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MT1001 Semester 1 2015-2016

1\. (a) supply function after 7% tax: $p = 2.75 + 0.21x$

before 7% tax: $p' = \frac{2.75+0.21x}{1+7\%}$

equilibrium: supply equals to demand

$$\frac{2.75+0.21x}{1+7\%} = \frac{4.28}{x} + 1.67$$

$x_1 = 7.49$ $\cancel{x_2 = -2.91}$ $(7.49, 2.24)$

(b) supply function before tax: $p = 1.3 + 0.75e^{0.25x}$

after tax: $p' = (1.3 + 0.75e^{0.25x}) \times (1+7\%)$

equilibrium is where $(1.3 + 0.75e^{0.25x}) \times (1+7\%) = \frac{4.28}{x} + 2.718$

$1.391 + 0.8025e^{0.25x} = \frac{4.28}{x} + 2.718$

let $f(x) = 0.8025e^{0.25x} - \frac{4.28}{x} - 1.327 = 0$

Newton's method: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f'(x) = 0.8025 \times 0.25e^{0.25x} + 4.28\frac{1}{x^2}$

$\because f(4) < 0$ $f(5) > 0$

$\therefore x_1 = 4 - \frac{f(4)}{f'(4)} = 4.26521$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 4.26461$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 4.26461$

equilibrium point $(4.26461, 3.72161)$

(c) $f'(p) = 0.75 \times 0.25e^{0.25x}$

$E = \frac{pf'(p)}{f(p)} = \frac{4 \times 0.75 \times 0.25 \times e^{0.25 \times 4}}{1.3 + 0.75 \times e^{0.25 \times 4}} = \frac{0.75e}{1.3 + 0.75e} = 0.61 < 1$ inelastic

2\. (a) (i) $C(x) = 7.1x + 95$

① Market A: Profit $p(x_1) = p_1x_1 - C(x_1) = 46.5x_1 - 2x_1^2 - 7.1x - 95$

$= -2x_1^2 + 39.4x_1 - 95$

Marginal Profit: $p'(x_1) = -4x_1 + 39.4$

② Market B: Profit $p(x_2) = p_2x_2 - C(x_2) = 62.5x_2 - 3x_2^2 - 7.1x_2 - 95$

$= -3x_2^2 + 55.4x_2 - 95$

Marginal Profit: $p'(x_2) = -6x_2 + 55.4$

(ii) $x_1 = \frac{39.4}{4}$ $x_2 = \frac{55.4}{6}$

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Total profit : $-2\times(\frac{39.4}{4})^2+39.4\times\frac{39.4}{4}-95-3\times(\frac{55.4}{6})^2+55.4\times\frac{55.4}{6}-95$

$= 259.81$

(b) $500{,}000 = R\times\frac{(1+\frac{4\%}{12})^5-1}{\frac{4\%}{12}}$

$R = 9933555$

3.(a) $\int\frac{1}{x+x^{n+1}}dx$

$=\int(\frac{1}{x(x^n+1)})dx$

$=\int(\frac{1}{x}-\frac{x^{n-1}}{x^n+1})dx$

$=\ln|x|-\frac{1}{n}\int\frac{nx^{n-1}}{x^n+1}dx+c'$

$=\ln|x|-\frac{1}{n}\ln|x^n+1|+c$, (c is a constant)

(b) $\frac{d}{dx}f(x)=\frac{d}{dx}(x\int_0^1 f(x)dx)$

$=\int_0^1 f(x)dx = \text{constant}$

let $f(x)=ax+b$

$f(x)=(x\int_0^1 f(x)dx)-1 \longrightarrow ax+b = x\int_0^1(ax+b)dx-1$

$= x[\frac{1}{2}ax^2+bx]_0^1-1$

$= x(\frac{a}{2}+b)-1$

$\therefore a=-2$, $b=-1$

$f(x)=-2x-1$

(c)(i) $FV=\int_0^T f(t)e^{r(T-t)}dt = e^{rT}\int_0^T f(t)e^{-rt}dt$

$e^{0.1\times3}\int_0^3 f(3)e^{-0.1\times3}dt = e^{0.3}\int_0^3 f(t)e^{-0.1t}dt$

$=e^{0.3}\int_0^3 e^{-0.1t^2-0.1t}dt$

Simpson's rule: $\int_a^b f(x)dx=\frac{\Delta x}{3}[f(x_1)+4f(x_2)+2f(x_3)+4f(x_4)+\cdots+4f(x_n)+f(x_{n+1})]$

$\because n=8$ $\therefore \Delta x=\frac{3-0}{8}=\frac{3}{8}$

$\int_0^3 e^{-0.1t^2-0.1t}dt=\frac{1}{8}[f(0)+4f(\frac{3}{8})+2f(\frac{6}{8})+4f(\frac{9}{8})+2f(\frac{12}{8})+4f(\frac{15}{8})+2f(\frac{18}{8})$
$+4f(\frac{21}{8})+f(3)]$

$=2.0273$

4.(a) $D=(u+2v)^2=(\frac{x}{yz}+2x+2\ln y+2e^z)^2$

$D_x= 2(\frac{x}{yz}+2x+2\ln y+2e^z)(\frac{1}{yz}+2)$

$D_y= 2(\frac{x}{yz}+2x+2\ln y+2e^z)(-\frac{x}{zy^2}+\frac{2}{y})$

$D_z= 2(\frac{x}{yz}+2x+2\ln y+2e^z)(-\frac{x}{yz^2}+2e^z)$

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(b) $g(x,y) = x^2+y^2-1=0$

$F(x,y,\lambda) = x^2y^2\sqrt{2-x^2-y^2} + \lambda(x^2+y^2-1)$

$$\begin{cases} F_x = 2xy^2\sqrt{2-x^2-y^2} + x^2y^2\dfrac{-2x}{2\sqrt{2-x^2-y^2}} + 2\lambda x = 0 \\ F_y = 2x^2y\sqrt{2-x^2-y^2} + x^2y^2\dfrac{-2y}{2\sqrt{2-x^2-y^2}} + 2\lambda y = 0 \\ F_\lambda = x^2+y^2-1=0 \end{cases}$$

$\therefore \quad x = \frac{\sqrt{2}}{2},\ y = \frac{\sqrt{2}}{2},\ \lambda = -\frac{3}{8}.$

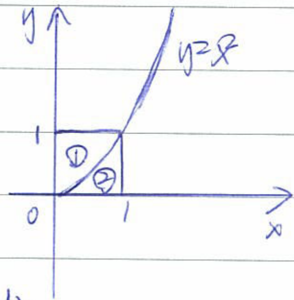
$P(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) = \frac{1}{4}.$

(c) ① $x^2<y<1$

$\int_0^1\int_{x^2}^1 x^2\,dy\,dx$

② $0<y<x^2$

$\int_0^1\int_0^{x^2} y\,dy\,dx$



$$\therefore \min\{y,x^2\} = \int_0^1\int_{x^2}^1 x^2\,dy\,dx + \int_0^1\int_0^{x^2} y\,dy\,dx$$
$$= \int_0^1 (x^2y|_{x^2}^1)dx + \int_0^1 (\tfrac{y^2}{2}|_0^{x^2})dx$$
$$= \int_0^1 (x^2-x^4)\,dx + \int_0^1 (\tfrac{x^4}{2})dx$$
$$= \tfrac{x^3}{3} - \tfrac{x^5}{5}\Big|_0^1 + \tfrac{x^5}{10}\Big|_0^1$$
$$= \tfrac{1}{3} - \tfrac{1}{5} + \tfrac{1}{10}$$
$$= \tfrac{7}{30}$$

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