MT1001

December 2014

1\. a. Equilibrium : Demand = Supply

$$\Rightarrow \frac{2750}{q} = \frac{3q}{20} + 15$$

$$\Rightarrow \frac{3q^2}{20} + 15q - 2750 = 0$$

$$q_1 = \frac{-15 + \sqrt{15^2 - 4\times\frac{3}{20}\times(-2750)}}{\frac{3}{20}\times 2} = 94.34$$

$$q_2 = \frac{-15 - \sqrt{15^2 - 4\times\frac{3}{20}\times(-2750)}}{\frac{3}{20}\times 2} < 0 \quad \text{(abandon)}$$

b. Equilibrium : Demand = (1+15%) Supply

$$\Rightarrow \frac{2750}{q} = (1+15\%)\times \frac{3q}{20} + 15$$

$$\Rightarrow 0.1725q^2 + 17.25 - 2750 = 0$$

$$q_1 = \frac{-17.25 + \sqrt{17.25^2 - 4\times 0.1725\times(-2750)}}{0.1725\times 2} = 85.80$$

$$q_2 = \frac{-17.25 - \sqrt{17.25^2 - 4\times 0.1725\times(-2750)}}{0.1725\times 2} < 0 \quad \text{(abandon)}$$

c. Equilibrium : $\frac{10500}{q^{1.7}} = \frac{3}{20}q + 15$

$$\Rightarrow \frac{10500}{q^{1.7}} - \frac{3}{20}q - 15 = 0$$

$$f(q) = \frac{10500}{q^{1.7}} - \frac{3}{20}q - 15$$

$$f'(q) = -17850q^{-2.7} - 0.15$$

Newton's method: $q_{n+1} = q_n - \frac{f(q_n)}{f'(q_n)}$

$f(38.5) > 0 \qquad f(39) < 0$

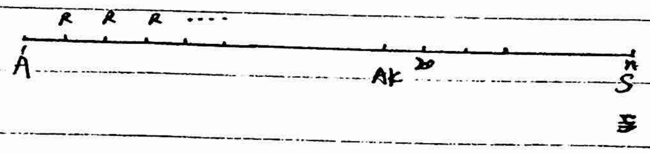
$$q_1 = 38.5 - \frac{f(38.5)}{f'(38.5)} = 38.87232$$

$$q_2 = q_1 - \frac{f(q_1)}{f'(q_1)} = 38.87655$$

$$q_3 = q_2 - \frac{f(q_2)}{f'(q_2)} = 38.87655$$

$\therefore$ The new equilibrium point is 38.87655

2. (a).



(b). $A = R\left(\frac{1-(1+r)^{-n}}{r}\right)(1+r)^{-k}$

$23,500,000 - 5,000,000 = 400,000\left(\frac{1-(1+\frac{0.036}{12})^{-n}}{\frac{0.036}{12}}\right)\left(1+\frac{0.026}{12}\right)^{-19}$

$n \approx 53$

$n' = 53 + 20 = 73$

(c). $18,500,000 = 400,000\left(\frac{1-(1+0.003)^{-53}}{0.003}\right)(1+0.003)^{-19} + X(1+0.003)^{-73}$

$X = 11813.80$

The rest is the interest.

3. (a). $\int \frac{dx}{x(2+x^8)}$

Assume $u = x^8$ $\quad du = 8x^7\,dx$

$u = x^8$

$\frac{1}{x^8}\cdot\frac{1}{x^8+2}$

$\int \frac{dx}{x(2+x^8)} = \frac{1}{8}\int \frac{1}{u}\cdot\frac{1}{u+2}\,du$

$= \frac{1}{8}\int\left(\frac{1}{u} - \frac{1}{u+2}\right)du$

$= \frac{1}{8}\left(\int \frac{1}{u}\,du - \int \frac{1}{u+2}\,du\right)$

$= \frac{1}{8}\ln x^8 - \frac{1}{8}\ln(x^8+2)$

$= \ln x - \frac{1}{8}\ln(x^8+2)$

(b). $\int_0^1 (x-1)^2 f(x)\,dx$

$= \frac{1}{3}(x-1)^3 f(x)\Big|_0^1 - \int_0^1 \frac{1}{3}(x-1)^3\cdot e^{-(x-1)^2}\,dx$

$= 0 - \frac{1}{3}\int_0^1 \frac{1}{2}(x-1)^2 e^{-(x-1)^2}\,d(x-1)^2$

$= \frac{1}{6}\int_0^1 (x-1)^2\cdot e^{-(x-1)^2}\,d(x-1)^2$ $\quad$ Assume $y = (x-1)^2$

$= \frac{1}{6}\int_0^1 y\cdot e^{-y}\,dy = -\frac{1}{6}y\cdot e^{-y}\Big|_0^1 + \frac{1}{6}\int_0^1 e^{-y}\,dy$

$= -\frac{1}{6e} - \frac{1}{6}\left(\frac{1}{e} - 1\right)$

3.(c). (i). $CS = \int_0^{1.03} [10e^{-0.1x^2} - 9]\, dx$

(ii). $\int_0^{1.03} [10e^{-0.1x^2} - 9]\, dx$

$= \frac{0.12875}{2} [f(0) + 2f(0.12875) + f(0.2575) + 2f(0.38625) + f(0.515) + 2f(0.64375)$
$+ f(0.7725) + 2f(0.90125) + f(1.03)]$

$= \frac{0.12875}{2} [1 + 1.9669 + 0.9329 + 1.7038 + 0.7383 + 1.1881 + 0.4207 + 0.4397 - 0.0066]$

$= 0.5397$

4. (a). $D = (\frac{x}{y} + 2x + 2\ln y)^2$

$D_x = 2(\frac{x}{y} + 2x + 2\ln y) \cdot (\frac{1}{y} + 2)$

$D_y = 2(\frac{x}{y} + 2x + 2\ln y) \cdot (-\frac{x}{y^2} + \frac{2}{y})$

(b). Assume $F(x,y,\lambda) = P(x,y) + \lambda g(x,y)$

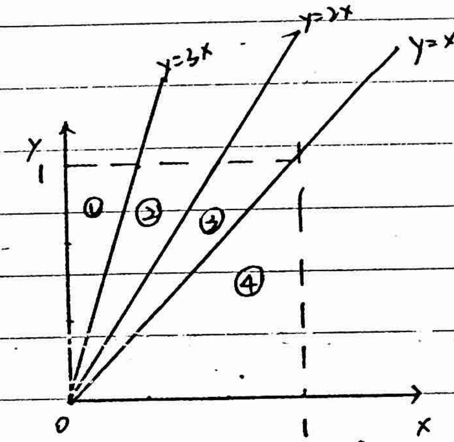
$= \frac{x+y}{x^2+y^2+1} + \lambda(x^2+y^2)$

$F_x(x,y,\lambda) = \frac{-x^2-2x+1}{(x^2+y^2+1)^2} + 2\lambda x = 0$

$F_y(x,y,\lambda) = \frac{-y^2-2y+1}{(x^2+y^2+1)^2} + 2\lambda y = 0$

$F_\lambda(x,y,\lambda) = x^2+y^2 = 0$

(c).



① $y-2x \geq x$ $\quad y \geq 3x$

② $y-2x \geq 0$ $\quad 2x \leq y < 3x$

③ $2x-y \leq x$ $\quad x \leq y < 2x$

④ $\cancel{2x-y > x}$ $\quad y < x$

$V = \int_0^1\int_0^x (2x-y)\, dy\, dx + \int_0^1\int_x^{2x} x\, dy\, dx + \int_0^1\int_{2x}^{3x} x\, dy\, dx + \int_0^1\int_{3x}^1 (y-2x)\, dy\, dx$

$= \int_0^1 (2x^2 - \frac{1}{2}x^2)\, dx + \int_0^1 x^2\, dx + \int_0^1 x^2\, dx + \int_0^1 (\frac{1}{2} - 2x + \frac{5}{2}x^2)\, dx$

$= \frac{1}{2}x^3\Big|_0^1 + \frac{2}{3}x^3\Big|_0^1 + (\frac{1}{2}x - x^2 + \frac{1}{2}x^3)\Big|_0^1$

$= \frac{7}{6}$