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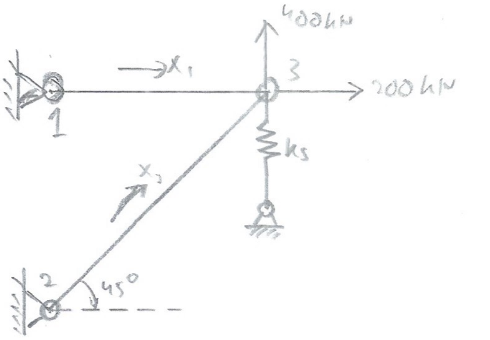
CV4101 PYP AY 2019/2020 Semester 1 Solution

1.



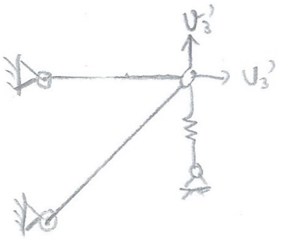
$E = 200\ kN/mm^2$

$L_1 = 3\ m$, $A_1 = 3000\ mm^2$

$L_2 = 3\sqrt{2}\ m$, $A_2 = 3000\sqrt{2}\ mm^2$

$k_s = 400\,000\ kN/m$

(a) DOF = U_3', V_3' //



(b) ·) Member 1 ($A_1E = 6\times10^5\ kN$, $L_1 = 3m$, $c_1 = 1$, $s_1 = 0$)

$$K_1' = \begin{array}{c} \left[\begin{array}{cc|cc} 2\times10^5 & 0 & -2\times10^5 & 0 \\ 0 & 0 & 0 & 0 \\ \hline -2\times10^5 & 0 & 2\times10^5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{l} \leftarrow U_1' \\ \leftarrow V_1' \\ \leftarrow U_3' \\ \leftarrow V_3' \end{array}$$

·) Member 2 ($A_2E = 6\sqrt{2}\times10^5\ kN$, $L_2 = 3\sqrt{2}\ m$, $c_1 = \frac{1}{\sqrt{2}}$, $s_1 = \frac{1}{\sqrt{2}}$)

$$K_2' = \left[\begin{array}{cc|cc} 1\times10^5 & 1\times10^5 & -1\times10^5 & -1\times10^5 \\ 1\times10^5 & 1\times10^5 & -1\times10^5 & -1\times10^5 \\ \hline -1\times10^5 & -1\times10^5 & 1\times10^5 & 1\times10^5 \\ -1\times10^5 & -1\times10^5 & 1\times10^5 & 1\times10^5 \end{array}\right] \begin{array}{l} \leftarrow U_2' \\ \leftarrow V_2' \\ \leftarrow U_3' \\ \leftarrow V_3' \end{array}$$

Assembly & Applying Boundary Conditions:

$$\left[\begin{array}{c} R_{1x'} \\ R_{1y'} \\ \hline R_{2x'} \\ R_{2y'} \\ \hline 200 \\ 400 - V_3' k_s \end{array}\right] = \left[\begin{array}{cc|cc|cc} 2\times10^5 & 0 & 0 & 0 & -2\times10^5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1\times10^5 & 1\times10^5 & -1\times10^5 & -1\times10^5 \\ 0 & 0 & 1\times10^5 & 1\times10^5 & -1\times10^5 & -1\times10^5 \\ \hline -2\times10^5 & 0 & -1\times10^5 & -1\times10^5 & 3\times10^5 & 1\times10^5 \\ 0 & 0 & -1\times10^5 & -1\times10^5 & 1\times10^5 & 1\times10^5 \end{array}\right] \left[\begin{array}{c} 0 \\ 0 \\ \hline 0 \\ 0 \\ \hline U_3' \\ V_3' \end{array}\right]$$

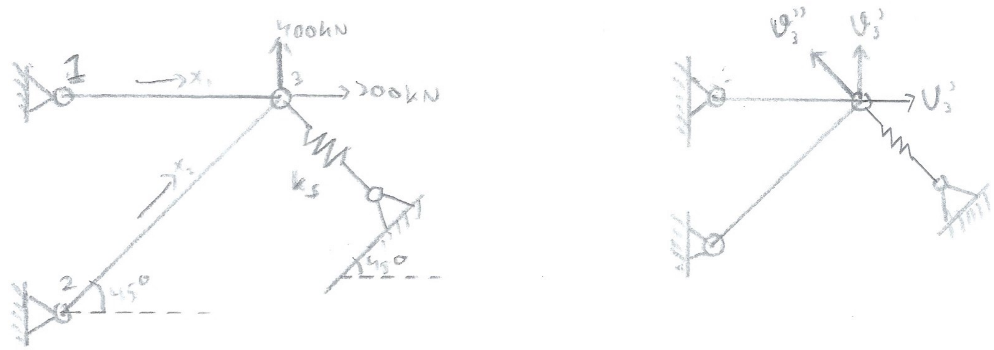
From Row 5 and Row 6, we have

$$\begin{bmatrix} 200 \\ 400 - 4\times10^5 v_3' \end{bmatrix} = \begin{bmatrix} 3\times10^5 & 1\times10^5 \\ 1\times10^5 & 1\times10^5 \end{bmatrix} \begin{bmatrix} u_3' \\ v_3' \end{bmatrix}$$

$$\Leftrightarrow \begin{cases} 200 = 3\times10^5 u_3' + 1\times10^5 v_3' \\ 400 = 1\times10^5 u_3' + 5\times10^5 v_3' \end{cases}$$

By calculator $\Leftrightarrow$ $u_3' = \frac{3}{7000}$ m $= 0.0004286$ m, $v_3' = \frac{1}{1400}$ m $= 0.0007143$ m

(c)



Let v_3'' be the displacement of joint 3 along the axial direction of the spring. Then, $v_3'' = -\frac{1}{\sqrt{2}} u_3' + \frac{1}{\sqrt{2}} v_3'$.

Thus, (5) the spring force is $F_s = -v_3'' k_s = \left(\frac{1}{\sqrt{2}} u_3' - \frac{1}{\sqrt{2}} v_3'\right)(4\times10^5\ \text{kN/m})$.

Hence, the assembled matrix and boundary condition becomes.

$$\begin{bmatrix} R_{1x'} \\ R_{1y'} \\ R_{2x'} \\ R_{2y'} \\ 200 - \frac{1}{\sqrt{2}} F_s \\ 400 + \frac{1}{\sqrt{2}} F_s \end{bmatrix} = \begin{bmatrix} 2\times10^5 & 0 & 0 & 0 & -2\times10^5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1\times10^5 & 1\times10^5 & -1\times10^5 & -1\times10^5 \\ 0 & 0 & 1\times10^5 & 1\times10^5 & -1\times10^5 & -1\times10^5 \\ -2\times10^5 & 0 & -1\times10^5 & -1\times10^5 & 3\times10^5 & 1\times10^5 \\ 0 & 0 & -1\times10^5 & -1\times10^5 & 1\times10^5 & 1\times10^5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ u_3' \\ v_3' \end{bmatrix}$$

From Row 5 and 6 we get

$$\begin{bmatrix} 200 - \left(\frac{1}{2} u_3' - \frac{1}{\sqrt{2}} v_3'\right)(4\times10^5) \\ 400 + \left(\frac{1}{2} u_3' - \frac{1}{2} v_3'\right)(4\times10^5) \end{bmatrix} = \begin{bmatrix} 3\times10^5 & 1\times10^5 \\ 1\times10^5 & 1\times10^5 \end{bmatrix} \begin{bmatrix} u_3' \\ v_3' \end{bmatrix}$$

$$\Leftrightarrow \begin{cases} 200 = 5\times10^5 u_3' - 1\times10^5 v_3' \\ 400 = -1\times10^5 u_3' + 3\times10^5 v_3' \end{cases}$$

By calculation $\Leftrightarrow$ $u_3' = \frac{1}{1400}$ m $= 0.0007143$ m

$v_3' = \frac{11}{7000}$ m $= 0.0015714$ m

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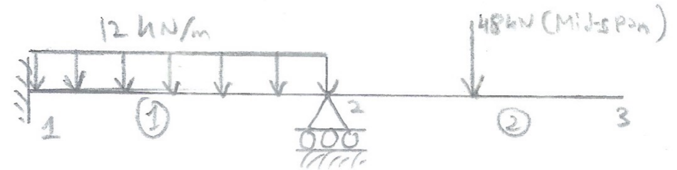
2.



$E = 200\ kN/mm^2$

$L_1 = L_2 = 2m$

$I_1 = I_2 = 8 \times 10^6\ mm^4$

(a) Using modified stiffness matrix whenever possible,

DOF = θ_2', v_3'

(b) ·) Member 1 ($EI_1 = 1600\ kN\cdot m^2$, $L_1 = 2m$)

$$k_1' = \begin{array}{c} \left[\begin{array}{cc|cc} 2400 & 2400 & -2400 & 2400 \\ 2400 & 3200 & -2400 & 1600 \\ \hline -2400 & -2400 & 2400 & -2400 \\ 2400 & 1600 & -2400 & 3200 \end{array}\right] \end{array} \begin{array}{l} \leftarrow v_1' \\ \leftarrow \theta_1' \\ \leftarrow v_2' \\ \leftarrow \theta_2' \end{array}$$

·) Member 2 ($EI_2 = 1600\ kN\cdot m^2$, $L_2 = 2m$)
(Modified Stiffness Matrix)

$$k_2' = \left[\begin{array}{cc|cc} 600 & 1200 & -600 & 0 \\ 1200 & 2400 & -1200 & 0 \\ \hline -600 & -1200 & 600 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \leftarrow v_2' \\ \leftarrow \theta_2' \\ \leftarrow v_3' \\ \leftarrow \theta_3' \end{array}$$

·) Member End Forces

*) Member 1 ($W = 12\ kN/m$, $L = 2m$)

$$F_{o1}' = \left[\begin{array}{c} 12 \\ 4 \\ \hline 12 \\ -4 \end{array}\right] \begin{array}{l} \leftarrow \\ \leftarrow \\ \leftarrow \\ \leftarrow \end{array}$$

*) Member 2 ($F = 48\ kN$, $L = 2m$) ($M_{k2} = 0$) Where k22

$$F_{o2}' = \left[\begin{array}{c} 33 \\ 18 \\ \hline 15 \\ 0 \end{array}\right]$$

Assembly & Applying Boundary Conditions:

$$\left[\begin{array}{c} R_{1y'} \\ M_{1\theta'} \\ \hline R_{2y'} \\ 0 \\ \hline 0 \\ 0 \end{array}\right] = \left[\begin{array}{cc|cc|cc} 2400 & 2400 & -2400 & 2400 & 0 & 0 \\ 2400 & 3200 & -2400 & 1600 & 0 & 0 \\ \hline -2400 & -2400 & 3000 & -1200 & -600 & 0 \\ 2400 & 1600 & -1200 & 5600 & -1200 & 0 \\ \hline 0 & 0 & -600 & -1200 & 600 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \left[\begin{array}{c} 0 \\ 0 \\ \hline 0 \\ \theta_2' \\ \hline v_3' \\ \theta_3' \end{array}\right] + \left[\begin{array}{c} 12 \\ 4 \\ \hline 45 \\ 14 \\ \hline 15 \\ 0 \end{array}\right]$$

From Row 4 and 5, we have

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5600 & -1200 \\ -1200 & 600 \end{bmatrix} \begin{bmatrix} \theta_2' \\ v_3' \end{bmatrix} + \begin{bmatrix} 14 \\ 15 \end{bmatrix}$$

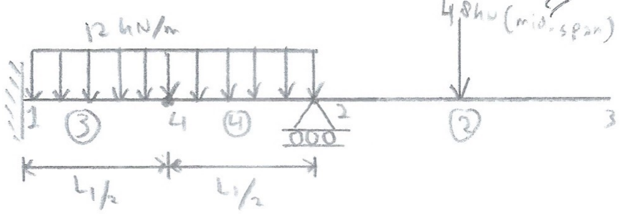
$$\Leftrightarrow \begin{cases} -14 = 5600\,\theta_2' - 1200\,v_3' \\ -15 = -1200\,\theta_2' + 600\,v_3' \end{cases}$$

By calculator

$$\Leftrightarrow \theta_2' = -\frac{11}{800} \text{ rad} = -0.01375 \text{ rad}$$

$$v_3' = -\frac{21}{400} \text{ m} = -0.0525 \text{ m}$$

(c)



$$L_3 = L_4 = \frac{L_1}{2} = \frac{2}{2} m = 1 m$$

$$I_3 = I_4 = I_1 = 8 \times 10^6 \text{ mm}^4$$

Let joint 4 be the midpoint of member 1. Also, let member 3 and 4 be the member 1-4 and member 4-2 respectively.

∴) Member 3 ($EI_3 = 1600$ kN·m, $L_3 = 1m$) & Member 4 ($EI_4 = 1600$ kN·m, $L_4 = 1m$)

$$k_3' = k_4' = \begin{bmatrix} 19200 & 9600 & -19200 & 9600 \\ 9600 & 6400 & -9600 & 3200 \\ -19200 & -9600 & 19200 & -9600 \\ 9600 & 3200 & -9600 & 6400 \end{bmatrix} \begin{array}{c|c} \leftarrow v_1' & v_4' \\ \leftarrow \theta_1' & \theta_4' \\ \leftarrow v_4' & v_2' \\ \leftarrow \theta_4' & \theta_2' \\ \text{Member 3} & \text{Member 4} \end{array}$$

∴) Member End Forces for Member 3 and 4.

*) Member 3 and 4 (W = 12 kN/m, L = 1m)

$$F_{03}' = F_{04}' = \begin{bmatrix} 6 \\ 1 \\ 6 \\ -1 \end{bmatrix}$$

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Assembly & Applying Boundary Conditions:

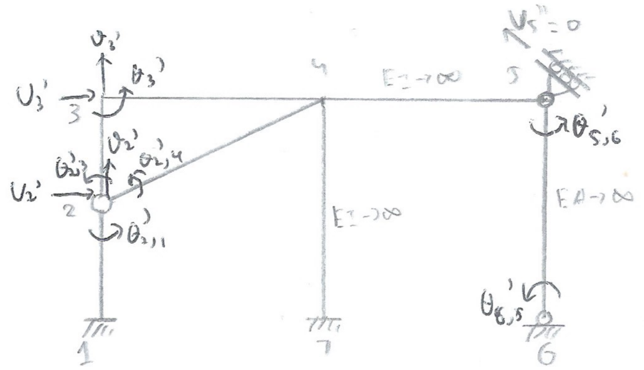
$$
\begin{bmatrix} R_{1y'} \\ M_{1(\theta')} \\ 0 \\ 0 \\ R_{2y'} \\ 0 \\ 0 \\ 0 \end{bmatrix}
=
\left[\begin{array}{cc|cc|cc|cc}
19200 & 9600 & -19200 & 9600 & 0 & 0 & 0 & 0 \\
9600 & 6400 & -9600 & 3200 & 0 & 0 & 0 & 0 \\
\hline
-19200 & -9600 & 38400 & 0 & -19200 & 9600 & 0 & 0 \\
9600 & 3200 & 0 & 12800 & -9600 & 3200 & 0 & 0 \\
\hline
0 & 0 & -19200 & -9600 & 19800 & -8400 & -600 & 0 \\
0 & 0 & 9600 & 3200 & -8400 & 5800 & -1200 & 0 \\
\hline
0 & 0 & 0 & 0 & -600 & -1200 & 600 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\begin{bmatrix} 0 \\ 0 \\ v_4' \\ \theta_4' \\ 0 \\ \theta_2' \\ v_3' \\ \theta_3' \end{bmatrix}
+
\begin{bmatrix} 6 \\ 1 \\ 12 \\ 0 \\ 39 \\ 17 \\ 15 \\ 0 \end{bmatrix}
$$

From Row 3, we have

$$0 = 0 + 0 + 38400\, v_4' + 0 + 0 + 9600\, \theta_2' + 0 + 0 + 12$$

$$\Leftrightarrow \quad v_4' = \frac{1}{38400}\left(-9600\left(-\frac{11}{800}\right) - 12\right) = \frac{1}{320}\ \text{m} = 0.003125\ \text{m}$$

3-(a)



$$\text{DOF} = U_2',\ v_2',\ \theta_{2,1}',\ \theta_{2,3}',\ \theta_{2,4}',\ U_3',\ v_3',\ \theta_3',\ \theta_{5,6}',\ \theta_{6,5}'$$

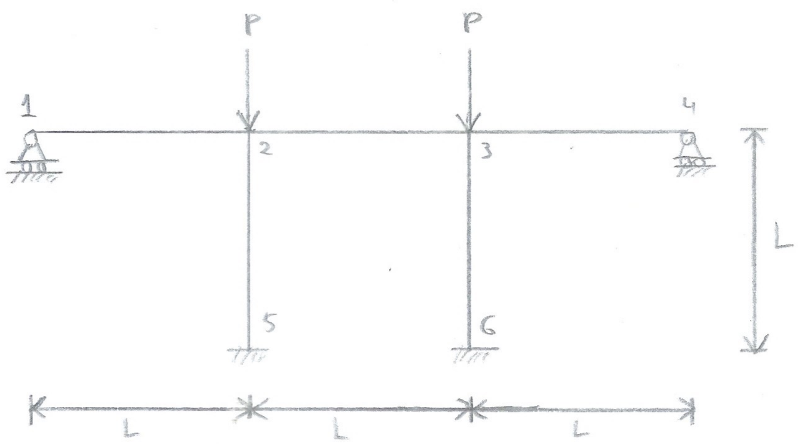
Reason:

-) Since member 4-7 has $EI \to \infty$, then $\theta_4' = 0$ and $U_4' = 0$.

-) Since member 5-6 has $EA \to \infty$, then $v_5' = 0$. But due to joint 5 is restrained by an inclined roller, then $U_5'' = 0$.

-) Since member 4-5 has $EI \to \infty$, then $\theta_4' = \theta_{5,4}' = 0$ and $v_4' = v_5'$.
(Here $v_5' = 0$, then $v_4' = 0$)

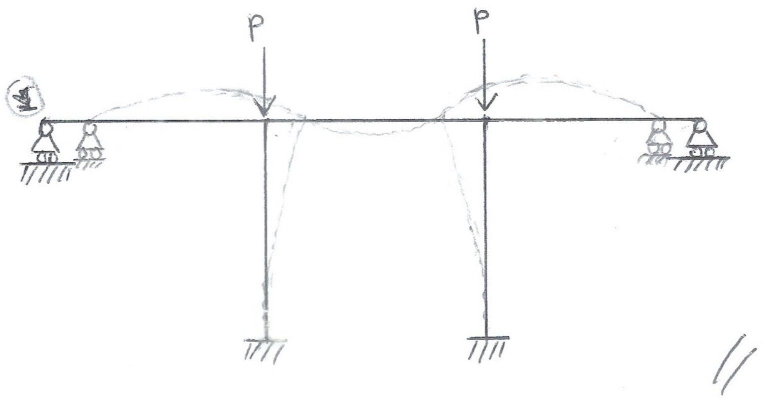
(b)



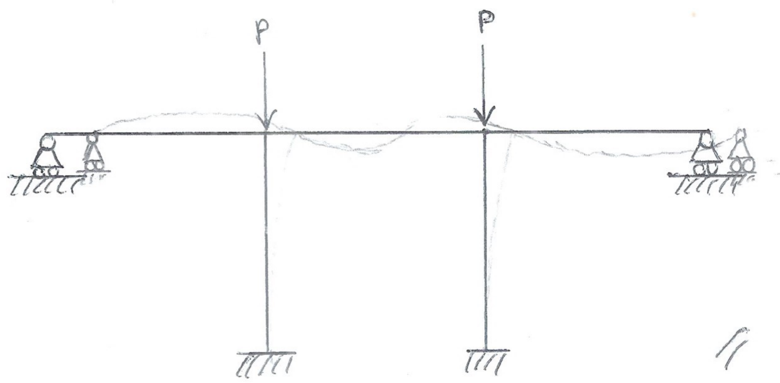
$EI \rightarrow$ constant for all members

Ignore axial deformation in all members.

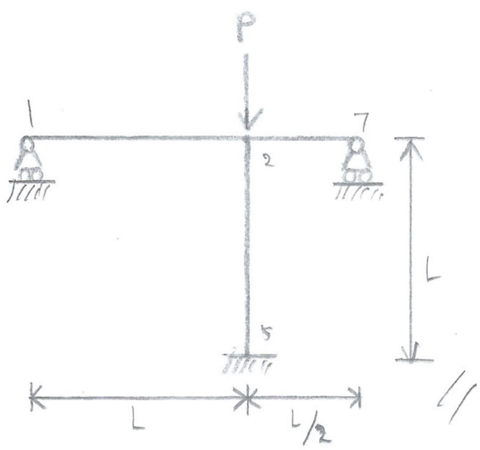
(i) •) Symmetric buckling shape



•) Anti-symmetric buckling shape



(ii)



Using modified stiffness when possible,

$DOF = \theta_2^1, U_2^2$

Let $k = \frac{EI}{L}$

Stiffness:

$\rightarrow$ Member 1-2 $= \frac{EI}{L} = k$

$\rightarrow$ Member 2-7 $= \frac{EI}{L/2} = 2k$

$\rightarrow$ Member 2-5 $= \frac{EI}{L} = k$

$$\rho = \frac{P}{P_E} = \frac{P}{\frac{\pi^2 EI}{L}}$$

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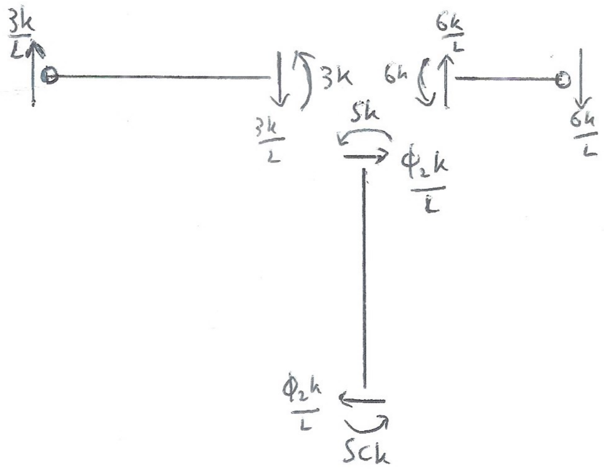
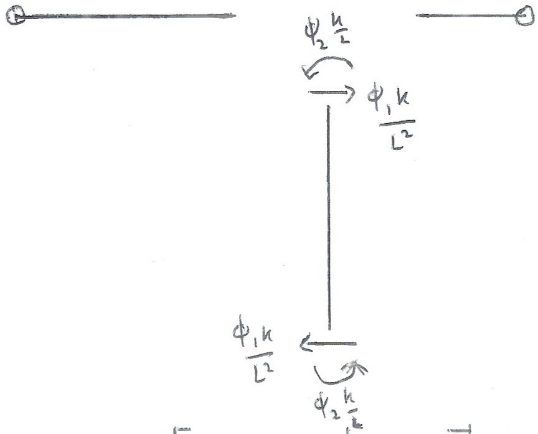
If $\theta_2' = 1$, $U_2' = 0$,



$$\Rightarrow \begin{bmatrix} (S+3+6)\,k \\ \dfrac{\phi_2 k}{L} \end{bmatrix} \begin{array}{l} \leftarrow \theta_2' \\ \leftarrow U_2' \end{array}$$

If $U_2' = 1$, $\theta_2' = 0$,



$$\Rightarrow \begin{bmatrix} \dfrac{\phi_2 k}{L} \\ \dfrac{\phi_1 k}{L^2} \end{bmatrix} \begin{array}{l} \leftarrow \theta_2' \\ \leftarrow U_2' \end{array}$$

Then, $K_{ff}' = \begin{bmatrix} (S+9)\,k & \dfrac{\phi_2 k}{L} \\ \dfrac{\phi_2 k}{L} & \dfrac{\phi_1 k}{L^2} \end{bmatrix}$

Non-trivial solution! $\left(|K_{ff}'| = 0 \right)$

$$|K_{ff}'| = \frac{k^2}{L^2}\left((S+9)\,\phi_1 - \phi_2^2 \right) = 0$$

where $\phi_1 = 2S(1+C) - \pi^2 \rho$

$\phi_2 = S(1+C)$

Let $f = (S+9)\,\phi_1 - \phi_2^2$. We will find ρ that makes $f = 0$ by trial & error.

Note: $0.5 < \rho < 1.0$

0.5 ↓ fix-free end; 1.0 ↓ fix-guided end

ρ	S	C	ϕ_1	ϕ_2	f
0.70	2.9809	0.7687	3.6359	5.2723	15.764
0.80	2.8159	0.8330	2.4274	5.1615	2.040
0.82	2.7822	0.8472	2.1855	5.1393	−0.662

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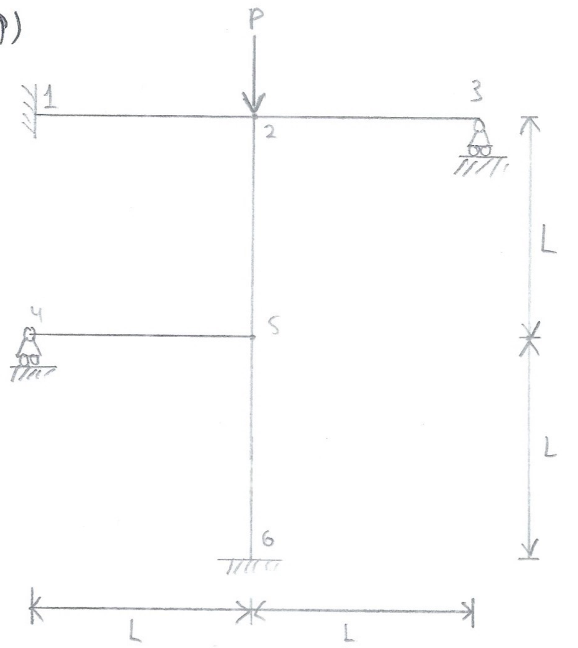
Then, $\rho = 0.80 + (0.82 - 0.80)\dfrac{(0 - 2.040)}{(-0.662 - 2.040)} = 0.8151$

$\therefore\ P_{cr} = \rho P_E = 0.8151 \dfrac{\pi^2 EI}{L^2}$ //

Note: Trial and Error on ρ takes a lot of time. Solving for ρ will not give you much mark. It is better to do the trial and error last

4.(a)



$EI \rightarrow$ constant (same) for all members.

Ignore axial deformations of all members.

Using modified stiffness whenever possible,

$DOF = \theta_2', U_5', \theta_5'$.

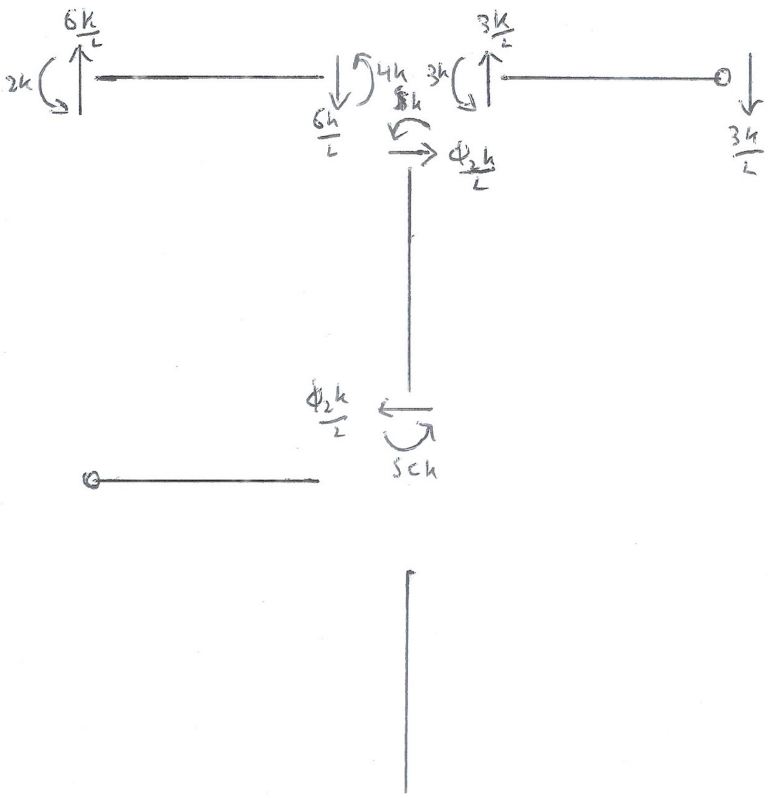
Let $k = \dfrac{EI}{L}$

Stiffness for all members $= \dfrac{EI}{L} = k$.

$P_{2-5} = P_{5-6} = P$

$P_{E,2-5} = P_{E,5-6} = \dfrac{\pi^2 EI}{L^2}$ $\Bigg\}$ $\rho_{2-5} = \rho_{5-6} = \rho$

i) If $\theta_2' = 1$, $U_5' = \theta_5' = 0$,



$$\Rightarrow \begin{bmatrix} (s+4+3)k \\ -\dfrac{\phi_2 k}{L} \\ sck \end{bmatrix} \begin{matrix} \leftarrow \theta_2' \\ \leftarrow U_5' \\ \leftarrow \theta_5' \end{matrix}$$

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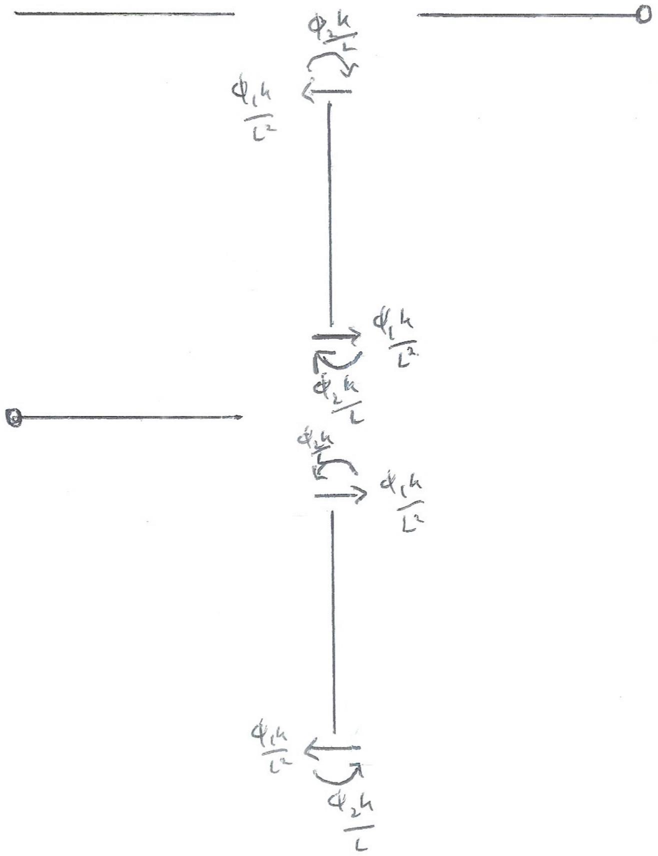
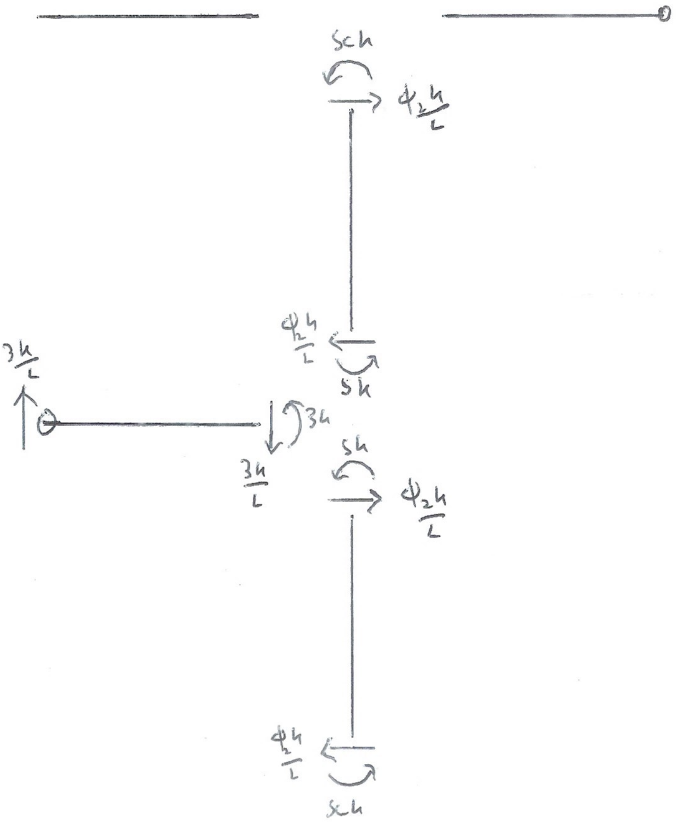
*) If $U_5' = 1$, $\theta_2' = \theta_5' = 0$,



$$\Rightarrow \begin{array}{cl} \left[\begin{array}{c} -\frac{\phi_2 k}{L} \\ \frac{\phi_1 k}{L^2} + \frac{\phi_1 k}{L^2} \\ \frac{\phi_2 k}{L} - \frac{\phi_2 k}{L} \end{array}\right] & \begin{array}{l} \leftarrow \theta_2' \\ \leftarrow U_5' \\ \leftarrow \theta_5' \end{array} \end{array}$$

*) If $\theta_5' = 1$, $\theta_2' = U_5' = 0$,



$$\Rightarrow \begin{array}{cl} \left[\begin{array}{c} sck \\ \frac{\phi_2 k}{L} - \frac{\phi_2 k}{L} \\ (s+s+3)k \end{array}\right] & \begin{array}{l} \leftarrow \theta_2' \\ \leftarrow U_5' \\ \leftarrow \theta_5' \end{array} \end{array}$$

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Then,
$$k_{ss}' = \begin{bmatrix} (s+7)k & -\dfrac{\phi_2 k}{L} & sck \\ -\dfrac{\phi_2 k}{L} & \dfrac{2\phi_1 k}{L^2} & 0 \\ sck & 0 & (2s+3)k \end{bmatrix}$$

Non-trivial solution: $\left(|k_{ss}'| = 0\right)$

$$|k_{ss}'| = \frac{k^3}{L^2}\Big((s+7)(2\phi_1)(2s+3) + 0 + 0 - (sc)^2(2\phi_1) - 0 - (2s+3)(-\phi_2)^2\Big) = 0$$

$$\Rightarrow (s+7)(2s+3)(2\phi_1) - (sc)^2(2\phi_1) - (2s+3)(\phi_2^2) = 0$$

Let $f = (s+7)(2s+3)(2\phi_1) - (sc)^2(2\phi_1) - (2s+3)(\phi_2^2)$

Where $\phi_1 = 2s(1+c) - \pi^2 \rho$

$\phi_2 = s(1+c)$

We will find ρ that makes $f = 0$ by trial and error.

Note: $\rho < 1.0 \rightarrow$ ($1.0 =$ fix - guided end)

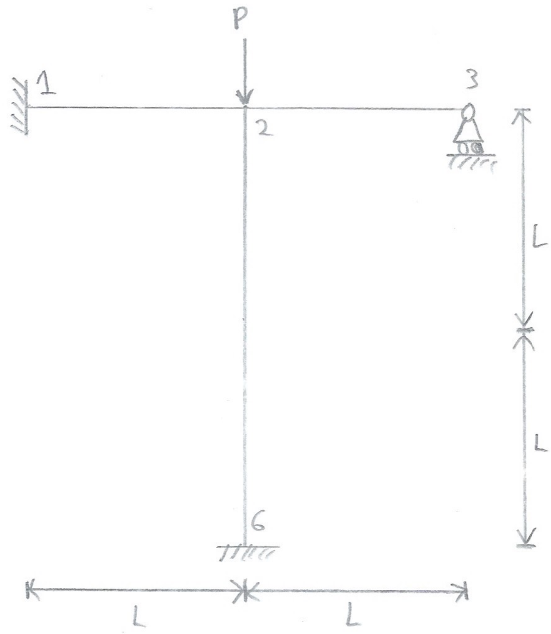
ρ	s	c	ϕ_1	ϕ_2	f
0.70	2.9809	0.7687	3.6359	5.2723	363.1461
0.80	2.8159	0.8330	2.4274	5.1615	154.6669
0.90	2.6449	0.9090	1.2156	5.0491	-31.0069
0.88	2.6797	0.8927	1.4585	5.0719	4.3207

Then, $\rho = 0.88 + (0.90 - 0.88)\dfrac{(0 - 4.3207)}{(-31.0069 - 4.3207)} = 0.8824$

$\therefore P_{cr} = \rho P_E = 0.8824 \dfrac{\pi^2 EI}{L^2}$ //

Note: Trial and Error on ρ takes a lot of time. Solving for ρ will not give you much mark. It is better to do the trial and error last.

(b)



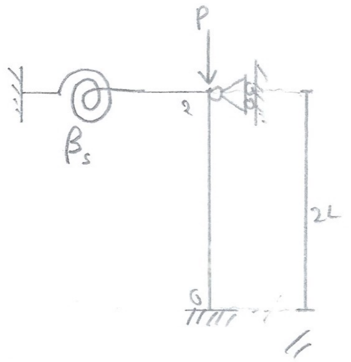
Ignore axial deformations of all members

All members have the same EI.

Note:

For this question (Part (b)), You have to make sure you have a correct sketch for Part (i), otherwise your answer in Part (ii) will be rubbish.

(i)

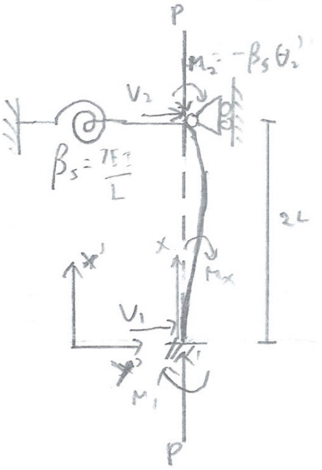


Since we ignore axial deformations and joint 1 is a fixed support, then joint 2 cannot move horizontally. (This the reason why there is a pin at joint 2)

From Part(a), member 1-2 and 2-3 acts as rotational spring.

$$M_{@2} = (3+4)\frac{EI}{L}\theta_2' = \beta_s \theta_2' \Leftrightarrow \beta_s = \frac{7EI}{L}$$

(ii)



Here, $\theta_2' = v'(x=2L)$

Boundary Conditions:

i) $v(x=0) = 0$

ii) $v'(x=0) = 0$

iii) $v(x=2L) = 0$

$$+\rightarrow \sum F_y = 0 \Leftrightarrow V_1 + V_2 = 0 \Leftrightarrow V_2 = V_1$$

$$+\circlearrowright \sum M_{joint2} = 0 \Leftrightarrow M_1 + M_2 - V_1(2L) = 0$$

$$M_1 = \beta_s v'(x=2L) + 2L\,V_1$$

$$M_x = -Pv - M_1 + V_1 x$$

$$\Leftrightarrow EIv'' + Pv = -\frac{7EI}{L}v'(x=2L) - 2LV_1 + V_1 x \quad \dots(1)$$

Let $\omega = \sqrt{\frac{P}{EI}}$, then (1) becomes

$$v'' + \omega^2 v = -\frac{7v'(x=2L)}{L} - \frac{2LV_1}{EI} + \frac{V_1 x}{EI}$$

Welli

Then, $\vartheta = A\sin(\omega x) + B\cos(\omega x) + \frac{1}{\omega^2}\left(-\frac{7\vartheta'(x=2L)}{L} - \frac{2LV_1}{EI} + \frac{V_1 x}{EI}\right)$

where A and B are constants to be determined.

Applying Boundary condition:

-) $\vartheta(x=0)=0$

$$0 = A\sin(0) + B\cos(0) + \frac{1}{\omega^2}\left(-\frac{7\vartheta'(x=2L)}{L} - \frac{2LV_1}{EI} + 0\right)$$

$$\Leftrightarrow B = \frac{1}{\omega^2}\left(\frac{7\vartheta'(x=2L)}{L} + \frac{2LV_1}{EI}\right)$$

-) $\vartheta'(x=0)=0$

$$0 = A\omega\cos(0) - B\omega\sin(0) + \frac{1}{\omega^2}\left(\frac{V_1}{EI}\right)$$

$$\Leftrightarrow A = -\frac{V_1}{\omega^3 EI}$$

-) $\vartheta(x=2L)=0$

$$0 = A\sin(2L\omega) + B\cos(2L\omega) + \frac{1}{\omega^2}\left(-\frac{7\vartheta'(x=2L)}{L} - \frac{2LV_1}{EI} + \frac{V_1(2L)}{EI}\right)$$

$$\Leftrightarrow 0 = -\frac{V_1}{\omega^3 EI}\sin(2\omega L) + \frac{1}{\omega^2}\left(\frac{7\vartheta'(x=2L)}{L} + \frac{2LV_1}{EI}\right)\cos(2\omega L) - \frac{7\vartheta'(x=2L)}{\omega^2 L}$$

$$\Leftrightarrow 7\left(1-\cos(2\omega L)\right)\vartheta'(x=2L) = \frac{V_1}{EI}\left(-\frac{L}{\omega}\sin(2\omega L) + 2L^2\cos(2\omega L)\right)$$

$$\Leftrightarrow \vartheta'(x=2L) = \frac{V_1L^2}{EI}\frac{\left(\frac{1}{\omega L}\sin(2\omega L) - 2\cos(2\omega L)\right)}{7\left(\cos(2\omega L)-1\right)} \quad \text{---(2)}$$

-) $\vartheta'(x=2L)$

$$\Rightarrow \vartheta'(x=2L) = A\omega\cos(2L\omega) - B\omega\sin(2L\omega) + \frac{1}{\omega^2}\left(\frac{V_1}{EI}\right)$$

$$\Leftrightarrow \vartheta'(x=2L) = \left(-\frac{V_1}{\omega^3 EI}\right)\omega\cos(2L\omega) - \left(\frac{1}{\omega^2}\left(\frac{7\vartheta'(x=2L)}{L} + \frac{2LV_1}{EI}\right)\right)\omega\sin(2L\omega) + \frac{1}{\omega^2}\left(\frac{V_1}{EI}\right)$$

$$\Leftrightarrow \vartheta'(x=2L)\left(1 + \frac{7\sin(2\omega L)}{\omega L}\right) = \frac{V_1L^2}{EI}\left(-\frac{1}{\omega^2L^2}\cos(2\omega L) - \frac{2}{\omega L}\sin(2\omega L) + \frac{1}{\omega^2L^2}\right)$$

---(3)

Willem

Subtitute $v'(x=2L)$ of (2) to (3), we get

$$\frac{V_1 L^2}{EI} \cdot \frac{\left(\frac{1}{wL}\sin(2wL) - 2\cos(2wL)\right)}{7(\cos(2wL) - 1)} \left(1 + \frac{7\sin(2wL)}{wL}\right)$$

$$= \frac{V_1 L^2}{EI} \left(-\frac{1}{(wL)^2}\cos(2wL) - \frac{2}{wL}\sin(2wL) + \frac{1}{(wL)^2}\right)$$

$$\Leftrightarrow \frac{V_1 L^2}{EI} \left[\left(\frac{1}{wL}\sin(2wL) - 2\cos(2wL)\right)\left(1 + \frac{7\sin(2wL)}{wL}\right) + 7\left(\cos(2wL) - 1\right)\left(\frac{1}{(wL)^2}\cos(2wL) + \frac{2}{wL}\sin(2wL) - \frac{1}{(wL)^2}\right)\right] = 0$$

$$\Leftrightarrow \frac{V_1 L^2}{EI} \left(\frac{\sin(2wL)}{wL} - 2\cos(wL) + \frac{7\sin^2(2wL)}{(wL)^2} - \frac{14\sin(2wL)\cos(2wL)}{wL} + \frac{7\cos^2(2wL)}{(wL)^2} + \frac{14\sin(2wL)\cos(2wL)}{wL} - \frac{7\cos(2wL)}{(wL)^2} - \frac{7\cos(2wL)}{(wL)^2} - \frac{14\sin(2wL)}{wL} + \frac{7}{(wL)^2}\right) = 0$$

$$\Leftrightarrow -\frac{14\cos(2wL)}{(wL)^2} - \frac{13\sin(2wL)}{wL} - 2\cos(2wL) + \frac{7\sin^2(2wL) + 7\cos^2(2wL) + 7}{(wL)^2} = 0$$

$$\Leftrightarrow -\frac{14\left(1 - 2\sin^2(wL)\right)}{(wL)^2} - \frac{13\left(2\sin(wL)\cos(wL)\right)}{wL} - 2\cos(2wL) + \frac{14}{(wL)^2} = 0$$

$$\Leftrightarrow \frac{28\sin^2(wL)}{(wL)^2} - \frac{26\sin(wL)\cos(wL)}{wL} - 2\left(\cos^2(wL) - \sin^2(wL)\right) = 0$$

$$\Leftrightarrow \frac{28}{(wL)^2} - \frac{26\cot(wL)}{wL} - 2\cot^2(wL) + 2 = 0$$

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Note: Question 4(b)(ii) is harder than what it looks like. The important this is you have to make sure the boundary conditions are correct and cleverly used.