NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2014-2015

CV4116 - COASTAL ENGINEERING

April / May 2015 **Time Allowed: 2½ hours**

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FOUR (4)** pages.
2. Answer **ALL** questions.
3. All questions carry equal marks.
4. All answers must be written in the answer book provided.
5. This is a Restricted Open Book Examination. Only **ONE (1) Sheet** of A4 paper with handwritten notes on both sides is allowed.
6. A separate set of tables and charts is issued together with the paper.

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1. (a) A record of sea waves was analysed using the zero down-crossing approach. The results are summarised in Table Q1:

Table Q1

	Range of Wave Height [m]	Number of Waves
0.06	0.00-0.12	21
0.18	0.12-0.24	57
0.3	0.24-0.36	72
0.42	0.36-0.48	63
0.54	0.48-0.60	42
0.66	0.60-0.72	24
0.78	0.72-0.84	12
0.9	0.84-0.96	9
1.02	0.96-1.08	0

(i) What is the total number of waves in this record? 300 waves

(ii) By approximating the different wave heights as the mid-value in the ranges, calculate H_{rms}, H_{33}, and H_{10}.

33.3333%

(12 Marks)

$$H_{rms} = \sqrt{\frac{1}{N}\sum H_i^2}$$
$$= \sqrt{\left(21(0.06)^2 + 57(0.18)^2 + 72(0.3)^2 + 63(0.42)^2 + 42(0.54)^2 + 24(0.66)^2 + 12(0.78)^2 + 9(0.9)^2 + 0(1.02)^2\right) \times \frac{1}{300}}$$
$$= \sqrt{54.378\left(\frac{1}{300}\right)}$$
$$= \sqrt{0.18126}$$
$$= 0.426 m$$

H_{33} = Avg of highest $\frac{1}{3}$ wave height
$$= \frac{0(1.02) + 9(0.9) + 12(0.78) + 24(0.66) + 42(0.54) + 13(0.42)}{100} = 0.614 m$$

H_{10} = Avg of highest 10% wave height
$$= \frac{0(1.02) + 9(0.9) + 12(0.78) + 9(0.66)}{30} = 0.78 m$$

Note: Question No. 1 continues on page 2

(b) The record of sea waves in Part (a) was later found to be the results of wind wave generation with a wind speed of 7.5 m/s over a very long fetch. What was the wave duration necessary to generate waves of such magnitude? What was the wave period? (7 Marks)

(c) Spectral analysis was also performed on the record of sea waves in Part (a). The following wave energy density function was obtained:

$$E(f)=\frac{A}{f^5}e^{-\frac{1}{f^4}}$$

Determine the coefficient A and its dimensional unit (Hint: the significant wave height should be the same for the two approaches). (6 Marks)

2\. (a) Show that the following potential function, ϕ, satisfies the governing Laplace Equation. Also show that the kinematic boundary condition at the water surface can be satisfied with this potential function given the dispersion relationship.

$$\phi=-\frac{H}{2}\frac{g\cosh k(d+y)}{\sigma\cosh kd}\sin(kx-\sigma t)$$

(8 Marks)

(b) A wave propagates perpendicularly towards a coastal area as shown in Figure Q2. The wave period is 10 seconds. Due to tidal fluctuation, **h varies between 1.5 to 2.0 m.**

(i) If the deep water wave height H_o = 2.14 m, estimate the breaking wave height and the region of wave breaking in terms of the x-coordinate during the tidal fluctuation, by assuming that $H_b/d_b = 0.78$.

(i) If however the deep water wave height varies throughout the tidal fluctuation, but breaking is consistently observed with H_b = 1.0 m on the plateau region (i.e. the positive x-coordinate) and not at the slope region, estimate the variation of deep water wave height that can lead to this occurrence.

(17 Marks)

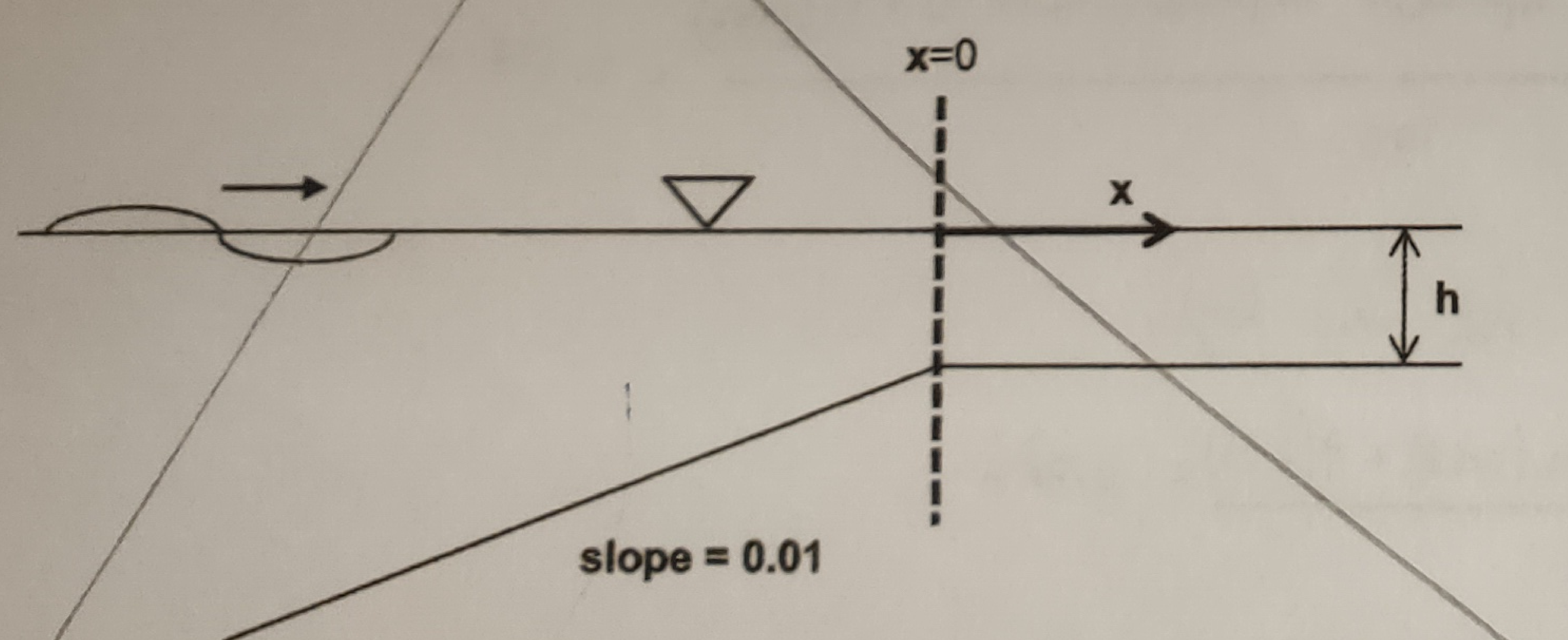

Figure Q2

1) (b) Wind speed, $U_0 = 7.5$ m/s

Wave duration = ?

T = ?

$H_{33\%} = 0.614$ m

use which H ?

Fig II-2-25 !!!

get around 8 hours

Fig II-2-26 ✓

$T \approx 3.35$

(c) $E(f)=\frac{A}{f^5}e^{-\frac{1}{f^4}}$

$E_i(\Delta f)_i = \frac{a_i^2}{2} = S_I$

$H_s = 4\sqrt{S_{m0}}$ $\quad T_s \approx \frac{1}{f_p}$

$H_s^2 = 16 S_{m0}$

$S_{m0} = \frac{16}{H_s^2}$

$S_{m0} = \int_0^\infty E(f)\,df$

$S_{m0} = \int_0^\infty \frac{A}{f^5}e^{-\frac{1}{f^4}}\,df$

$\frac{16}{H_s^2} = \frac{A}{4}\int_0^\infty \frac{4e^{-\frac{1}{f^4}}}{f^5}\,df$

$\frac{d}{df}(e^{-f^{-4}}) = e^{-f^{-4}}\cdot 4f^{-5} = \frac{4e^{-\frac{1}{f^4}}}{f^5}$

$\frac{16}{0.614^2} = \frac{A}{4}\left[e^{-\frac{1}{f^4}}\right]_0^\infty$

$\frac{64}{0.614^2} = A[1-0]$

$A = 169.763$

(b) The record of sea waves in Part (a) was later found to be the results of wind wave generation with a wind speed of 7.5 m/s over a very long fetch. What was the duration necessary to generate waves of such magnitude? What was the wave period?

(7 Marks)

(c) Spectral analysis was also performed on the record of sea waves in Part (a). The following wave energy density function was obtained:

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Determine the coefficient A and its dimensional unit (Hint: the significant wave height should be the same for the two approaches).

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$$\phi = -\frac{H}{2}\frac{g\cosh k(d+y)}{\sigma\cosh kd}\sin(kx-\sigma t)$$

(8 Marks)

(b) A wave propagates perpendicularly towards a coastal area as shown in Figure Q2. The wave period is 10 seconds. Due to tidal fluctuation, h varies between 1.5 to 2.0 m.

(i) If the deep water wave height H_o = 2.14 m, estimate the breaking wave height and the region of wave breaking in terms of the x-coordinate during the tidal fluctuation, by assuming that $H_b/d_b = 0.78$.

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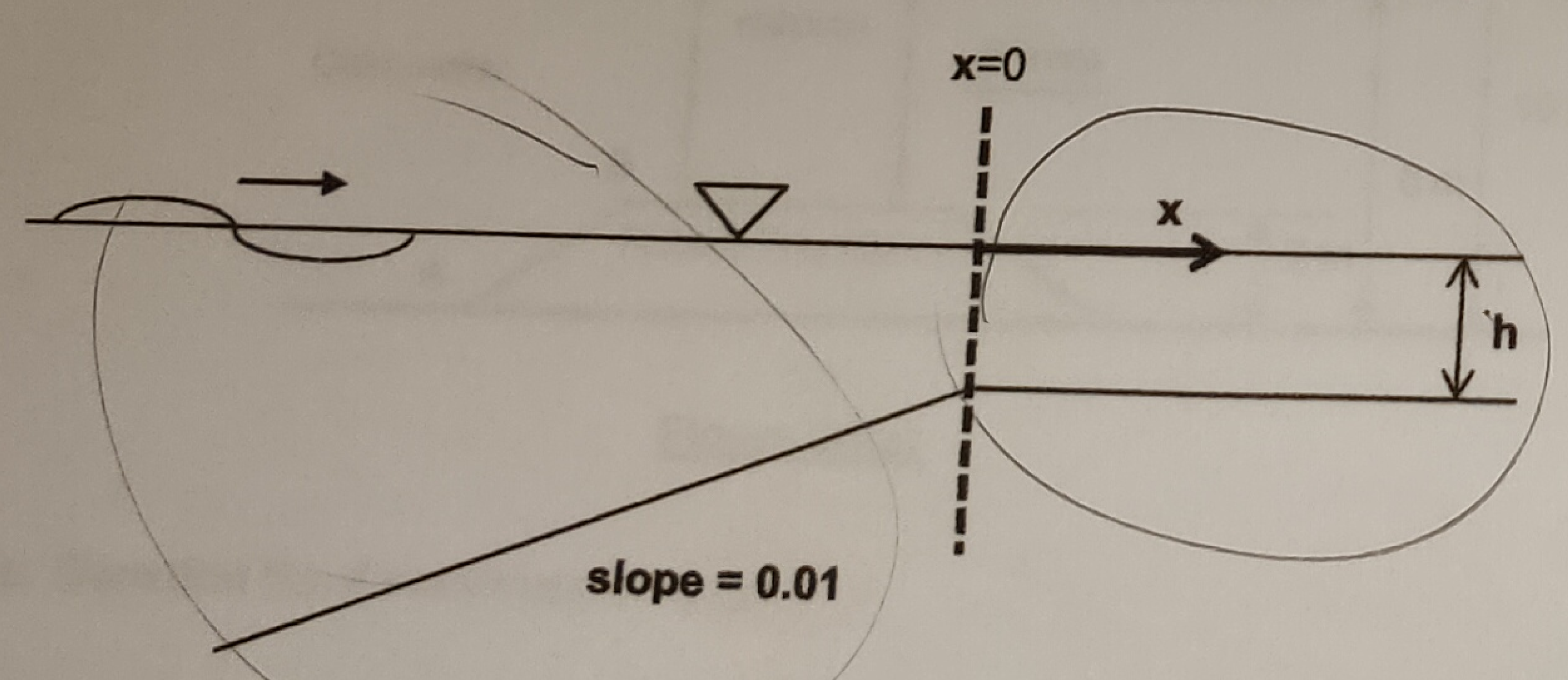

Figure Q2

$k = \frac{2\pi}{L}$

2)(a) $\phi = -\frac{H}{2}\frac{g\cosh[k(d+y)]}{\sigma\cosh(kd)}\sin(kx-\sigma t) = -\frac{Hg}{2\sigma}\frac{\cosh[k(d+y)]}{\cosh(kd)}\sin(kx-\sigma t)$

Governing eq: Laplace equation

$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$

$\phi = F(y) \times \sin(kx-\sigma t)$

Sub ⇒ $-k^2 F(y) + F(y) + \frac{d^2F}{dy^2} = 0$

$F = Ae^{ky} + Be^{-ky}$

use boundary condition to determine A & B.

Kinematic Free surface boundary condition

$V|_{y=0} = \frac{Hg}{2\sigma}k\frac{\sinh[k(d+0)]}{\cosh[kd]}\sin(kx-\sigma t)$

$= \frac{Hg}{2\sigma}k\tanh(kd)\sin(kx-\sigma t)$

$\frac{\partial\eta}{\partial t} = \frac{\partial}{\partial t}\left[\frac{H}{2}\cos(kx-\sigma t)\right]$

$= \frac{H\sigma}{2}\sin(kx-\sigma t)$

⇒ In order to satisfy the condition

$\frac{Hg}{2\sigma}k\tanh(kd) = \frac{H\sigma}{2}$

i.e. $\sigma^2 = gk\tanh[kd]$

- wave length is a function of water depth

Dispersion relationship

$\sigma^2 = gk\tanh[kd]$

$\left(\frac{2\pi}{T}\right)^2 = g\left(\frac{2\pi}{L}\right)\tanh\left(2\pi\frac{d}{L}\right)$

• Deep water: $\frac{d}{L} \geq \frac{1}{2}$

$\tanh(kd) \simeq 1$

$\frac{L_o}{T} = C_o = \frac{\sigma}{k_o} = \frac{gT}{2\pi}$

- phase speed depends only on wave period

• Intermediate water $\left(\frac{1}{2} > \frac{d}{L} > \frac{1}{20}\right)$

• Shallow water: $\frac{d}{L} \leq \frac{1}{20}$; $\tanh(kd) \simeq kd$

$\rightarrow \frac{L}{T} = C = \frac{\sigma}{k} = \sqrt{gd}$

☆ Phase speed independent only on wave

(b) T = 10 s

$H_o = 2.14$

$\theta_o = 0°$?

$H = K_s K_r H_o$

$\frac{H}{gT^2} = 0.0015$, $K_s K_r = 1.5$

$\frac{d}{gT^2} = 0.0020$, $K_s K_r = 1.4$

(ii)

3. The observed wave conditions at an offshore location comprise waves with a deepwater significant wave height H_o of 0.8 m and wave period T of 5 s. The waves approach the shore from the northwest direction while the bottom contours are parallel and slope northwards towards increasing depth.

(a) What is the significant wave height at a location closer to shore where the water depth is 7.5 m?

(4 Marks)

(b) Calculate the corresponding unrefracted deepwater waveheight. Describe physically what does this wave represent and indicate from which direction will it propagate from.

(4 Marks)

(c) A jetty supported by vertical circular piles of diameter 0.5 m is proposed for this location with water depth of 7.5 m. Determine the maximum horizontal wave force acting on each pile. Assume a C_D of 1.0 and a C_M of 1.15 for the pile, and that the seawater specific weight is 10 kN/m³.

(10 Marks)

(d) An underwater cylindrical storage tank of diameter 3 m and height 4 m is further proposed to be built at this location. The tank is to be mounted on the sea bed with its axis vertical. Using suitable expressions from linear wave theory, describe how you can determine the horizontal force induced by the surface waves on the tank. While calculations are not needed, state all significant assumptions used.

(7 Marks)

4. (a) A caisson wall founded on a rubble foundation as shown in Figure Q4(a) is proposed for a site where the significant wave height H_s is 1.5 m and the wave period T is 7 s. Using Miche-Rungren's method, determine the maximum horizontal force acting on the seaward side of the caisson wall. Assume calm water exists on the lee side and that the specific weight of seawater is 10 kN/m³.

(13 Marks)

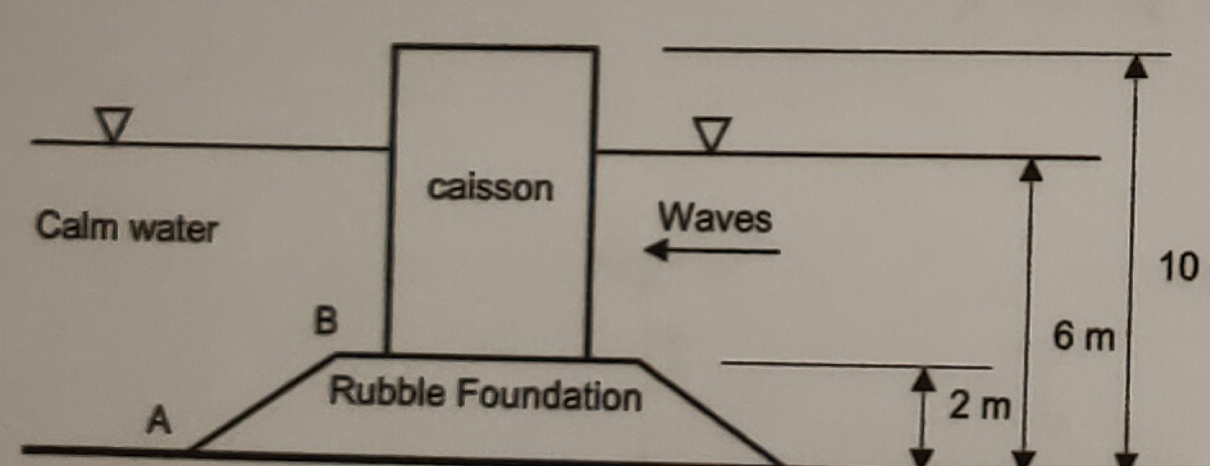

Figure Q4(a)

Note: Question No. 4 continues on page 4

(a) Given $H_o = 0.8$ m, $T = 5$ s, $d = 7.5$ m

$\frac{d}{gT^2} = \frac{7.5}{g(5)^2} = 0.031$

CEM Fig II-3-6: $K_R K_S = 1.07$, $\theta = 13°$

$H_s = (K_R K_S) H_o = 1.07(0.8) = 0.856$

$H_s/d = 0.856/7.5 = 0.114 < 0.78$ (Non breaking)

(b) $H_o' = K_R H_o$ — get from CEM Fig II-3-6
$= 0.86(0.8)$
$= 0.69$ m

(c) Jetty: D = 0.5 m, d = 7.5 m, $C_D = 1.0$, $C_M = 1.15$, $\rho g = 10000$ N/m³

Max horizontal force, $F_m = \phi \rho g C_D H^2 D$ (Use Dean stream theory) → $W = \frac{C_M D}{C_D H}$

$H_o = 0.8$, T = 5 s

$\frac{d}{gT^2} = \frac{7.5}{g(5)^2} = 0.031$, CEM Fig II-1-7: $\frac{H}{H_o} = 1.0$, $H = 1.0 H_o = 0.8$ m ?

Fig VI-5-130 Expect non-breaking: $\frac{H}{gT^2} = 0.021$, $H = 0.021(9.81)(5)^2 = 5.15$ m

use which one? ? — can't use b/c of linear theory?

OR From (a), get $H_s = 0.856$ m (Non-breaking)
Jetty → Rigid → Design wave, $H = 1.67 H_s = 1.67(0.856) = 1.43$ m ?
I use this

$= \frac{1.15(0.5)}{1.0(1.43)} = 0.4$

$\frac{H}{gT^2} = \frac{1.43}{g(5)^2} = 0.00583$, $\frac{d}{gT^2} = 0.031$

CEM Fig VI-5-132 W = 0.1 → get $\phi_m = 0.3$
CEM Fig VI-5-133 W = 0.5 → get $\phi_m = 0.36$
Interpolate: $\phi_m =$? — I can't see the graph value

$F_m = \phi \rho g C_D H^2 D = (10^4)(1.0)(1.43)^2(0.5) = $ N

(d) D = 3 m, h = 4 m
Want Linear Wave theory, Airy Theory
max inertial F: $F_{im} = C_M \rho g \frac{\pi D^2}{4} H K_{im}$
max drag force: $F_{Dm} = C_D \rho g \frac{DH^2}{2} K_{Dm}$
$\frac{d}{gT^2}$ — CEM Fig II-5-126 get K_{im}, II-5-127 get K_{Dm}

? Assumption used: Form drag ...
- L = wave length, d = depth water, H = wave height
- Simplification on hyperbolic forms if waves are in shallow or deep water

3. The observed wave conditions at an offshore location comprise waves with a deepwater significant wave height H_o of 0.8 m and wave period T of 5 s. The waves approach the shore from the northwest direction while the bottom contours are parallel and slope northwards towards increasing depth.

(a) What is the significant wave height at a location closer to shore where the water depth is 7.5 m?

(4 Marks)

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4. (a) A caisson wall founded on a rubble foundation as shown in Figure Q4(a) is proposed for a site where the significant wave height H_s is 1.5 m and the wave period T is 7 s. Using Miche-Rungren's method, determine the maximum horizontal force acting on the seaward side of the caisson wall. Assume calm water exists on the lee side and that the specific weight of seawater is 10 kN/m³.

(13 Marks)

$X = 1.0$ all

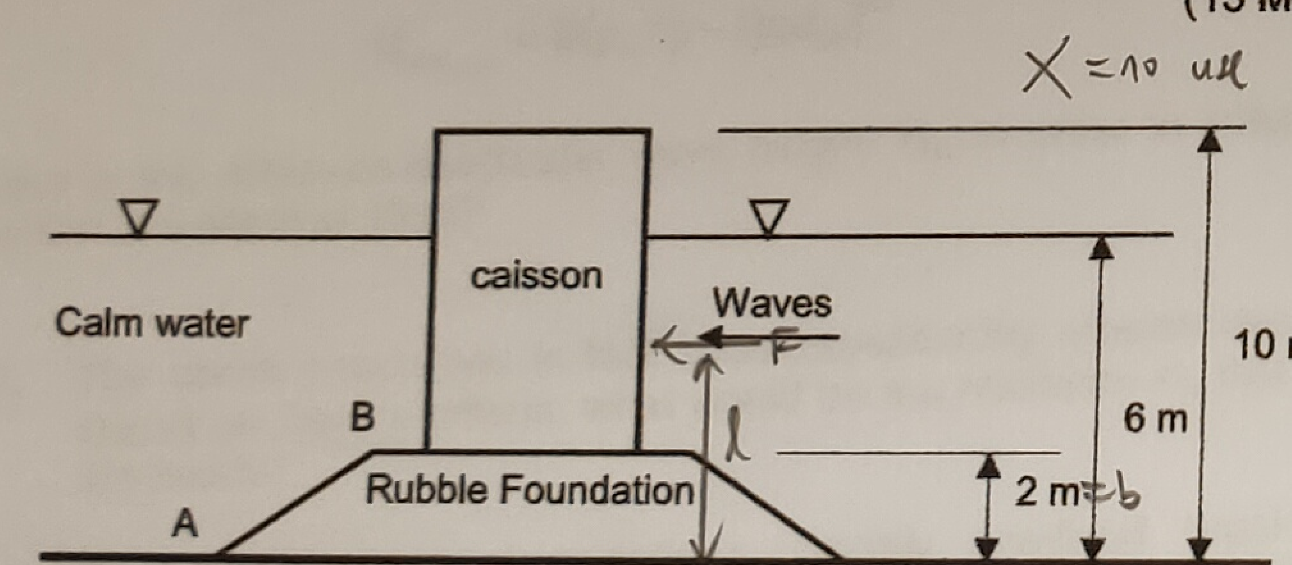

Figure Q4(a)

$M|_A = Fl$

$M|_B = F(l-b)$

$= Fl - Fb$

$= M|_A - Fb$

Note: Question No. 4 continues on page 4

4)(a) Rubble foundation,

$H_s = 1.5$ m

$d = 6$ m

$T = 7$ s

Rigid: $H_{1/100} = 1.67 H_s$

$= 1.67(1.5)$

$= 2.505$

$= 2.5$ m $= H_i$

CEM Fig 7.9, Non-breaking wave ($\chi = 1.0$)

$\frac{H_i}{gT^2} = \frac{2.5}{g(7)^2} = 0.0052$, $\frac{H_i}{d} = \frac{2.5}{6} = 0.42$

get $\frac{h_o}{H_i} = 0.48$

$h_o = 0.48(2.5) = 1.2$ m

Depth from bottom to crest, y_c

$= d + h_o + \left(\frac{1+\chi}{2}\right)H_i$

$= 6 + 1.2 + \left(\frac{1+1}{2}\right)(2.5)$

$= 9.7$ m

As $\chi = 1.0$

SPM 7.91 & SPM 7.92

$\frac{H_i}{gT^2} = 0.0052$, $\frac{H_i}{d} = 0.42$

$\frac{F_c}{\rho g d^2} = 0.48 \rightarrow F_c = 173$ kN/m

~~$\frac{F_t}{\rho g d^2} = -0.24 \rightarrow F_t = 86$ kN/m~~

$\frac{M_c}{\rho g d^3} = 0.32 \rightarrow M_c = 691$ kNm/m

~~$\frac{M_t}{\rho g d^3} = -0.11 \rightarrow M_t = 238$ kNm/m~~

~~∴ Max Force, $F = F_c - (F_t)$ $= 173 + 86$ $= 259$ kN/m~~

~~Max Moment $= M_c - (M_t)$ $= 691 + 238$ $= 929$ kNm/m~~

Include hydrostatic on one-side

$F_{Tc} = F_c + \frac{1}{2}\rho g d^2$

$= 173 + \frac{1}{2}(10000)(6)^2$

$= 353$ kN/m

$M_{Tc} = M_c + \frac{1}{6}\rho g d^3$

$= 691 + \frac{1}{6}(10000)(6)^3$

$= 1051$ kNm/m

∴ Rubble foundation

$b = 2$ m

$y_c = 9.7$ m

$\frac{b}{y_c} = \frac{2}{9.7} = 0.21$

CEM Fig 7-97: $1 - r_f = 0.62$ ⟶ Max Total force, $F''_{Tc} = 0.62 F_{Tc} = 219$ kN/m

$1 - r_m = 0.88$ ⟶ Total Moment, $M''_{Tc} = 0.88 M_{Tc} = 925$ kNm/m

Moment about top foundation at B =

$M''_{Tc}|_B = M''_{Tc}|_A - F''_{Tc}\, b$

$= 925 - 219(2)$

$= 487$ kN m/m

 (b) (i)

(b) (i) A L-shape breakwater protects a harbour from incident waves over a range of incidence angles including waves having waveheight of 1 m and incident at an angle β of 60° as shown in Figure Q4(b). Determine the length of the breakwater arm PQ so that the diffracted waveheight is 0.2 m at point A for this incident wave condition. Assume that the reflection coefficient C_R along the breakwater and along the harbour are both zero and that the wavelength is 50 m.

(ii) With the aid of sketches, briefly describe how you can estimate the waveheight at point A if the reflection coefficients along the harbour and along the breakwater arm QR are now small and non-zero.

(iii) Based on the breakwater layout shown, do you think the predominant wave direction would likely be at this angle of β? Provide your reasons.

(12 Marks)

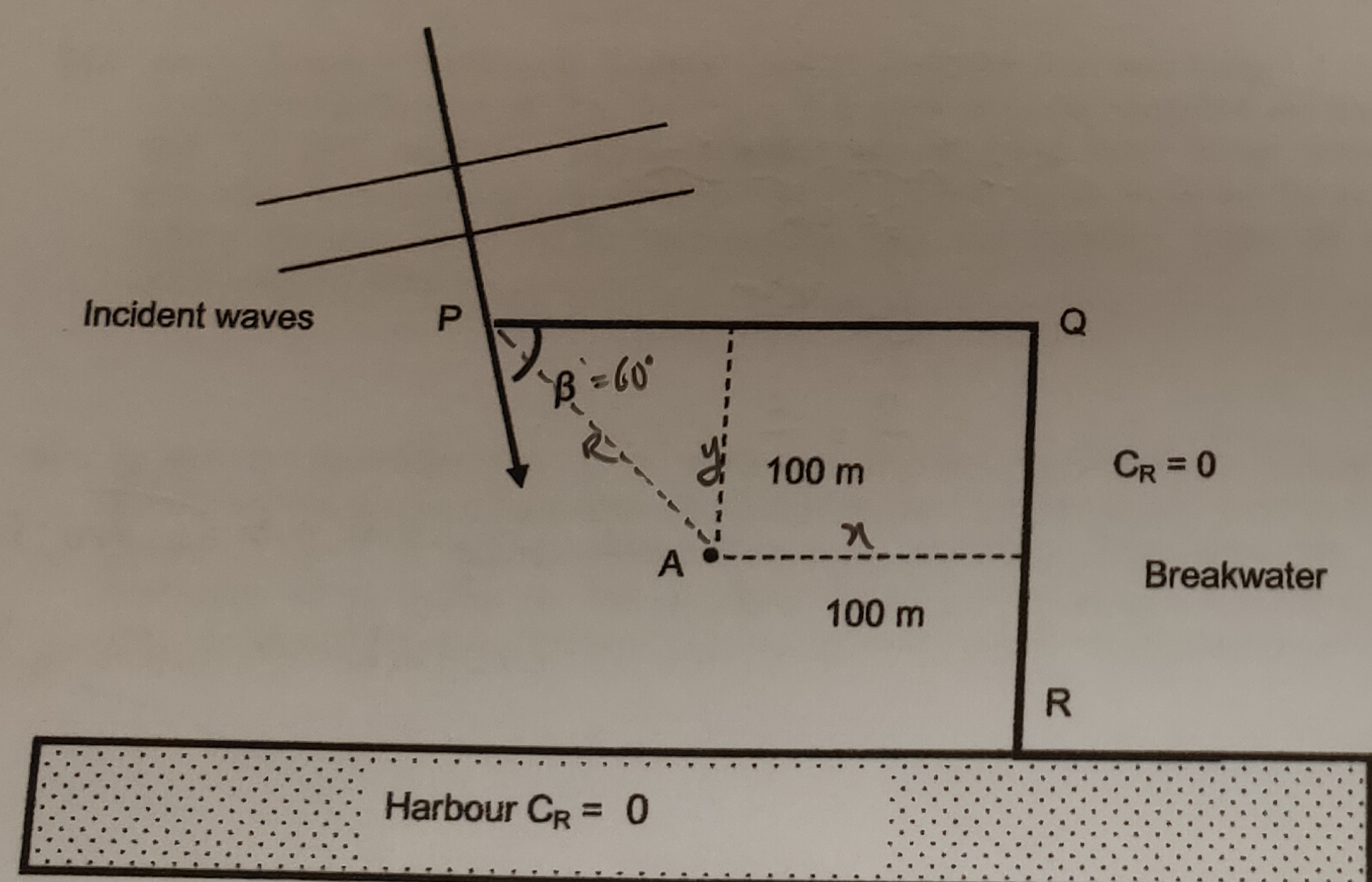

Figure Q4(b)

END OF PAPER

$H = 1m$

$\beta = 60°$

$C_R = 0$

$PQ = ?$

$H_A = 0.2m$

$H_A = K' H_i$

$0.2 = K'(1)$

$K' = 0.2$

let $L = L_0 = 50m$

$y = 100$, $\frac{y}{L} = \frac{100}{50} = 2$, $K' = 0.2$

SPM-Fig 2-33, get $\frac{x}{L} = 4.7$

$x = 4.7 \times L$

$x = 4.7(50)$

$= 235m$

$PQ = 235 + 100$

$= 335m$

(ii)