambda = 0 \\ F_\lambda = 0.3x + 0.6y - 12000 = 0 \end{cases}$$

$\therefore x = 10000 \quad y = 15000$

$N(10000, 15000) = 216864.4809$

(c)



① $x \le y$: $z = y - x$ $\quad x \le y \le 1$

$\int_0^1 \int_x^1 (y-x)\,dy\,dx$

$= \int_0^1 \left[\frac{1}{2}y^2 - xy\right]_x^1 dx$

$= \int_0^1 \left(\frac{1}{2} - x - \frac{1}{2}x^2 + x^2\right) dx$

$= \int_0^1 \left(\frac{1}{2}x^2 - x + \frac{1}{2}\right) dx$

$= \frac{1}{6}$

② $x \ge y$. $z = x - y$.

similar to ① $V = \frac{1}{6}$

$\therefore \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$