MT1001 2016u2017 $q \in [1, 450]$

1. (a) $\begin{cases} \text{supply} & 44p = 1.1q + 396 \rightarrow p = \dfrac{1.1q+396}{44} & ① \\ \text{demand} & pq = 7500 \rightarrow 1.1q^2 + 396q - 7500 \times 44 = 0 & ② \end{cases}$

$\therefore \quad q_1 = 396.54 \qquad q_2 = -756.54 < 0$ Reject

$\therefore \quad p = 18.91$

$\therefore$ equivalent point is $(\underset{\text{Quantity}}{396.54}, \underset{\text{Price}}{18.91})$

(b) supply $44 P_{original} = 1.1q + 396 \qquad P_{original} = \dfrac{1.1q+396}{44}$

$\begin{cases} P_{new} = P_{original} \times (1+5\%) = \dfrac{1.05}{44}(1.1q+396) & ① \\ \text{Demand} \quad pq = 7500 & ② \end{cases}$

$\therefore \quad q = 384.02 \qquad (q_2 = -744.02 < 0$ Reject$)$

$\therefore \quad p = 19.53$

(c) $\begin{cases} \text{Demand} & p^{0.5} = \dfrac{86.6}{q^{0.75}} \\ \text{Supply} & 44p = 1.1q + 396 \end{cases}$

$\therefore \quad 44 \dfrac{86.6^2}{q^{1.5}} = 1.1q + 369 \qquad \dfrac{86.6^2 \times 44}{q^{1.5}} - 1.1q - \cancel{369}\, 396 = 0 \quad (*)$

Let $f(q) = \dfrac{86.6^2 \times 44}{q^{1.5}} - 1.1q - 396 \qquad q \in [1, 450)$

$f(q)$ is continuous

$f(80) = -22.838036 < 0$
$f(75) = 29.538430 > 0$ $\Rightarrow$ equation $(*)$ must have a root between $q = 75$ and $q = 80$

$\therefore$ there is at least one equilibrium point in the given interval

Newton Method. $\quad x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$f'(q) = 86.6^2 \times 44 \times (-1.5) \times q^{-2.5} - 1.1 = -494970.96\, q^{-2.5} - 1.1$

let $x_0 = 75 \quad f(75) = 29.538430 \quad f'(75) = -11.260769$

$x_1 = 75 - \dfrac{f(75)}{f'(75)} = 77.623127$

$f(x_1) = 1.12 \qquad f'(x_1) = -10.424$

$x_2 = x_1 - \dfrac{f(x_1)}{f(x_1)} = 77.730571$

$f(x_2) = 1.23 \times 10^{-3} \qquad f'(x_2) = -10.39180$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} = 77.730689$

$f(x_3) = 4.07 \times 10^{-6} \qquad f'(x_3) = -10.392$

$x_4 = x_3 - \dfrac{f(x_3)}{f'(x_3)} = 77.730689 = x_3$

$\therefore\ q \approx 77.73069$

2. (a) $5\,m = 5$ million $= 5 \times 10^6 \qquad r = 0.04 \qquad$ month interest $r^* = \dfrac{0.04}{12}$

		①	②	③	(127)	128
month	0	15	16	17	141	142
	5m / −25m	0.2	0.2		0.2	

(b) discount all cash flow to the 15th month

$(25-5) \times \left(1+\frac{0.04}{12}\right)^{15} = 0.2 + \dfrac{0.2}{1+\frac{0.04}{12}} + \dfrac{0.2}{\left(1+\frac{0.04}{12}\right)^2} + \cdots + \dfrac{0.2}{\left(1+\frac{0.04}{12}\right)^x}$

$21.02367 = 0.2\left(1 + \frac{300}{301} + \left(\frac{300}{301}\right)^2 + \cdots + \left(\frac{300}{301}\right)^x\right)$

$21.02367 = 0.2\, \dfrac{1-\left(\frac{300}{301}\right)^{x+1}}{1-\frac{300}{301}}$

$\left(\frac{300}{301}\right)^{x+1} = 0.650770$

$x+1 = 129.09 \qquad x = 128.09$

~~$x \approx 128$~~ the number of 0.2 million payments is 127

②

$$21.02367 - 0.2 \times \frac{1-\left(\frac{300}{301}\right)^{128}}{1-\frac{300}{301}} = 0.142960$$

the last payment is $0.14296 \times \left(1+\frac{0.04}{12}\right)^{127} = 0.218152468 = 0.2 + X$

$$X = \$0.018152468 \times 10^6 = 18152.468.$$

3 (a). let $t = 1+\sqrt{e^x}$

$$e^x = (t-1)^2$$

$$e^x dx = 2(t-1)\,dt$$

$$dx = \frac{2(t-1)}{e^x}dt = \frac{2}{t-1}dt$$

$$\int \frac{dx}{1+e^{\frac{x}{2}}} = \int \frac{1}{t}\frac{2}{t-1}dt = 2\int\left(\frac{1}{t-1}-\frac{1}{t}\right)dt = 2\left[\ln(t-1) - \ln(t)\right] + C$$

$$= 2\ln\sqrt{e^x} - 2\ln(1+\sqrt{e^x}) + C$$

$$= x - \ln(e^x + 1 + 2\sqrt{e^x}) + C$$

(b) $\int_0^1 x f''(2x)\,dx = x \cdot \frac{f'(2x)}{2}\Big|_0^1 - \int_0^1 \frac{f'(2x)}{2}dx$

$$x\frac{f'(2x)}{2}\Big|_0^1 = \frac{f'(2)}{2} = 2.5 \qquad (f'(2)=5)$$

$$\int_0^1 \frac{f'(2x)}{2}dx = \frac{f(2x)}{4}\Big|_0^1 = \frac{f(2)-f(0)}{4} = \frac{3}{4}$$

$$\therefore \int_0^1 x f''(2x)\,dx = 2.5 - 0.75 = 1.75$$

(c) i. $p = 9$ $\quad 9 = 10e^{-0.1\sqrt{x}}$ $\quad x = \left(\frac{\ln 0.9}{-0.1}\right)^2 = 1.1101$

Consumer surplus $= \int_0^{1.11}\left(10e^{-0.1\sqrt{x}} - 9\right)$

ii $n=8$ $\quad \Delta x = \frac{1.11-0}{8} = 0.13875$

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$f(x) = 10e^{-0.1\sqrt{x}} - 9$

number, n	x_n	$f(x_n)$	simpson coefficient k	$k f(x_n)$
0	0	1	1	1
1	0.13875	0.63436	4	2.53744
2	0.2775	0.48685	2	0.97370
3	0.41625	0.37520	4	1.50079
4	0.555	0.28209	2	0.56418
5	0.69375	0.20083	4	0.80331
6	0.8325	0.12797	2	0.25595
7	0.97125	0.06149	4	0.24594
8	1.11	3.58×10^{-5}	1	3.58×10^{-5}

$$\text{Simpson Estimate} = \frac{\Delta x}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+\cdots+4f(x_7)+f(x_8)\right]$$

$$= \frac{\Delta x}{3}\sum_{n=0}^{8} k f(x_n) = \frac{0.13875}{3} \times 7.88135 = 0.36451$$

$$\approx 0.3645$$

4. a. $$\frac{\partial D}{\partial x} = \frac{\partial D}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial D}{\partial v}\frac{\partial v}{\partial x}$$

$$= \frac{1}{\cancel{2}}\cancel{2}(2u+v)^{-\frac{1}{2}}\cdot\frac{z}{y} + \frac{1}{2}(2u+v)^{-\frac{1}{2}}\cdot \ln y$$

$$= (2u+v)^{-\frac{1}{2}}\frac{z}{y} + \frac{1}{2}(2u+v)^{-\frac{1}{2}}\ln y$$

$$\frac{\partial D}{\partial y} = \frac{\partial D}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial D}{\partial v}\frac{\partial v}{\partial y}$$

$$= (2u+v)^{-\frac{1}{2}}\left(\frac{-xz}{y^2}\right) + \frac{1}{2}(2u+v)^{-\frac{1}{2}}\left(\frac{x}{y}+e^{z}\right)$$

$$\frac{\partial D}{\partial z} = \frac{\partial D}{\partial u}\frac{\partial u}{\partial z} + \frac{\partial D}{\partial v}\frac{\partial v}{\partial z}$$

$$= (2u+v)^{-\frac{1}{2}}\frac{x}{y} + \frac{1}{2}(2u+v)^{-\frac{1}{2}}\, ye^{z}$$

b. $P(x,y) = -(x^2+y^2-6x-8y) = -\left((x-3)^2+(y-4)^2-25\right)$

$= 25-(x-3)^2-(y-4)^2$ $\qquad (x-3)^2 \geq 0 \quad (y-4)^2 \geq 0$

≤ 25

$P(x,y) = 25$ (maximum) when $x=3$, $y=4$

$3^2+4^2 \leq 25$ ✓

$\therefore$ maximum profit is 25 when $x=3$ $y=4$