NO:................................ DATE:................................

CV4101 - Structural Analysis III 17/18

1. a) 2 DOFs - u_4', v_4'

b) Member 1: $c=1$, $s=0$, $\frac{EA}{L}=400\,000$ Member 2: $c=0$, $s=1$, $\frac{EA}{L}=400\,000$

$$k_1' = \begin{pmatrix} \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & 400\,000 & 0 \\ \times & \times & 0 & 0 \end{pmatrix} \qquad k_2' = \begin{pmatrix} \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & 0 & 0 \\ \times & \times & 0 & 400\,000 \end{pmatrix}$$

Member 3: $c=-\frac{\sqrt{2}}{2}$, $s=\frac{\sqrt{2}}{2}$, $\frac{EA}{L}=400\,000$

$$k_3' = \begin{pmatrix} \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & 200\,000 & -200\,000 \\ \times & \times & -200\,000 & 200\,000 \end{pmatrix}$$

Assembling

$$\begin{bmatrix} R_{1x'} \\ R_{1y'} \\ R_{2x'} \\ R_{2y'} \\ R_{3x'} \\ R_{3y'} \\ -400 \\ -300 \end{bmatrix} = \begin{pmatrix} & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & 600\,000 & -200\,000 \\ & & -200\,000 & 600\,000 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ u_4' \\ v_4' \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -400 \\ -300 \end{pmatrix} = \begin{pmatrix} 600\,000 & -200\,000 \\ -200\,000 & 600\,000 \end{pmatrix} \begin{pmatrix} u_4' \\ v_4' \end{pmatrix}$$

$$\therefore \begin{pmatrix} u_4' \\ v_4' \end{pmatrix} = \begin{pmatrix} -9.375 \times 10^{-4} \\ -8.125 \times 10^{-4} \end{pmatrix} m$$

c) To obtain maximum displacement at joint 4, member 3 must be zero-force member.

So, $u_4' = v_4'$ for zero-force member in member 3.

$$\begin{pmatrix} -P\cos\beta \\ -P\sin\beta \end{pmatrix} = \begin{pmatrix} 600\,000 & -200\,000 \\ -200\,000 & 600\,000 \end{pmatrix} \begin{pmatrix} u_4' \\ v_4' \end{pmatrix} \quad \text{and} \quad u_4' = v_4'$$

will get $\sin\beta = \cos\beta$

$\therefore \beta = 45^\circ$ or 225°

NO:.................................... DATE:....................................

2. a) 2 DOFs – θ_2', v_3'

b) Member 1: $\frac{4EI}{L} = 1600$, $\frac{6EI}{L^2} = 2400$ Member 2: $\frac{3EI}{L} = \frac{3EI}{L^2} = \frac{3EI}{L^3} = 1200$

$$K_1' = \begin{pmatrix} \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & -2400 \\ \times & \times & \times & 1600 \end{pmatrix}, F_{01}' = \begin{pmatrix} \times \\ \times \\ 12 \\ -3 \end{pmatrix} \qquad K_2' = \begin{pmatrix} \times & \times & \times & 0 \\ \times & 1200 & -1200 & 0 \\ \times & -1200 & 1200 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, F_{02}' = \begin{pmatrix} \times \\ 2 \\ 6 \\ 0 \end{pmatrix}$$

Assembling

$$\begin{bmatrix} R_{1y}' \\ M_{1\theta}' \\ R_{2y}' \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{array}{cc} \text{column 4} & \text{column 5} \\ \downarrow & \downarrow \end{array} \begin{pmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & 2800 & -1200 & \\ & & & -1200 & 1200 & \\ & & & & & \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ \theta_2' \\ V_3' \\ \times \end{pmatrix} + \begin{pmatrix} \times \\ \times \\ \times \\ -1 \\ 6 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2800 & -1200 \\ -1200 & 1200 \end{pmatrix} \begin{pmatrix} \theta_2' \\ V_3' \end{pmatrix} + \begin{pmatrix} -1 \\ 6 \end{pmatrix}$$

$$\therefore \begin{pmatrix} \theta_2' \\ V_3' \end{pmatrix} = \begin{pmatrix} -3.125 \times 10^{-3} \\ -8.125 \times 10^{-3} \end{pmatrix} \begin{matrix} \text{rad} \\ \text{m} \end{matrix}$$

c) V and M at joint 3 must obtained from member forces in member 1.

$V = -2400 \times -3.125 \times 10^{-3} + 12$

$= 19.5$ kN

$M = 1600 \times -3.125 \times 10^{-3} - 3$

$= -8$ kN.m

$F_3 = -24$ kN

$F_4 = 0$

$F_5 = 19.5$ kN

$F_6 = -8$ kN.m

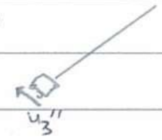
$u_5 = 0$

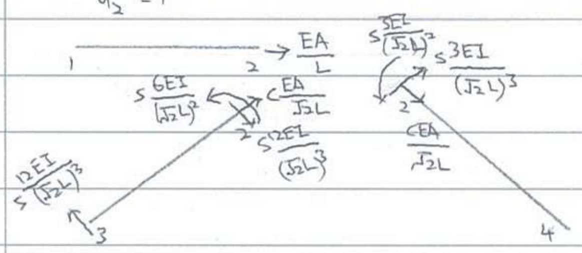
$u_6 = -3.125 \times 10^{-3}$ rad

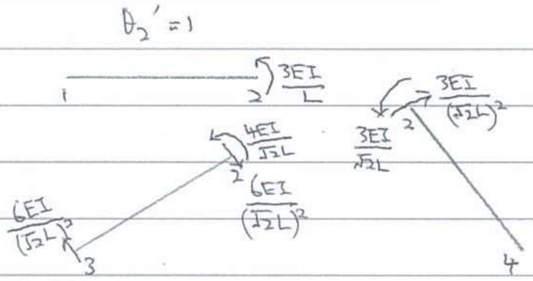
NO:.................... DATE:....................

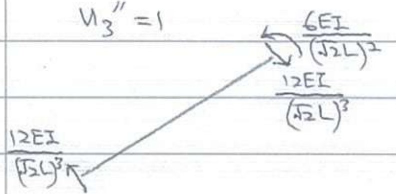
3. a) As u_2' and v_2' are zero, there will be no axial deformation in member 1 and 2 for the two-member frame. Normally, Each frame member contains both axial force and bending. In this two-member frame, frame members can experience bending effects only. This characteristic resemble characteristic of beam as beam ignore axial deformation. The structure in Figure Q3(b) is a beam. So, rotation at joint 2 will be the same for the two structures, because the frame in Figure Q3(a) have bending effects only ~~and~~ that is the same as beam in Figure Q3(b).

b) 3 DOFs – u_2', θ_2', u_3''



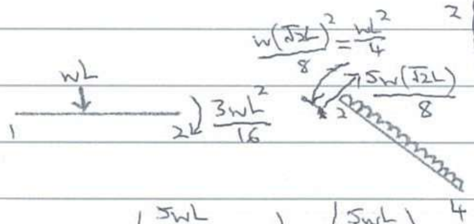






$$k_{ff}' = \begin{pmatrix} \frac{EA}{L} + s^2\frac{12EI}{(\sqrt{2}L)^3} + s^2\frac{3EI}{(\sqrt{2}L)^3} + 2c^2\frac{EA}{\sqrt{2}L} & s\frac{6EI}{(\sqrt{2}L)^2} + s\frac{3EI}{(\sqrt{2}L)^2} & s\frac{12EI}{(\sqrt{2}L)^3} \\ s\frac{6EI}{(\sqrt{2}L)^2} + s\frac{3EI}{(\sqrt{2}L)^2} & \frac{3EI}{L} + \frac{7EI}{\sqrt{2}L} & \frac{6EI}{(\sqrt{2}L)^2} \\ s\frac{12EI}{(\sqrt{2}L)^3} & \frac{6EI}{(\sqrt{2}L)^2} & \frac{12EI}{(\sqrt{2}L)^3} \end{pmatrix} \begin{matrix} \leftarrow u_2' \\ \leftarrow \theta_2' \\ \leftarrow u_3'' \end{matrix}$$

$$2 \begin{pmatrix} 4.3588\frac{EI}{L^3} & 3.1820\frac{EI}{L^2} & 3\frac{EI}{L^3} \\ 3.1820\frac{EI}{L^2} & 7.9497\frac{EI}{L} & 3\frac{EI}{L^2} \\ 3\frac{EI}{L^3} & \frac{3EI}{L^2} & 4.2426\frac{EI}{L^3} \end{pmatrix}$$



$$F_{ok}' = \begin{pmatrix} \frac{5wL}{8} \\ \frac{wL^2}{4} - \frac{3wL^2}{16} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{5wL}{8} \\ \frac{wL^2}{16} \\ 0 \end{pmatrix}$$

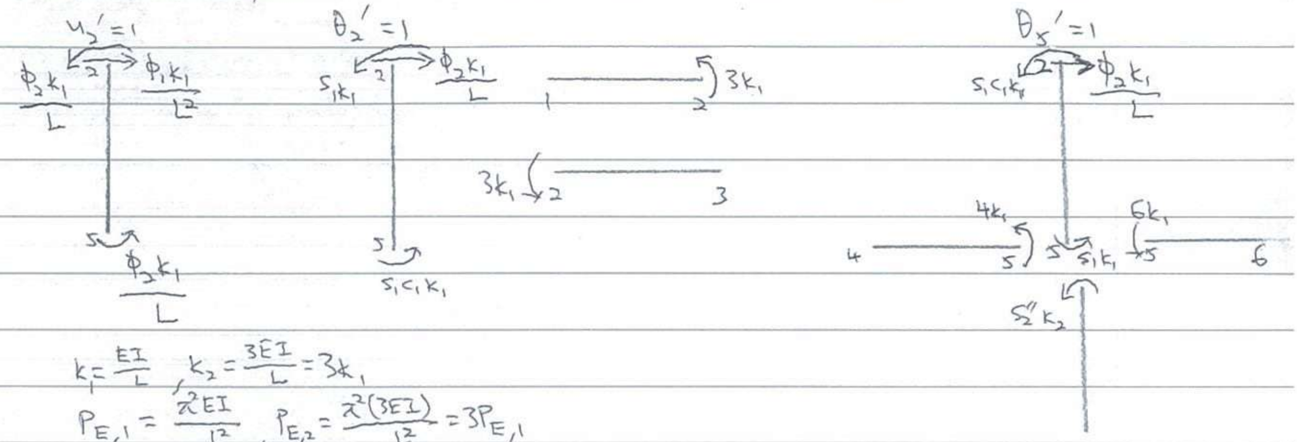
$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = k_{ff}' \begin{pmatrix} u_2' \\ \theta_2' \\ u_3'' \end{pmatrix} + F_{ok}'$$

$$\begin{pmatrix} u_2' \\ \theta_2' \\ u_3'' \end{pmatrix} = \begin{pmatrix} -0.3001 \\ 0.0439/L \\ 0.1812 \end{pmatrix} \frac{wL^4}{EI}$$

Pentex

NO: DATE:

4. a) 3DOFs – u_2', θ_2', θ_5'



$k_1=\frac{EI}{L}$, $k_2=\frac{3EI}{L}=3k_1$

$P_{E,1}=\frac{\pi^2 EI}{L^2}$, $P_{E,2}=\frac{\pi^2(3EI)}{L^2}=3P_{E,1}$

$P_1=P_2=P$, $\rho_1=3\rho_2$

$$k_{ff}'=\begin{pmatrix} \frac{\phi_1 k_1}{L^2} & \frac{\phi_2 k_1}{L} & \frac{\phi_2 k_1}{L} \\ \frac{\phi_2 k_1}{L} & (6+s_1)k_1 & s_1 c_1 k_1 \\ \frac{\phi_2 k_1}{L} & s_1 c_1 k_1 & (10+s_1+3s_2'')k_1 \end{pmatrix}\begin{matrix} \leftarrow u_2' \\ \leftarrow \theta_2' \\ \leftarrow \theta_5' \end{matrix}$$

$|k_{ff}'|=0$, $\therefore (6\phi_1+s_1\phi_1-\phi_2^2)(10+s_1+3s_2'')+2s_1c_1\phi_2^2-\phi_2^2(6+s_1)-s_1^2c_1^2\phi_1=0$

$\rho_1=0.66$, $\rho_2=0.22$, $s_1=3.0453$, $c_1=0.7456$, $s_2''=2.5359$

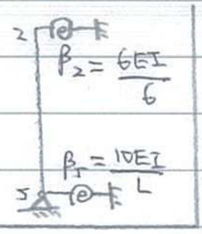
$f(s,c)=37.1256$

$\rho_1=0.68$, $\rho_2=0.227$, $s_1=3.0132$, $c_1=0.7570$, $s_2''=2.5207$

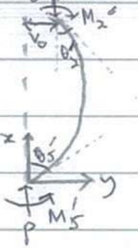
$f(s,c)=-2.6301$

$\rho_{cr}=\frac{0-37.1256}{-2.6301-37.1256}(0.68-0.66)+0.66=0.679$, $\therefore P_{cr}=0.679\frac{\pi^2 EI}{L^2}$

b) i)



ii)



$M_2'=\frac{6EI}{L}\theta_2'$, $M_5'=\frac{10EI}{L}\theta_5'$

$EIv''=-Pv+M_5'$, $\boxed{EIv''(L)=M_2'}$

$v=A\sin\omega x+B\cos\omega x+\frac{M_5'}{P}$, $\omega=\sqrt{\frac{P}{EI}}$

$=A\sin\omega x+B\cos\omega x+\frac{10}{\omega^2 L}\theta_5'$

$v'=A\omega\cos\omega x-B\omega\sin\omega x$

$-Pv_0+\frac{10EI}{L}\theta_5'=\frac{6EI}{L}\theta_2'$

$\boxed{v_0=(10\theta_5'-6\theta_2')/\omega^2 L}$

$v(0)=0$, $B=-\frac{10}{\omega^2 L}\theta_5'$

$v'(0)=\theta_5'$, $A=\frac{\theta_5'}{\omega}$

$v'(L)=-\theta_2'$, $\theta_5'\cos\omega L+\frac{10}{\omega L}\theta_5'\sin\omega L=-\theta_2'$

$v(L)=v_0$, $\frac{\theta_5'}{\omega}\sin\omega L-\frac{10}{\omega^2 L}\theta_5'\cos\omega L+\frac{10}{\omega^2 L}\theta_5'=v_0$

$\theta_5'\omega L\sin\omega L-10\theta_5'\cos\omega L+\cancel{10\theta_5'}=\cancel{10\theta_5'}-6\theta_2'$

$\theta_5'\varphi\sin\varphi-10\theta_5'\cos\varphi=6\left(\theta_5'\cos\varphi+\frac{10}{\varphi}\theta_5'\sin\varphi\right)$, cancel θ_5'

will get $\underline{\varphi^2-16\varphi\cot\varphi-60=0}$, $\varphi=\omega L$, $\omega=\sqrt{\frac{P}{EI}}$

Kok De Sheng