NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2017-2018

CV4116 - COASTAL ENGINEERING

April / May 2018 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FOUR (4)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. All answers must be written in the answer book provided.
5. This is a Restricted Open Book Examination. Only **ONE (1) Sheet** of A4 paper with notes on both sides is allowed.
6. A separate booklet of tables and charts is issued together with the paper. Do not write on this booklet.

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1. (a) There are three missing data in the series of wave measurements (accurate to one decimal place) in the following table:

Wave	H(m)	Wave	H(m)	Wave	H(m)
1	3.6	11	1.2	21	3.4
2	2.3	12	3.6	22	1.7
3	2.8	13	1.8	23	2.7
4	3.2	14	1.1	24	3.4
5	0.9	15	0.6	25	0.8
6	1.6	16	3.5	26	0.5
7	4.3	17	4.5	27	3.3
8	2.4	18	0.5	28	?? x
9	2.7	19	2.1	29	?? y
10	4.7	20	4.5	30	?? z
	28.5		23.4		15.8

However, it is known that the mean wave height is 2.6 m, the significant wave height is 4.1 m and H_{10} is 4.6 m, respectively. Compute the values of the three missing data. $H_{1/3}$

(13 Marks)

(b) Show, by derivation, that the potential energy density due to the wave motion is equal to $\rho g H^2/16$. Also briefly discuss how one can derive the total energy density for the wave motion.

(7 Marks)

Note: Question No. 1 continues on page 2

1) (a)

4.7, 4.5, 4.5, 4.3, 3.6, 3.6, 3.5, 3.4

$H_{10} = 4.6 = \frac{1}{3}(4.7+4.5+x) \rightarrow 13.8 = 4.7+4.5+x$

$x = 4.6m$

$H_{\frac{1}{3}} = 4.1 = \frac{1}{10}[4.7+4.5+4.5+4.3+3.6+3.6+3.5+3.4+x+y]$

$41 = 32.1 + x + y$

$y = 4.3$

$x+y+z = 7.3$

$\frac{28.5+23.4+15.8+x+y+z}{30} = 2.6$

$x+y+z+67.7 = 78$

$z = 1.4$

$\therefore x = 4.6$ m, $y = 4.3$ m, $z = 1.4$ m

(b) Consider a column of water with Δx

Centroid height $h_c = \frac{\eta+d}{2}$

$\Delta P.E. = \text{Mass} \times g \times h_c = \rho(\eta+d)\,\Delta x \times g \times \frac{\eta+d}{2} = \rho g(\eta+d)\left(\frac{\eta+d}{2}\right)\Delta x$

Total P.E. over one wave length, $PE|_{wave} = \int_0^L \rho g(\eta+d)\left(\frac{\eta+d}{2}\right)dx$

Without wave motion, $PE|_{still\ water} = \int_0^L \rho g d\left(\frac{d}{2}\right)dx$, when $\eta = 0$

Thus, PE due to wave motion, $E_p = PE|_{wave} - PE|_{still\ water}$

$= \int_0^L \rho g\left(\frac{\eta^2}{2} + \eta d\right)dx$

$\eta = \frac{H}{2}\cos(kx-\sigma t) = \frac{H}{2}\cos(kx)$ assume $t=0$

$\therefore E_p = \int_0^L \rho g\left(\frac{H^2}{8}\cos^2(kx) + \frac{Hd}{2}\cos(kx)\right)dx = \int_0^L \rho g\frac{H^2}{8}\left(\frac{1}{2}+\frac{1}{2}\cos(2kx)\right)dx = \rho g\frac{H^2}{16}L$

$\therefore$ Energy density, $\bar{E}_p = \frac{E_p}{L} = \rho g\frac{H^2}{16}$

The total density to the wave motion can be calculated by adding the kinematic energy density to the potential energy density.

(c) Describe qualitatively three different types of wave energy capturing systems for renewable energy generation that are being explored around the world.

(5 Marks)

2. (a) A wave train with a wave period of 10 seconds is propagating perpendicularly towards the shore. The deep water wave height Ho is equal to 2.1 m and the bed contours are parallel to the shore.

(i) Compute the range of transitional water depth (i.e. between deep water, d1, and shallow water, d2 as shown in Figure Q2) for this wave train.

(ii) Calculate the range of shoaling coefficient in this transitional water depth range.

(iii) What is the magnitude of the horizontal velocity at the sea bed level at the two locations with d1 and d2, as well as at the location half-way in between?

(iv) Will wave breaking likely occur within this transitional zone if one uses the wave breaking criterion of H/d =0.78? Explain your reasoning.

(18 Marks)

(b) Instead of perpendicular approach, the wave train in Part 2(a) now approaches the shore with uniform bottom contours at a deep water angle of 60 degree between the wave crest and bottom contours. Will the range of transitional water depth change? Again assess the possibility of wave breaking in this depth range with the criterion of h/d = 0.78. Explain your reasoning.

Assume $K_r K_s H_0 = 0.78 d_b$

(7 Marks)

X, constant wall slope ???

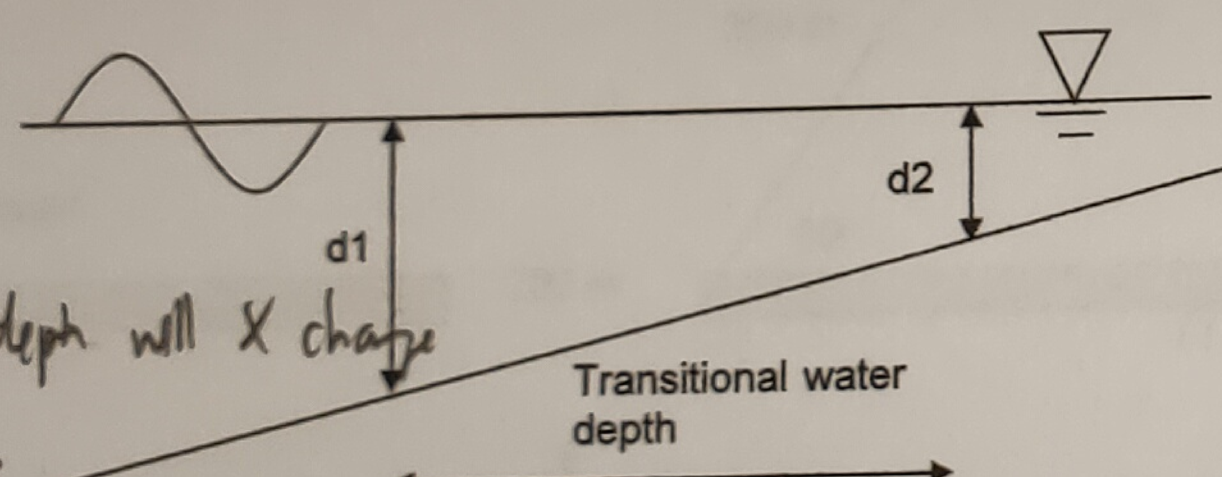

Figure Q2

(b) the range of transitional water depth will X change

$\frac{d_2}{gT^2} = 0.0025, \quad \theta_0 = 60^\circ$

$K_s K_r = 0.92$

$H = (K_s K_r) H_0 = 0.92(2.1) = 1.932\,m$

$\frac{H}{d_2} = \frac{1.932}{2.45} = 0.79 > 0.78$ (0.79 more closer to 0.78)

Most likely in transitional depth $\frac{H}{d} < 0.78$, wave will X break

2) (a) $T = 10\,s$

$H_0 = 2.1\,m$

(i) Deep water: $L_0 = \frac{gT^2}{2\pi} = \frac{g(10)^2}{2\pi} = 156.13$

$\frac{d_1}{L} > \frac{1}{2}$

$d_1 > \frac{1}{2}(L) = 78\,m$

shallow water: $L = T\sqrt{gd} = 10\sqrt{g(d)} = 31.3\sqrt{d}$

$\frac{d_2}{L} < \frac{1}{20}$

$\frac{d_2}{31.3\sqrt{d_2}} < \frac{1}{20}$

$d_2^{0.5} < 1.565$

$d_2 < 2.45$

∴ Range of transitional water: $2.45\,m \le d \le 78$

(ii) $K_s = \sqrt{\frac{1}{\tanh(kd)\left[1 + \frac{2kd}{\sinh(2kd)}\right]}}$, where $k = \frac{2\pi}{L}$

Deep water: $K_s = \sqrt{\frac{1}{\tanh(0.0403 \times 78)\left[1 + \frac{2(0.0403 \times 78)}{\sinh(2kd)}\right]}} = 0.99$

$k = \frac{2\pi}{L} = \frac{2\pi}{156.13} = 0.0403$

Shallow water: $K_s = \sqrt{\frac{1}{\tanh(0.128 \times 2.45)\left[1 + \frac{2(\;)}{\sinh(\;)}\right]}} = 1.30$

$k = \frac{2\pi}{L} = \frac{2\pi}{31.3\sqrt{2.45}} = 0.128$

∴ Range of $K_s = 0.99 \le K_s \le 1.30$

(iii) • At deep water, $d_1 = 78\,m$

$u_1 = \frac{\pi H}{T} e^{\left(\frac{2\pi z}{L_0}\right)} \cos\theta$

$= \frac{\pi(2.1)}{10} e^{\left(\frac{2\pi 78}{156.13}\right)}$

$= 0.028\,m/s$

~~ux transition formula~~

• At shallow water, $\frac{d_2}{gT^2} = \frac{2.45}{g(10)^2} = 0.0025$, $\theta_0 = 0^\circ$

CEM Fig II-3-6: get $K_R K_s = 1.35$

$H = (K_R K_s) H_0 = 1.35 \times 2.1 = 2.835\,m$

$u_2 = \frac{H}{2}\sqrt{\frac{g}{d}} \cos\theta = \frac{2.835}{2}\sqrt{\frac{g}{2.45}} = 2.84\,m/s$

• At half way in between

$d = \frac{d_1 + d_2}{2} = \frac{78 + 2.45}{2} = 40.225\,m$

$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$

$= \frac{g(10)^2}{2\pi} \tanh\left(\frac{2\pi \times 40.225}{L}\right)$

$= 146.5$ (Iterate)

$\frac{d}{gT^2} = \frac{40.225}{g(10)^2} = 0.041, \quad \theta_0 = 0^\circ$

get $K_s K_R = 0.935$

$H = (K_s K_R) H_0 = 0.935 \times 2.1 = 1.9635\,m$

$u = \frac{H}{2} \frac{gT}{L} \frac{\cosh\left[\frac{2\pi(z+d)}{L}\right]}{\cosh\left(\frac{2\pi d}{L}\right)}$

$= \frac{1.9635 g(10)}{2(146.5)} \frac{\cosh\left[\frac{2\pi(-40.225 + 40.225)}{146.5}\right]}{\cosh\left(\frac{2\pi(40.225)}{146.55}\right)}$

$= 0.23\,m/s$

(iv) $\frac{H}{d_2} = \frac{2.835}{2.45} = 1.16 > 0.78$

∴ wave will break near to the shore in transitional zone.

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3. (a) Waves are normally incident onto a long breakwater which has a gap opening of 200 m as sketched in Figure Q3. The waves have wave height H of 0.7 m and wave period T of 6 s. Determine the diffracted wave height at point A located at 300 m along the 60° line with respect to the right arm of breakwater. Assume a constant water depth of 5 m. State any assumptions used in your answer.

Determine H at A = ?

(10 Marks)

(b) What is the encounter probability in any given year of a 100-year wave (i.e. wave with T_r = 100 years). Hence derive the expression for the encounter probability P_e of a T_r return period wave over a structure design lifetime of L years.

(7 Marks)

(c) Given Morison's equation and applying linear waves in deep water, determine the expression for the maximum drag force acting on a vertical circular cylinder in terms of the wave and cylinder parameters. You can assume that the cylinder extends more than one wavelength vertically into the water.

(8 Marks)

(b) Encounter prob in any year $= \frac{1}{T_r} = \frac{1}{100}$

Prob X encounter in any year $= 1 - \frac{1}{100}$

Prob X encounter in any year over L years $= \left(1-\frac{1}{100}\right)^L$

Encounter probability over L years $= 1 - \left(1-\frac{1}{100}\right)^L$

(c)

Useful formula:

Morison's equation for a circular cylinder

$$f = f_I + f_D = \frac{\pi}{4} C_M \rho D^2 \frac{du}{dt} + \frac{1}{2} C_D D \rho |u| u$$

(a)

$$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$$
$$= \frac{g(6)^2}{2\pi} \tanh\left(\frac{2\pi \times 5}{L}\right)$$
$$= 38.09 \text{ m (transitional wave)}$$

$\frac{R}{L} = \frac{300}{38.09} = 7.88$, $\beta = 60°$, get $K' = 0.14$ (Fig 2-33)

$H_A = K' H_i = 0.14 \times 0.7$
$= 0.098$ m

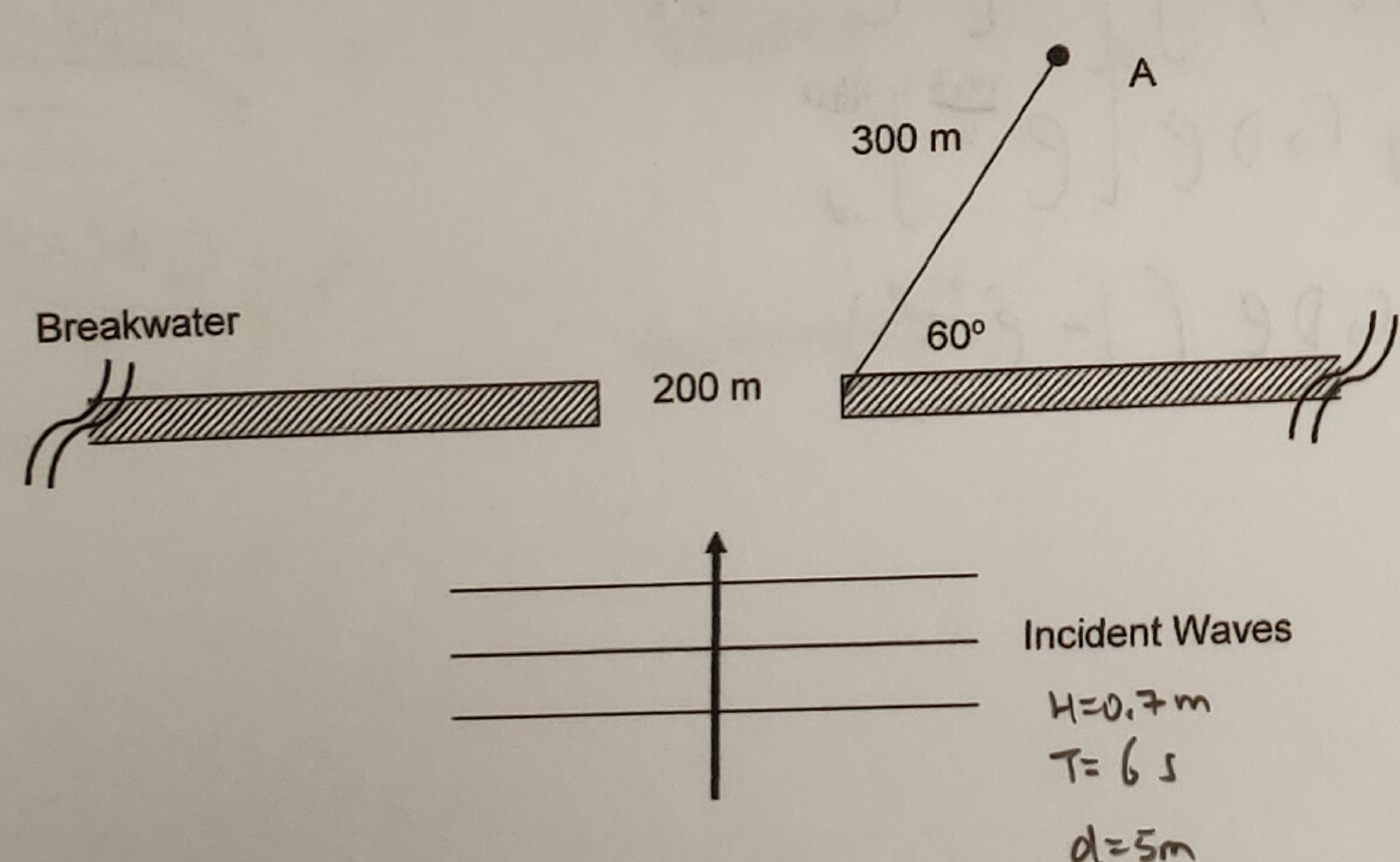

Figure Q3

Left break water: $R = \sqrt{(200+300\cos 60)^2 + (300\sin 60)^2} = 435.89$, $\frac{R}{L} = \frac{435.89}{38.09} = 11.44$

Left break water in transitional zone !!!
(X [illegible])

3. (a) Waves are normally incident onto a long breakwater which has a gap opening of 200 m as sketched in Figure Q3. The waves have wave height H of 0.7 m and wave period T of 6 s. Determine the diffracted wave height at point A located at 300 m along the 60° line with respect to the right arm of breakwater. Assume a constant water depth of 5 m. State any assumptions used in your answer.

(10 Marks)

(b) What is the encounter probability in any given year of a 100-year wave (i.e. wave with T_r = 100 years). Hence derive the expression for the encounter probability P_e of a T_r return period wave over a structure design lifetime of L years.

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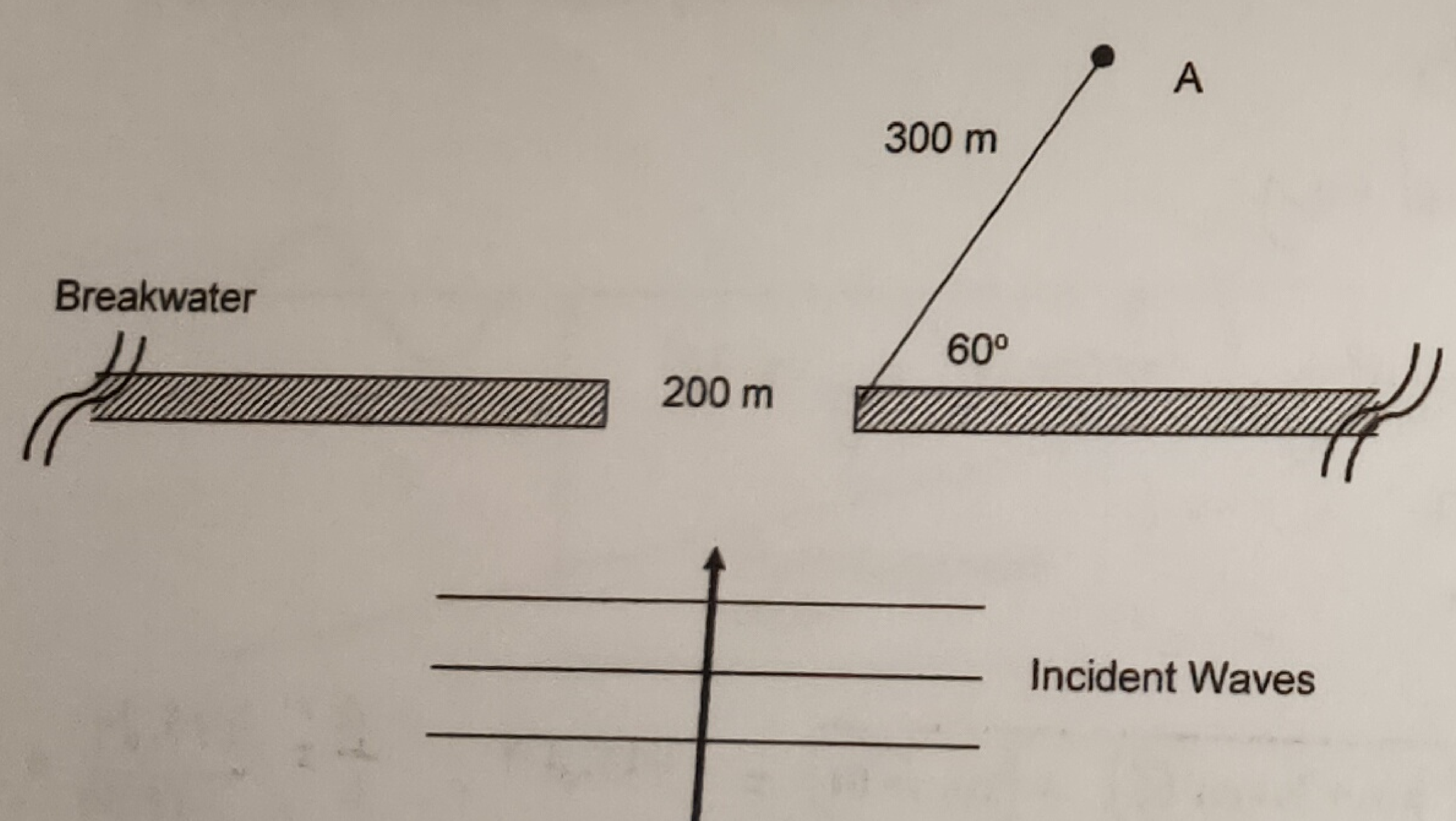

Figure Q3

3)(c) $F_D = \int_{-d}^{0} f_D\, dz$

$= \int_{-d}^{0} \frac{1}{2} C_D D\rho |u| u \, dz$

$= \frac{1}{2}\int_{-d}^{0} C_D \rho \frac{\pi^2 H^2}{T^2} e^{\frac{4\pi z}{L}} |\cos\theta| \cos\theta \, dz$

$u = \frac{\pi H}{T} e^{\frac{2\pi z}{L}}$

$F_{DM} = \frac{1}{2} C_D \rho \frac{\pi^2 H^2}{T^2} \int_{-d}^{0} e^{\frac{4\pi z}{L}} z$

$= \frac{1}{8} C_D \rho \frac{\pi^2 H^2}{T^2} \left[e^{\frac{4\pi z}{L}}\right]_{-d}^{0}$

$= \frac{1}{8} C_D \rho \frac{\pi^2 H^2}{T^2} \left(1 - e^{-\frac{4\pi d}{L}}\right)$

$\int e^{\frac{4\pi z}{L}}$

$F_D = \int_{-d}^{0} f_D\, dz$

$= \int_{-d}^{0} \frac{1}{2} C_D D \rho |u| u \, dz$

$F_D = \int_{-d}^{0} \frac{1}{2} C_D D \rho |u| u \, dz \quad e^{\frac{4\pi z}{L}} \cdot \frac{4\pi}{L}$

$u = \frac{\pi H}{T} e^{\frac{2\pi z}{L}}$

$F_D = \int_{-d}^{0} \frac{1}{2} C_D D \rho \frac{\pi^2 H^2}{T^2} e^{\frac{4\pi z}{L}} dz$

$= C_D D\rho H^2 g \left(\frac{1}{8}\right) \int_{-d}^{0} \frac{4\pi}{L} e^{\frac{4\pi z}{L}} dz$

$= \frac{1}{8} H^2 g C_D D \rho \left[e^{\frac{4\pi z}{L}}\right]_{-d}^{0}$

$= \frac{1}{8} H^2 g C_D D \rho \left(1 - e^{-\frac{4\pi d}{L}}\right)$

$F_D = \int_{-d}^{0} \frac{1}{2} C_D D \rho \frac{\pi^2 H^2}{T^2} e^{\frac{4\pi z}{L}} dz$

$= \frac{1}{2} \frac{C_D D \rho \pi^2 H^2}{T^2} \int_{-d}^{0} e^{\frac{4\pi z}{L}} dz$

$= \frac{1}{2} \frac{C_D D \rho \pi^2 H^2}{T^2} \frac{L}{4\pi} \int_{-d}^{0} \frac{4\pi}{L} e^{\frac{4\pi z}{L}} dz$

$= \frac{1}{8} \frac{C_D D \rho \pi H^2}{T^2} \left[e^{\frac{4\pi z}{L}}\right]_{-d}^{0}$

$= \frac{1}{8} \frac{C_D D \rho \pi H^2}{T^2} \left(1 - e^{\frac{4\pi d}{L}}\right)$

4. (a) A caisson wall is built on a rubble foundation as shown in Figure Q4. Waves are normally incident with significant wave height H_s of 1.5 m and wave period T of 7 s. Using Miche-Rungren's method, determine the maximum wave force acting on the seaward side of the caisson wall. Assume calm water exists on the lee side and that the specific weight of seawater is 10 kN/m³.

(13 Marks)

(b) Quarry stones are to be used for the rubble foundation in Part 4(a). Determine the minimum mass of the quarry stones needed using 2,600 kg/m³ as the stone density.

(6 Marks)

(c) Sketch the pressure profile acting along the base of the caisson assuming that the rubble foundation is porous. Suggest a way how this pressure profile can be calculated. Note no actual calculations are needed for this part.

(6 Marks)

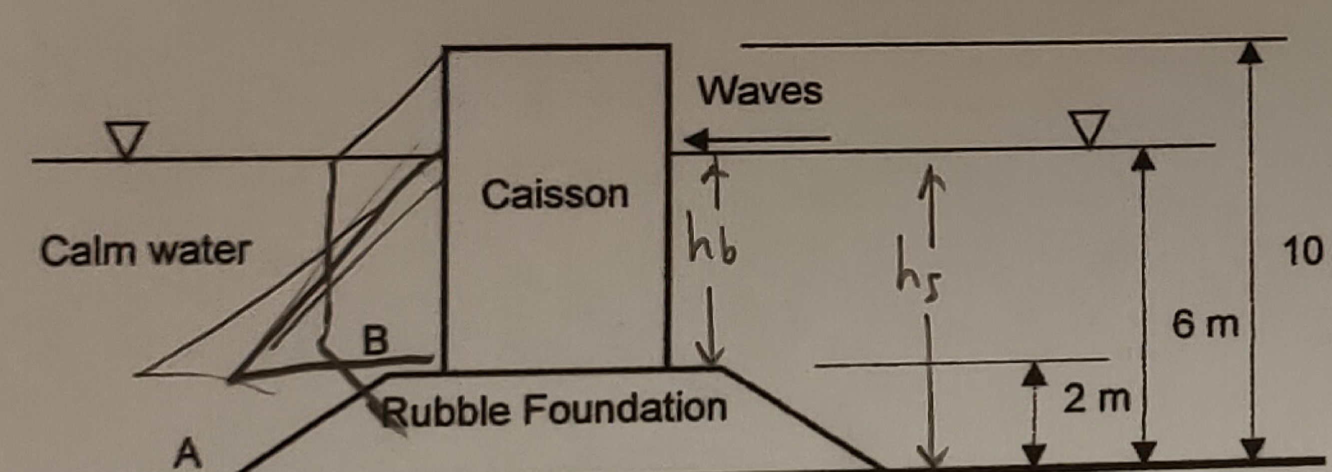

Figure Q4

END OF PAPER

4(a) $H_s = 1.5$ m
$T = 7$ s

$H_{\frac{1}{100}} = 1.67 H_s = 1.67(1.5) = 2.505$ m

$H_i = 2$ m

CEM Fig 7.90

$\frac{H_i}{gT^2} = \frac{1.005}{g(7)^2} = 0.0021$, $\frac{H_i}{d} = \frac{1.005}{6} = 0.1675 = 0.17$

get $\frac{H_0}{H_i} = 0.2$

$H_0 = 0.2 H_i = 0.2(1.005) = 0.2$ m

$y_c = d + h_0 + \left(\frac{1+\chi}{2}\right) H_i$
$= 6 + 0.2 + \left(\frac{1+1}{2}\right)1$
$= 7.2$ m < 10 (no overtopping)

$\chi = 1.0$

SPM 7.91

$\frac{H_i}{gT^2} = 0.0021$, $\frac{H_i}{d} = 0.17$

$\frac{F_c}{\rho g d^2} = 0.16 \rightarrow F_c = 57.6$ kN/m

$\frac{M_c}{\rho g d^3} = 0.1 \rightarrow M_c = 216$ kNm/m

Including hydrostatic on one side
$F_{TC} = F_c + \frac{1}{2}\rho g d^2$
$= 57.6 + 180$
$= 237.6$ kN/m

$M_{TC} = M_c + \frac{1}{6}\rho g d^3$
$= 216 + 360$
$= 576$ kNm/m

(b) Quarry stone

Hudson: $M = \frac{\rho_s H^3}{K_D\left(\frac{\rho_s}{\rho_w} - 1\right)^3 \cot\alpha}$

Slope ???

Rubble foundation: $H_{1/10} = 1.27(H_s) = 1.27(1.5) = 1.905$ m

$\frac{h_b}{h_s} = \frac{4}{6} = 0.667$

CEM VI-5.45 → get $N_s^3 = 20$

$M = \frac{\rho_s H^3}{N_s^3\left(\frac{\rho_s}{\rho_w} - 1\right)^3} = \frac{2600(1.905)^3}{20\left(\frac{2600}{1000} - 1\right)^3}$

$= 219$ kg

(c)

Rubble foundation
$b = 2$ m
$y_c = 7.2$ m
$\frac{b}{y_c} = \frac{2}{7.2} = 0.278$

Max
CEM Fig 7-97: $1 - r_f = 0.52 \rightarrow F''_{TC} = 0.52 F_{TC} = 0.52(237.6) = 123.6$ kN/m
$1 - r_m = 0.8 \rightarrow M''_{TC} = 0.8 M_{TC} = 0.8(576) = 460.8$ kNm/m

$M_{TC}|_A = 460.8$ kNm/m

$M_{TC}|_B = 460.8 - 123.6(2)$
$= 193.6$ kNm/m