Q1

(a) Supply : $P = \frac{1.1q+396}{44}$ demand : $P = \frac{7500}{q}$

$\frac{1.1q+396}{44} = \frac{7500}{q}$

$1.1q^2 + 396q - 330000 = 0$

$q = \frac{-396 \pm \sqrt{396^2 - 4\times1.1\times(-330000)}}{4\times1.1}$

$q_1 = 396.54$ $q_2 = -756.54$ (Rej)

$P = 18.91$

$\therefore$ Equilibrium point $(396.54, 18.91)$

(b) New supply : $P = \frac{1.1q+396}{44} \times (1+5\%) \Rightarrow P = \frac{1.155q+415.8}{44}$

$\frac{1.155q+415.8}{44} = \frac{7500}{q}$

$1.155q^2 + 415.8q - 330000 = 0$

$q = \frac{-415.8 \pm \sqrt{415.8^2 + 4\times1.155\times330000}}{4\times1.155}$

$q_1 = 384.02$ $q_2 = -744.02$ (Rej)

$P = 19.53$

$\therefore$ New Equilibrium Point $(384.02, 19.53)$

(c) New demand : $P^{0.5} = \frac{86.6}{q^{0.75}} \Rightarrow P = \frac{7499.56}{q^{1.5}}$

$\frac{7499.56}{q^{1.5}} = \frac{1.1q+396}{44}$

$1.1q^{2.5} + 396q^{1.5} - 329980.64 = 0$

$f(q) = 1.1q^{2.5} + 396q^{1.5} - 329980.64$

$f'(q) = 2.75q^{1.5} + 594q^{0.5}$

$f(77) = -5184$ $f(78) = 1920$ $f(77) < f(q) < f(78)$

$\therefore q_0 = 77$

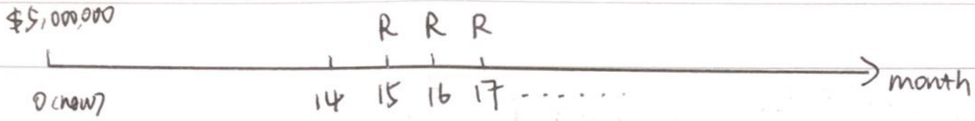
$q_1 = 77 - \frac{f(77)}{f'(77)} = 77.73333$

$q_2 = 77.73333 - \frac{f(77.73333)}{f'(77.73333)} = 77.73069$

$q_3 = 77.73069 - \frac{f(77.73069)}{f'(77.73069)} = 77.73069$

$\therefore q = 77.73069$ $P = 10.94327$

Q2(a)



(b) $k=14$

$$25{,}000{,}000 - 5{,}000{,}000 = 200{,}000 \times \frac{1-(1+\frac{0.04}{12})^{-n}}{\frac{0.04}{12}} \times (1+\frac{0.04}{12})^{-14}$$

$$n = 129.09 \approx 129$$

(c) $$(25{,}000{,}000 - 5{,}000{,}000)(1+\frac{0.04}{12})^{129+14} = 200{,}000\,\frac{(1+\frac{0.04}{12})^{129}-1}{\frac{0.04}{12}} + X$$

$$X = 18884.35$$

Q3(a) let $t=\sqrt{e^x}$ $\quad t^2 = e^x$

$$2t\,dt = e^x dx$$

$$2t\,dt = t^2 dx$$

$$dx = \frac{2}{t}dt$$

$$\int \frac{1}{1+\sqrt{e^x}}\,dx$$

$$= \int \frac{1}{1+t}\cdot\frac{2}{t}\,dt$$

$$\frac{2}{t(1+t)} = \frac{A}{t} + \frac{B}{t+1}$$

$$2 = A(t+1) + Bt$$

$$2 = (A+B)t + A$$

$$A=2 \quad A+B=0 \quad B=-2$$

$$\therefore \int \frac{2}{t} - \frac{2}{t+1}\,dt$$

$$= 2\ln|t| - 2\ln|t+1| + C$$

$$= 2\ln|\sqrt{e^x}| - 2\ln|\sqrt{e^x}+1| + C$$

(b) $$\int_0^1 x f''(2x)\,dx$$

let $u=x \quad u'=1$

$v = f'(2x) \quad v' = f''(2x)$

$$\therefore = [x f'(2x)]_0^1 - \int_0^1 f'(2x)\,dx$$
$$= f'(2x) - 0 - [f(2x)]_0^1$$
$$= f'(2x) - (f(2) - f(0))$$
$$= 5 - (3-2)$$
$$= 4$$

(c)(i) $9 = 10e^{-0.1\sqrt{x}}$

$$e^{0.1\sqrt{x}} = \frac{10}{9}$$
$$0.1\sqrt{x} = \ln\frac{10}{9}$$
$$x = 1.1100$$
$$CS = \int_0^{1.1100} (10e^{-0.1\sqrt{x}} - 9)\,dx$$

(ii) $\Delta x = \frac{1.11}{8} = 0.1388$

$x_1 = 0$ $\quad f(x_1) = 1$

$x_2 = 0.1388$ $\quad f(x_2) = 0.634295$

$x_3 = 0.2776$ $\quad f(x_3) = 0.4867618$

$x_4 = 0.4164$ $\quad f(x_4) = 0.37508866$

$x_5 = 0.5552$ $\quad f(x_5) = 0.2819657322$

$x_6 = 0.6940$ $\quad f(x_6) = 0.2006895$

$x_7 = 0.8328$ $\quad f(x_7) = 0.1278229$

$x_8 = 0.9716$ $\quad f(x_8) = 0.061324$

$x_9 = 1.1104$ $\quad f(x_9) = -0.001350$

$$CS = \int_0^{1.11} (10e^{-0.1\sqrt{x}} - 9)\,dx$$
$$= \frac{0.1388}{3}(f(x_1) + 4f(x_2) + 2f(x_3) + 4f(x_4) + 2f(x_5) + 4f(x_6) + 2f(x_7) + 4f(x_8) + f(x_9))$$
$$= 0.3645$$

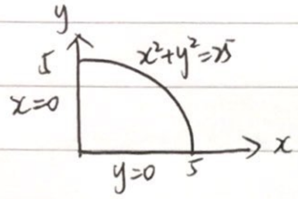
Q4 (a) $D = \left(\frac{2xz}{y} + x\ln y + ye^{z}\right)^{\frac{1}{2}}$

$D_x = \frac{1}{2}\left(\frac{2xz}{y} + x\ln y + ye^{z}\right)^{-\frac{1}{2}}\left(\frac{2z}{y} + \ln y\right)$

$D_y = \frac{1}{2}\left(\frac{2xz}{y} + x\ln y + ye^{z}\right)^{-\frac{1}{2}}\left(-\frac{2xz}{y^2} + \frac{x}{y} + e^{z}\right)$

$D_z = \frac{1}{2}\left(\frac{2xz}{y} + x\ln y + ye^{z}\right)^{-\frac{1}{2}}\left(\frac{2x}{y} + ye^{z}\right)$

(b)



$P(x,y) = -x^2 - y^2 + 6x + 8y \quad x^2+y^2 \le 25$

1.) interior points

$P_x = -2x + 6 = 0 \qquad x = 3$

$P_y = -2y + 8 = 0 \qquad y = 4$

$P_{xx} = -2 \quad P_{yy} = -2 \quad P_{xy} = 0 \quad P_{xx}P_{yy} - P_{xy}^2 = 4 - 0 = 4 > 0$ max

$\therefore P(3,4) = 25$ max

2.) Boundary Points

a) When $y=0 \quad P(x,0) = -2x^2 + 6x$

$P_x = -2x + 6 = 0 \qquad x = 3$

$P_{xx} = -2 < 0$ max

$\therefore P(3,0) = 9$ max

End points $P(0,0) = 0 \qquad P(5,0) = 5$

b) when $x = 0 \quad P(0,y) = -y^2 + 8y$

$P_y = -2y + 8 = 0 \qquad y = 4$

$P_{yy} = -2 < 0$ max

$\therefore P(0,4) = 16$ max

End point $P(0,5) = 15$

c) when $x^2 + y^2 = 25 \Rightarrow x^2 + y^2 - 25 = 0$

$P(x,y,\lambda) = -x^2 - y^2 + 6x + 8y + \lambda(x^2 + y^2 - 25)$

$P_x = -2x + 6 + 2\lambda x = 0 \qquad x = \frac{6}{2-2\lambda}$

$P_y = -2y + 8 + 2\lambda y = 0 \qquad y = \frac{8}{2-2\lambda}$

$P_\lambda = x^2 + y^2 - 25 = 0$

$$\left(\frac{6}{2-2\lambda}\right)^2+\left(\frac{8}{2-2\lambda}\right)^2-25=0$$

$$\frac{36+64}{(2-2\lambda)^2}=25$$

$$(2-2\lambda)^2=4$$

$$2-2\lambda=\pm 2$$

$$\lambda=0 \text{ or } 2$$

$\lambda=0 \quad x=3 \quad y=4$

$\lambda=2 \quad x=-3 \quad y=-4$ (Rej. $\because x,y>0$)

$\therefore P(x,y)=25$

x,y	(3,4)	(3,0)	(0,0)	(5,0)	(0,4)	(0,5)
P	25	9	0	5	16	15

$\therefore P(3,4)=25$

(C) $z=\sqrt{x^2-2xy+y^2}=\sqrt{(x-y)^2}$

① $x-y\geq 0 \qquad 0\leq y\leq x$

② $y-x\geq 0 \qquad x\leq y\leq 1$

$$\int_0^1\int_0^x (x-y)\,dy\,dx+\int_0^1\int_x^1 (y-x)\,dy\,dx$$

$$=\int_0^1 \left(xy-\tfrac{1}{2}y^2\right)_0^x dx+\int_0^1\left(\tfrac{1}{2}y^2-xy\right)_x^1 dx$$

$$=\int_0^1\left(x^2-\tfrac{1}{2}x^2\right)dx+\int_0^1\left(\tfrac{1}{2}-x-\tfrac{1}{2}x^2+x^2\right)dx$$

$$=\int_0^1\left(\tfrac{1}{2}x^2\right)dx+\int_0^1\left(\tfrac{1}{2}-x+\tfrac{1}{2}x^2\right)dx$$

$$=\left(\tfrac{1}{6}x^3\right)_0^1+\left(\tfrac{1}{2}x-\tfrac{1}{2}x^2+\tfrac{1}{6}x^3\right)_0^1$$

$$=\tfrac{1}{6}+\tfrac{1}{2}-\tfrac{1}{2}+\tfrac{1}{6}$$

$$=\tfrac{1}{3}$$

Done by Miao Mengjie