MH2814 No.: 2014/2015 sem 1

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1a i $P(A) = P(B) = P(C) = \frac{1}{3}$ ①

$P(A \cap B) = P(A \cap C) = P(B \cap C) = 0.1$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= 3(\frac{1}{3}) - 3(0.1) + P(A \cap B \cap C)$

$= 0.7 + P(A \cap B \cap C)$

$\therefore P(A \cap B \cap C) = P(A \cup B \cup C) - 0.7$

$\because P(A \cup B \cup C) \leq 1$

$\therefore P(A \cap B \cap C) \leq 0.3$

$\therefore$ Cannot equal 0.4

ii There are 3 red balls and 3 blue balls in box 1, 2 & 3

Let A: event picked red ball from box 1, balls picked are put in box 3

B: event picked red ball from box 2, balls picked are put in box 3

C: event picked red ball from box 3

$P(A) = P(B) = P(C) = \frac{3}{6} = \frac{1}{2}$

→ A and B are independent

→ $P(B \cap C) = P(R | R_2) P(R_2)$

$= (\frac{4}{7})(\frac{1}{2})$

$= \frac{2}{7}$ $P(B \cap C) \neq P(B) \cdot P(C)$

$\therefore$ B and C are dependent

$\therefore$ conditions satisfied

$P(A \cap C) = P(R | R_1) P(R_1)$

$= (\frac{4}{7})(\frac{1}{2})$

$= \frac{2}{7} \neq P(A) \cdot P(C)$

$\therefore$ A and C are dependent.

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②

1b) Let X be event even number appear

$$P(X \geq 6) = P(X=6) + P(X=7)$$

$$= \frac{(3)^6(3)(7)}{6^7} + \frac{3^7}{6^7}$$

$$= 0.0625$$ #

odd

(7 ways to arrange)

∴ multiply by factor of 7

ii Let X be event two 6's in a row and no other 6's

6

$$P(X) = \frac{(1)^2(5)^5(6)}{6^7}$$

$$= \cancel{0.060}\ 0.0669796$$ #

2 a) Let E_i be i^{th} suits did not appear

$$P(E_1 \cup E_2 \cup E_3) = \sum_{k=1}^{3}(-1)^{k+1} \sum_{1 \leq i_1 < i_2 < i_3 \leq 3} P(E_{i_1} \cap E_{i_2} \cap \dots)$$ (Inclusion-exclusion principle)

$$= \sum_{k=1}^{3}(-1)^{k+1} \sum_{1 \leq i_1 < i_2 < i_3 \leq 3} \frac{\binom{52-13k}{7}}{\binom{52}{7}}$$

$$= \sum_{k=1}^{3}(-1)^{k+1} \frac{\binom{52-13k}{7}}{\binom{52}{7}} \binom{4}{k}$$

$$= 0.43042$$

∴ probability all suits occur at least once $= 1 - 0.43042$

$$= 0.56958$$ #

ii

$$= \frac{\binom{13}{4}\binom{13}{1}^3\binom{4}{1}}{\binom{52}{7}}$$

number of suits for the one suits of 4 cards

$$= 0.046967$$ #

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2b

$p = 0.01$

$n = 50$

$\lambda = np = 0.5$

Because $np \le 5$ and $n \ge 50$

It is sufficient to use Poisson distribution #

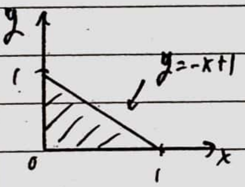
~~$P(X \ge 2)$~~

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(X = 2) = \frac{e^{-0.5}(0.5)^2}{2!}$$

$$= 0.07582 \ \#$$

3 i



$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, dy\, dx = 1$$

$$k\int_0^1\int_0^{-x+1} x + y \ dy\, dx = 1$$

$$k\int_0^1 xy + \frac{y^2}{2}\Big|_0^{-x+1} dx = 1$$

$$k\int_0^1 -\frac{1}{2}x^2 + \frac{1}{2}\, dx = 1$$

$$k\left[-\frac{x^3}{6} + \frac{1}{2}x\Big|_0^1\right] = 1$$

$$k = 3 \ \#$$

ii $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$

$$= 3\int_0^{-x+1} x + y \ dy$$

$$= 3\left[xy + \frac{y^2}{2}\Big|_0^{-x+1}\right]$$

$$= 3\left[x(-x+1) + \frac{(-x+1)^2}{2}\right]$$

$$= \frac{3}{2} - \frac{3}{2}x^2 \ \#$$

$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\, dx$

$$= 3\int_0^{-y+1} x + y \ dx$$

$$= 3\left[\frac{x^2}{2} + xy\Big|_0^{-y+1}\right]$$

$$= \frac{3}{2} - \frac{3}{2}y^2 \ \#$$

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④

III $E(X) = \int_{-\infty}^{\infty} x f_X(x)\, dx$

$= \int_0^1 x\left(\frac{3}{2} - \frac{3}{2}x^2\right) dx$

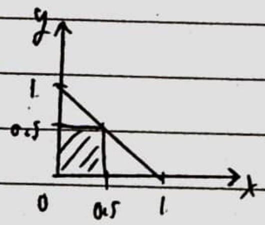
$= 0.375$ #

$E(Y) = \int_{-\infty}^{\infty} y f(y)\, dy$

$= \int_0^1 y\left(\frac{3}{2} - \frac{3}{2}y^2\right) dy$

$= 0.375$ *

IV $P(X \le 0.5, Y \le 0.5)$



$= 3\int_0^{0.5}\int_0^{0.5} x + y \, dy\, dx$

$= 3\int_0^{0.5} xy + \frac{y^2}{2}\Big|_0^{0.5} dx$

$= 3\int_0^{0.5} \frac{x}{2} + \frac{1}{8}\, dx$

$= 3\left[\frac{x^2}{4} + \frac{1}{8}x\Big|_0^{0.5}\right]$

$= 0.375$ #

V $f(x,y) = 3(x+y)$

$f_X(x) \cdot f_Y(y) = \left(\frac{3}{2} - \frac{3}{2}x^2\right)\left(\frac{3}{2} - \frac{3}{2}y^2\right)$

Choose $(X = 0.5, Y = 0.5)$

$f(0.5, 0.5) = 3$

$f_X(0.5) f_Y(0.5) = 1.266$

$\cancel{f(x,y) \ne f_X(x}$ $f(0.5, 0.5) \ne f_X(0.5) f_Y(0.5)$

$\therefore$ X and Y are dependent. *

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4a

$$f(x) = \begin{cases} p, & x=1 \\ 1-p, & x=0 \end{cases}$$ Bernoulli distribution

$M(t) = E(e^{tx})$

$= e^{t(0)}(1-p) + e^{t(1)}(p)$

$= 1 - p + e^{t}p$

$M'(t) = e^{t}p$

$M'(0) = E(X) = p$ # (mean)

$M''(t) = e^{t}p$

$M''(0) = E(X^2) = p$

$Var(x) = E(x^2) - [E(x)]^2$

$= p - p^2$

$= p(1-p)$ # (Variance)

$M'''(t) = e^{t}p$

$M'''(0) = E(X^3) = p$ # (third moment)

b i $E(x) = 0 \cdot \frac{a}{2} + 1 \cdot \frac{1-a}{4} + 2 \cdot \frac{3(1-a)}{4} + 3 \cdot \frac{a}{2}$

$= \frac{7}{4} - \frac{a}{4}$

$\frac{\Sigma x}{N} = \frac{2+1+2+1+3+3}{8} = \frac{12}{8}$

$E(x) = \frac{\Sigma x}{N}$

$\frac{7}{4} - \frac{a}{4} = \frac{12}{8}$

$\hat{a} = 1$ #

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5 a i Average of data set $= \frac{2.3 + 4.26}{2}$

$= 3.28$ #

ii $100(1-\alpha) = 99$

$\alpha = 0.01$

$\frac{\alpha}{2} = 0.005$

$P(Z \geq Z_{\frac{\alpha}{2}}) = \frac{\alpha}{2}$

$\cancel{P(Z \geq Z_{0.005}) = 0.005}$

$\phi(Z_{0.005}) = 0.995$

$Z_{0.005} = 2.575$

Confidence interval for $\mu = [\bar{x} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \bar{x} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}]$

$= [3.28 - (2.575)(\frac{2}{\sqrt{16}}), 3.28 + (2.575)(\frac{2}{\sqrt{16}})]$

$= [1.9925, 4.5675]$ #

b For T to be unbiased estimate of μ

$E(T) = \mu$

$\cancel{E(a(X_1 + X_2 + \dots + X_n) + b) = \mu}$

$aE(X_1 + X_2 + \dots + X_n) + b = \mu$

$an\mu + b = \mu$

$\therefore a = \frac{1}{n}, b = 0$ #

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4b))

$$lik(a|x_1,\ldots,x_n) = f(x_1,\ldots,x_n|a)$$

$$= \prod f(x_i|a)$$

$$= \left(\frac{a}{2}\right)^2\left(\frac{1-a}{4}\right)^2\left(\frac{3(1-a)}{4}\right)^2\left(\frac{a}{2}\right)^2$$

$$\ell = \sum \ln \left(\frac{a}{2}\right)^2\left(\frac{1-a}{4}\right)^2\left(\frac{3(1-a)}{4}\right)^2\left(\frac{a}{2}\right)^2$$

$$= 2\ln\left(\frac{a}{2}\right) + 2\ln\left(\frac{1-a}{4}\right) + 2\ln\left(\frac{3(1-a)}{4}\right) + 2\ln\left(\frac{a}{2}\right)$$

$$\frac{d\ell}{da} = \frac{2}{a} + \frac{2(-1)}{1-a} + \frac{2(-1)}{1-a} + \frac{2}{a}$$

$$\frac{d\ell}{da} = 0$$

$$\frac{4}{a} = \frac{4}{1-a}$$

$$\hat{a} = 0.5 \ \#$$