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MH2814 PYP YR 2015 DEC

Before I start, I have to point out some errors of exam Q.

- Question 1(b): Assume that the building will not be rebuilt after ---
- Question 4(b): ··· The finding were that the sample mean is 13.8 km/Liter
- Question 5(c): ··· estimate the correlation coefficient ρ.
- Question 5(d)(ii) ··· confidence interval of the mean water consumption for the given population size.

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Q1. (a)(i) Poisson Distribution, $\lambda = 0.2$

$$P(X=1) = \frac{(0.2)^1}{1!} \cdot e^{-0.2} = 16.375\%.$$

(ii) $P(X \geq 2) = 1 - P(X=0) - P(X=1) = 1 - \frac{(0.2)^0}{0!} \cdot e^{-0.2} - 16.375\% = 1.75\%.$

(b)(i) Earthquakes are independent.

$P(X \geq 2) = 1.75\%$,

Probability distribution should be geometric distribution

Parameters: X is Geom(0.0175)

(ii) $E(X) = \frac{1}{P} = \frac{1}{0.0175} = 57.14$ years

(iii) $P(X \leq 3) = P(X=1) + P(X=2) + P(X=3)$

$= 0.0175 + 0.0175 \cdot (1-0.0175)^1 + 0.0175 \cdot (1-0.0175)^2$

$= 5.16\%$

(iv) $P(X>2) = 1 - P(X=0) - P(X=1) \overset{-P(X=2)}{=} 1.75\% - P(X=2) = 1.75\% - \frac{(0.2)^2}{2!} \cdot e^{-0.2}$

$P(X>2) = 0.1125\%$

$P(\text{Geo})$ of $x \leq 3 = P(x=1) + P(x=2) + P(X=3)$ at $P = 0.1125\%$

$P(\text{Geo})$ of $x \leq 3 = 0.1125\% + 0.1125\% \cdot (1-0.1125\%)^1 + 0.1125\% \cdot (1-0.1125\%)^2 < 1\%$

Therefore, it's sufficient to build the building to withhold 3 years with $< 1\%$ of collapse rate. Because P of collapse is less that required which is 1%.

Q2 (a) $E(x) = 14mm$, $\sigma = 1.2mm$

A bearing has diameter of at least 12.8mm:

$P(X \geq 12.8mm) = 1 - P(X \leq 12.8mm) = 1 - \Phi\left(\frac{12.8-14}{1.2}\right) = 1 - \Phi(-1)$

$\because \Phi(1) = 0.8413$, $\therefore$ From the theorem, $\Phi(-1) = 1 - \Phi(1) = 0.1587$

$\therefore P(X \geq 12.8mm) = 1 - \Phi(-1) = \Phi(1) = 84.13\%$

(b) (i) Independent, distribution will be binomial distribution

Parameter: $Bin(3, 84.13\%)$

(ii) $P(x \geq 2) = 1 - P(x=0) - P(x=1)$

$= 1 - \binom{3}{0} \cdot 84.13\%^0 \cdot (1-84.13\%)^{3-0} - \binom{3}{1} \cdot 84.13\%^1 \cdot (1-84.13\%)^{3-1}$

$= 93.31\%$

(c)(i) $\because P(X \geq 2)$ of $Bin(3; P) = 1 - \binom{3}{0} \cdot P^0 \cdot (1-P)^{3-0} - \binom{3}{1} \cdot P^1 \cdot (1-P)^{3-1}$

$\therefore P(X \geq 2) = 1 - 1 \cdot 1 \cdot (1-P)^3 - 3 \cdot P \cdot (1-P)^2$

$\therefore P(X \geq 2) = 1 - (1-P)\cdot(1-P)^2 - 3P \cdot (1-P)^2$

$\therefore P(X \geq 2) = 1 - (1-P+3P) \cdot (1-2P+P^2)$

$\therefore P(X \geq 2) = 1 - (1+2P) \cdot (1-2P+P^2)$

$\therefore P(X \geq 2) = 1 - (1 - \cancel{2P} + P^2 + \cancel{2P} - 4P^2 + 2P^3)$

$\therefore P(X \geq 2) = 1 - 1 - P^2 + 4P^2 - 2P^3 = -2P^3 + 3P^2$

$\because P(X \geq 2)$ in question compute that $P(X \geq 2) = 0.972$

$\therefore -2P^3 + 3P^2 = 0.972$

$\therefore 2P^3 - 3P^2 + 0.972 = 0$ End prove.

(ii) $\because (P-0.9) \cdot (2P^2 - 1.2P - 1.08) = 0. \Rightarrow \begin{cases} \text{either } P-0.9=0 & ① \\ \text{or } 2P^2 - 1.2P - 1.08 = 0. & ② \end{cases}$

$\therefore$ ①: $P - 0.9 = 0$, $P = 0.9$

②: $2P^2 - 1.2P - 1.08 = 0$

$P = \frac{1.2 \pm \sqrt{(-1.2)^2 + 4 \cdot 1.08 \cdot 2}}{2 \cdot 2} \Rightarrow \begin{cases} P_1 = 1.0937 \\ P_2 = -0.4937 \end{cases}$

$\because$ Possibility $0 \leq P \leq 1$ $\therefore$ Possible value is $P = 0.9$.

(iii) $\because 0.9 = 1 - \Phi\left(\frac{d-14}{1.2}\right) \Rightarrow \Phi\left(\frac{d-14}{1.2}\right) = 0.1$

$\because \Phi(1.282) = 0.9 \Rightarrow \Phi(-1.282) = 1 - \Phi(1.282) = 1 - 0.9 = 0.1$

$\therefore \frac{d-14}{1.2} = -1.282 \Rightarrow d = 12.4616mm$

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Q3. (a) From the chart we can list:

$$\begin{cases} 4\cdot P + 8\cdot 0.2 + 12\cdot 0.3 + 16\cdot q = 12 \\ P + 0.2 + 0.3 + q = 1 \quad \text{(total } P = 1) \end{cases}$$

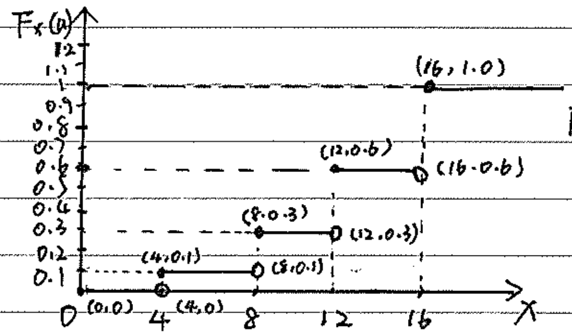
$$\Rightarrow \begin{cases} P = 0.1 \\ q = 0.4 \end{cases}$$

(b) $Var(X) = \sum_{all\ x} (x - E(x))^2 \cdot P(X = x)$

$\Rightarrow Var(X) = (4-12)^2 \cdot 0.1 + (8-12)^2 \cdot 0.2 + (12-12)^2 \cdot 0.3 + (16-12)^2 \cdot 0.4$

$\therefore Var(X) = 16.$

(c)&(d) CDF, $F_X(a) = \sum_{x=a} (P(X=x))$, graph shown:



$F_X(a)$.

$$\Rightarrow F_X(a) \begin{cases} 0 & (x < 4) \\ 0.1 & (4 \le x < 8) \\ 0.3 & (8 \le x < 12) \\ 0.6 & (12 \le x < 16) \\ 1.0 & (x \ge 16) \end{cases}$$

(e) The assumption made are:

- Assume X is rondomly select from the number of x above
- We assume p follow poisson distribution
- It proximate the item as normal distribution with σ of the group $= \frac{16}{\sqrt{64}} = 2$, $E(\bar{x}) = \mu$)

(Im Not sure for this 3(e) is correct or not).

Q4 (a)(i) $H_0: \mu = 14$ km/liter, accept the car fuel efficient standard
$H_1: \mu < 14$ km/liter, reject the car fuel efficient standard

(ii) The appropriate way is to use Z distribution because we already know the S.D.

(iii) test statistic: $Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{13.8 - 14}{1.2/\sqrt{10}} = -0.52$

(iv) Rejion of rejection $Z_{\alpha = 5\%} = -Z_{0.95} = -1.645$

(v) Null hypothesis is accepted due to test statistic does not lie in region of rejection $(-0.52 > -1.645)$

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(b) (i) Null $H_0: \mu = 14.0$ km/liter, accept the car fuel efficiency
$H_1: \mu < 14.0$ km/liter, reject the car fuel efficiency

(ii) Should use z distribution due to known S.D and large $n > 30$.

(iii) Test statistic: $z = \frac{13.8 - 14}{1.5/\sqrt{225}} = -2$

(iv) region of rejection $z_{(\alpha=5\%)} = -z_{0.95} = -1.645$

(v) Should accept the alternative due to the test statistic lies in the region of rejection $(-2 < -1.645)$

Q5. (a) (i) $\bar{x} = (14+16+27+42+39+50+83) \div 7 = 38.71$

(ii) S_x^2 for x, because we must calculate the non-bias S_x^2,

$S_x^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n \cdot \bar{x}^2\right) = \frac{1}{7-1} \cdot (14^2+16^2+27^2+42^2+39^2+50^2+83^2 - 7 \cdot (38.71)^2)$

$= \frac{1}{6} \cdot (13855 - 10489.2487)$

$= 560.96$

(iii) $\bar{y} = (2+5+7+9+10+13+20) \div 7 = 9.43$.

(iv) $S_Y^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 - n \cdot \bar{y}^2\right) = \frac{1}{7-1}(2^2+5^2+7^2+9^2+10^2+13^2+20^2 - 7 \cdot 9.43^2)$

$S_Y^2 = 828 \cdot \frac{1}{6} = 138$

(b) $\because \hat{\beta} = \frac{(\sum xy) - n \cdot \bar{x} \cdot \bar{y}}{(\sum x^2) - n \cdot \bar{x}^2}$, $\sum xy = 14 \cdot 2 + 16 \cdot 5 + 27 \cdot 7 + 42 \cdot 9 + 39 \cdot 10 + 50 \cdot 13 + 83 \cdot 20$

$\Rightarrow \sum xy = 3375$

$\because \sum x^2 = 14^2+16^2+27^2+42^2+39^2+50^2+83^2 = 13855$

$\therefore \hat{\beta} = \frac{3375 - 7 \cdot 38.71 \cdot 9.43}{13855 - 7 \cdot 38.71^2} = 0.243$

$\therefore \hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} = 9.43 - 0.243 \cdot 38.71 = 0.0019$

$\therefore E(Y|X=x) = 0.0019 + 0.243x$

(c) $S_Y^2|_{x=x} = \frac{\sum (y-\hat{y})^2}{n-2} \Rightarrow \because \sum (y-\hat{y})^2 = [2-(0.0019+0.243 \cdot 14)]^2 + [5-(0.0019+0.243 \cdot 16)]^2 + \dots + [20-(0.0019+0.243 \cdot 83)]^2$

$\Rightarrow \sum (y-\hat{y})^2 = 5.87$

$\therefore S_Y^2|_{x=x} = \frac{5.87}{7-2} = \frac{5.87}{5} = 1.17$

$\rho = \sqrt{1 - \frac{\sum (y-\hat{y})^2}{\sum (y-\bar{y})^2}} \Rightarrow \because \sum (y-\bar{y})^2 = (2-9.43)^2 + (5-9.43)^2 + \dots + (20-9.43)^2 = 205.7143$

$\rho = \sqrt{1 - \frac{5.87}{205.7143}} = 0.986$

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(d) (i) The expected consumption follow $y = \hat{\alpha} + \hat{\beta} x$

$\because x = 50$,

$y = 0.0019 + 0.243 \cdot 50 = 12.1519$ (10^6 liter)

(ii) confidence interval: $d = t_{0.99} \cdot S_Y|_{x=x} \cdot \sqrt{\frac{1}{n} + \frac{(x-\bar{x})^2}{\sum x_i^2 - n \cdot \bar{x}^2}}$

$\because$ Sample $n = 7$, $\therefore d.o.f = 5$, $x = 50$

$\Rightarrow t_{0.99}$ at $d.o.f = 5 = \cancel{2.57}\ 3.365$

$\Rightarrow d = 3.365 \cdot \sqrt{1.17} \cdot \sqrt{\frac{1}{7} + \frac{(50-38.71)^2}{(13855 - 10489.2487)}} = 1.547$

$\therefore$ confidence interval (98%) $= (12.1519 - 1.547,\ 12.1519 + 1.547)$

$d: (10.6045, 13.6993)$.

At last, best wishes for your exam! :)