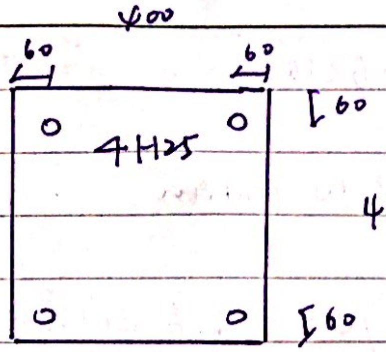
LIUC0045@e.ntu.edu.sg
Liu Chang

CV4120
S1 1819
Nov/Dec 18

1.



$f_{cd} = \frac{0.85}{1.5} \cdot 30 = 17 \; N/mm^2$

$f_{yd} = \frac{1}{1.15} 500 = 434.8 \; N/mm^2$

$d = 340 \, mm$

(a)

$M_{oe} = 0.6 M_{02} + 0.4 M_{01} = 0.6 \times 80 + 0.4 \times 20 = 56 \, KN \cdot m$

$> 0.4 M_{02} = 0.4 \times 80 = 32 \, KN \cdot m$

$M_i = N_{Ed} L_0 / 400 = 1500 \times 6.3 / 400 = 23.625 \, KN \cdot m$

$M_{0Ed} = M_{oe} + M_i = 32 + 23.625 = 55.625 \, KN \cdot m$

$\frac{1}{r_0} = \frac{f_{yd}}{E_s \cdot 0.45 d} = \frac{434.8}{200{,}000 \cdot 0.45 \cdot 340} = 1.421 \times 10^{-5}$

$n = \frac{N_{Ed}}{A_c \cdot f_{cd}} = \frac{1500 \times 10^3}{400^2 \cdot 17} = 0.5515$

$n_u = 1 + A_s f_{yd} / A_c f_{cd} = 1 + \frac{1964 \times 434.8}{400^2 \cdot 17} = 1.314$

$n_{bal} = 0.4$ for symmetrically reinforced section

$K_r = \frac{n_u - n}{n_u - n_{bal}} = \frac{1.314 - 0.5515}{1.314 - 0.4} = 0.8342 \leq 1$

$\lambda = \frac{L_0}{i} = \frac{L_0}{0.2887 h d} = \frac{6300}{0.2887 \cdot 400} = 54.555$

$\beta = 0.35 + f_{ck}/200 - \lambda/150 = 0.1363$

$K_\varphi = 1 + \beta \varphi_{ef} = 1 + 0.1363 \times 0.87 = 1.1186$

$\frac{1}{r} = K_r K_\varphi \cdot \frac{1}{r_0} = 0.8342 \times 1.1186 \times 1.421 \times 10^{-5} = 1.326 \times 10^{-5}$

$e_2 = 0.1 \frac{1}{r} \cdot l_0^2 = 0.1 \times 1.326 \times 10^{-5} \times 6300^2 = 52.63$ mm

$M_2 = N e_2 = 1500 \times 52.63 = 78.94$ KN·m

$M_{0Ed} + M_2 = 55.625 + 78.94 = 134.6$ KN·m

$M_{02} = 80$ KN·m

$M_{01} = 20$ KN·m.

$\Rightarrow M_{Ed} = 134.6$ KN·m.

(b) $m = \frac{E_s}{E_{cm}} = \frac{200}{34} = 5.88$

$\rho = \frac{A_s}{bd} = \frac{2513}{300 \times 390} = 2.15\%$, $\rho' = \frac{A_s'}{bd} = \frac{982}{300 \times 390} = 0.839\%$

$x_u = \frac{0.5h^2/d + (m-1)(\rho d + \rho' d')}{h/d + (m-1)(\rho + \rho')} = 292$ mm

$I_u = \frac{bh^3}{12} + bh(0.5h - x_u)^2 + (m-1)A_s \cdot (d - x_u)^2 + (m-1)A_s'(d' - x_u)^2$

$= 3.24 \times 10^9$ mm^4

$x_c/d = -m\rho - (m-1)\rho' + \sqrt{[m\rho + (m-1)\rho']^2 + 2[m\rho + (m-1)\rho' d'/d]}$

$= 0.374$

$x_c = 146$ mm.

$I_c = \frac{b x_c^3}{3} + m A_s (d - x_c)^2 + (m-1) A_s' (d' - x_c)^2$

$= 1.23 \times 10^9$ mm^4

$\frac{1}{r_u} = \frac{M_{Ed}}{E_{cm} I_u} = \frac{150 \times 10^6}{34{,}000 \times 3.24 \times 10^9} = 1.362 \times 10^{-6}$

$\frac{1}{r_c} = \frac{M_{Ed}}{E_{cm} I_c} = \frac{150 \times 10^6}{34{,}000 \times 1.23 \times 10^9} = 3.5868 \times 10^{-6}$

$M_r = \frac{I_u}{h - x_u} \cdot f_{ctm} = \frac{3.24 \times 10^9}{450 - 292} \cdot 3.2 = 65.62$ KN·m.

$\zeta = 1 - \beta \frac{M_r}{M_{Ed}} = 1 - 1 \times \left(\frac{65.62}{150}\right)^2 = 0.8086$

$\frac{1}{r_b} = \zeta \frac{1}{r_c} + (1 - \zeta) \frac{1}{r_u} = 3.161 \times 10^{-6}$

$\delta_b = \frac{5}{48} \frac{1}{r_b} \cdot L^2 = 26.67$ mm

2(a)

(i) Shear wall system : shear walls only , Stiff at base & decreased stiffness with height

Frame-shear wall system : rigid frame + shear walls
↳ rigid frame is stiff at upper levels ~~to~~
↳ combine good characteristics from both rigid frame and shear walls , suitable for higher buildings.

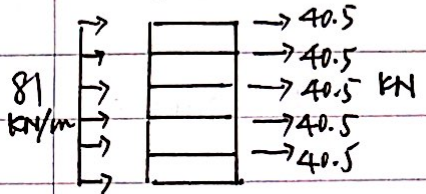
(ii) ∘ reduce shear lag effect in "flanges" ⇒ increase the effectiveness of the section
∘ ~~redu~~ increase column spacing ⇒ architecturally pleasing

(b) Notional inclination $\theta = \frac{1}{410}$

$W_K = 1.2 \times 45 = 54$ KN/m
$G_{K,h} = 9 \times 25 \times 45 \times 1/410 = 24.7$ KN
$Q_{K,h} = 2.5 \times 25 \times 45 \times 1/410 = 6.86$ KN

Loading case 1 :

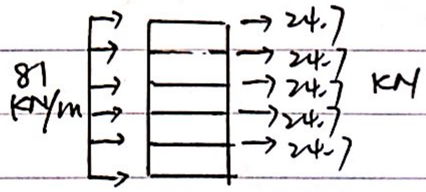
$1.5 W_K = 1.5 \times 54 = 81$ KN/m
$1.35 G_{K,h} + 1.5 \times 0.7 Q_{K,h} = 40.5$ KN



$M_{Ed} = [40.5 \times (3+6+9+12+15) + 81 \times 15 \times \frac{15}{2}] \times \frac{1}{2}$
$= 5467.5$ KN·m

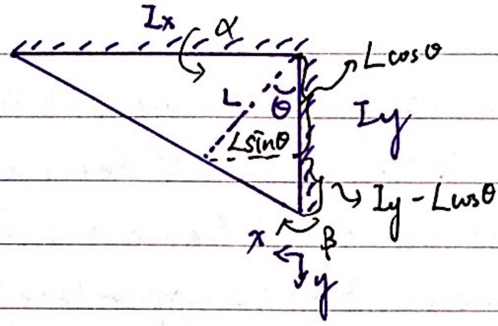
Loading case 2

$1.5 W_K = 81$ KN/m
$1.0 G_{K,h} = 24.7$ KN



$M_{Ed} = [24.7 \times (3+6+9+12+15) + 81 \times 15 \times \frac{15}{2}] \times \frac{1}{2}$
$= 5112$ KN·m.

3(a)



(i) Assume rotation along x axis is α, along y axis is β

$$\Delta = \alpha \cdot L\cos\theta = \beta \cdot L\sin\theta$$

$$\Rightarrow \beta = \frac{\alpha\cos\theta}{\sin\theta}$$

$$W_e = \left(\frac{1}{2}\cdot I_x \cdot I_y\right)\cdot\Delta\cdot\frac{1}{3}W_u = \frac{1}{2}I_x I_y\cdot\alpha L\cos\theta\cdot\frac{1}{3}W_u = \frac{\alpha}{6}I_x I_y\cos\theta\cdot W_u$$

$$W_i = L\sin\theta\cdot\alpha\cdot m + L\cos\theta\cdot\beta\cdot m$$

$$= L\sin\theta\cdot\alpha\cdot m + L\cos\theta\cdot\frac{\alpha\cos\theta}{\sin\theta}\cdot m$$

$$= \alpha m L\left(\sin\theta + \frac{\cos^2\theta}{\sin\theta}\right)$$

Let $W_e = W_i \Rightarrow \frac{1}{6}I_x I_y\cos\theta W_u = m\left(\sin\theta + \frac{\cos^2\theta}{\sin\theta}\right)$

$$\Rightarrow W_u = \frac{6m}{I_x I_y}(\tan\theta + \cot\theta)$$

$$\frac{dW_u}{d\theta} = \frac{6m}{I_x I_y}\left[\frac{d\tan\theta}{d\theta} + \frac{d\cot\theta}{d\theta}\right]$$

$$= \frac{6m}{I_x I_y}(\sec^2\theta - \csc^2\theta) = 0$$

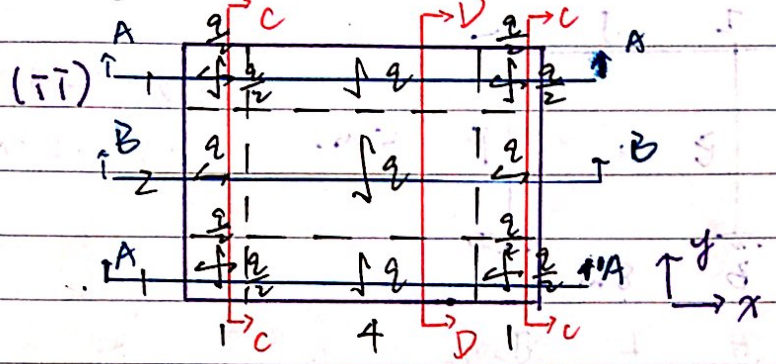
$$\Rightarrow \theta = 45^\circ$$

(ii) $\Rightarrow W_{u,min} = \frac{12m}{I_x I_y}$.

3(b)

(i) $q = 1.35 \times 25 \times 0.15 + 1.5 \times 5 = 12.5625\ KN/m^2$

(ii)



x direction: middle strip : B-B

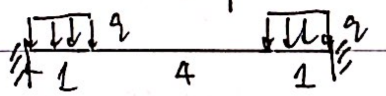
edge strip : A-A

y direction: middle strip: D-D

edge strip : C-C

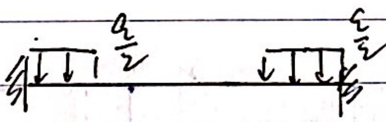
(iii) $\frac{m_{xs}}{m_{xf}} = 2$ in x-direction

Middle strip B-B:



$$\begin{cases} m_x = \frac{q \cdot 1^2}{2} = \frac{1}{2} q \\ m_{xs} = \frac{2}{3} \cdot m_x = \frac{1}{3} q = 4.2\ KN \cdot m/m \\ m_{xf} = \frac{1}{3} m_x = \frac{1}{6} q = 2.1\ KN \cdot m/m \end{cases}$$

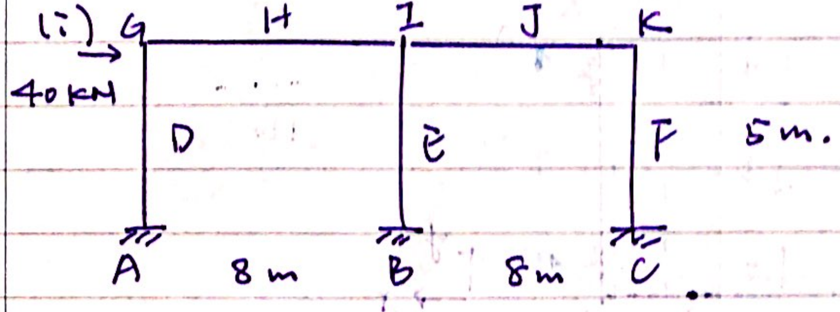
Edge strip D-D:



$$\begin{cases} m_{xs} = 2.1\ KN \cdot m/m \\ m_{xf} = 1.05\ KN \cdot m/m \end{cases}$$

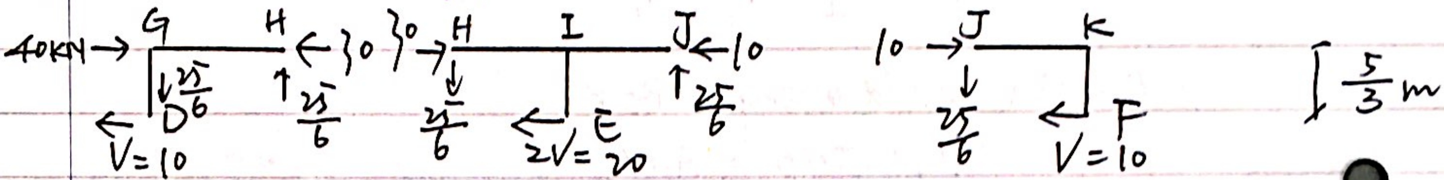
4(a) Ground columns fully fixed $\Rightarrow \frac{2}{3}$ full height · location of contra-flexure point.

(i)



$40 = V + 2V + V$

$\Rightarrow V = 10\text{ kN}$



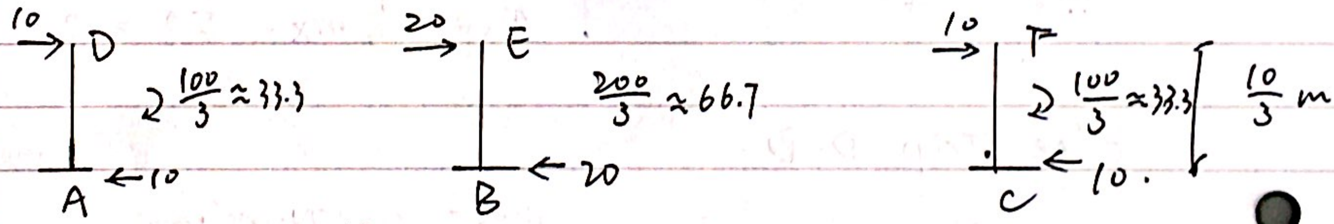
$\circlearrowleft \Sigma M_D = -40 \times \frac{5}{3} + 30 \times \frac{5}{3} + F_H \times 4 = 0$

$\Rightarrow F_H = \frac{25}{6} \approx 4.17\text{ kN}$

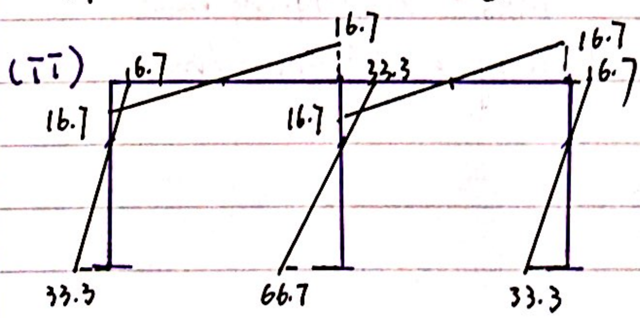
$\circlearrowleft \Sigma M_I = \frac{25}{6} \times 4 - 20 \times \frac{5}{3} + F_J \times 4 = 0$

$\Rightarrow F_J = \frac{25}{6} \approx 4.17\text{ kN}$

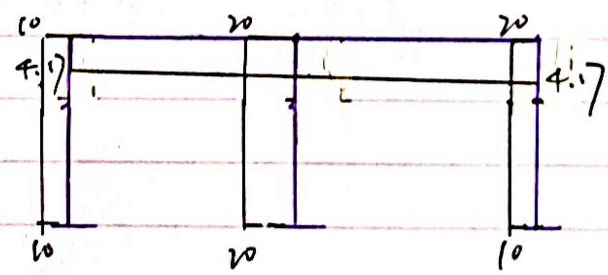
$\circlearrowleft \Sigma M_F = \frac{25}{6} \times 4 - 10 \times \frac{5}{3} = 0 \Rightarrow$ Calculation correct.



(ii)

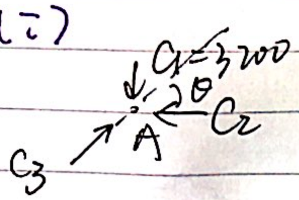


Bending Moment (kN.m)



Shear Force (kN)

4(b)(i)

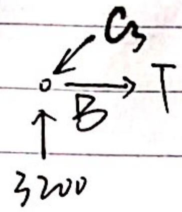


$\tan\theta = \frac{950}{900} = \frac{19}{18}$

$\uparrow \Sigma F_y = C_3 \cdot \sin\theta - C_1 = 0 \Rightarrow C_3 = 4408\ kN\ (C)$

$\rightarrow \Sigma F_x = C_3 \cos\theta - C_2 = 0 \Rightarrow C_2 = 3032\ kN\ (C)$

(ii)



$\rightarrow \Sigma F_x = T - C_3 \cdot \cos\theta = 0 \Rightarrow T = 3032\ kN\ (T)$

$A_s = \frac{T}{f_{yd}} = \frac{3032 \times 10^3}{\frac{500}{1.15}} = 6973\ mm^2$

(iii)

A: C-C-C : $\sigma_{Rd,max} = k_1 \cdot (1 - f_{ck}/250) f_{cd}$

$= 1 \times (1 - 40/250) \cdot 0.85 \cdot \frac{40}{1.5}$

$= 19.04\ MPa$

- $\sigma_{c1} = \frac{C_1}{500^2} = 12.8\ MPa$ ✓ < 19.04
- $\sigma_{c2} = \frac{C_2}{600 \cdot 2 \cdot 500} = 5.05\ MPa$ ✓ < 19.04
- * $\sigma_{c3} = \frac{C_3}{W_{c3} \cdot 500} = 12.5\ MPa$ ✓ < 19.04

B: C-C-T : $\sigma_{Rd,max} = k_2 (1 - f_{ck}/250) f_{cd}$

$= 0.85 (1 - 40/250)\ 0.85 \cdot \frac{40}{1.5}$

$= 16.2\ MPa.$

- $\sigma_B = \frac{3200}{500^2} = 12.8\ MPa$ ✓ < 16.2 ✓
- $\sigma_{c3} = 12.5\ MPa < 16.2$. ✓

* $W_{c3} = (250 \times 2) \times \cos\theta + 500 \times \sin\theta = 707\ mm$