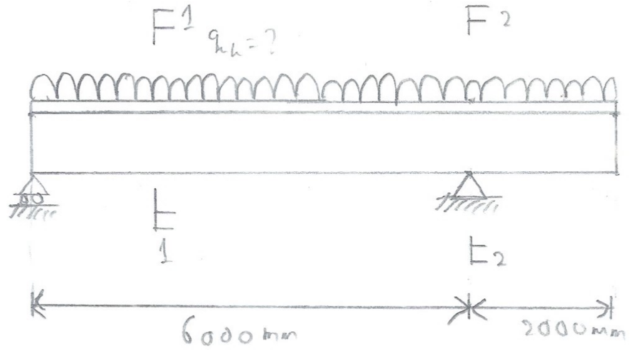
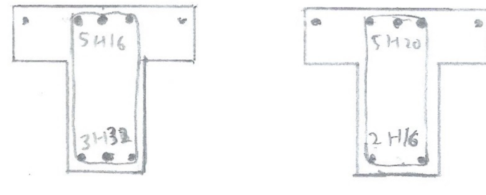
CV3011 PYP AY2019/2020 Semester 1 Solution

William Kho

1.





Section 1-1 Section 2-2

Note: There is a typo on the structure.

At the beginning of the exam, Prof. Li Bing announced that there should have been <u>a roller at the left end</u> of the beam.

$f_{ck} = 40\ N/mm^2$

$f_{yk} = 500\ N/mm^2$

Concrete unit weight $= 24\ kN/m^3$

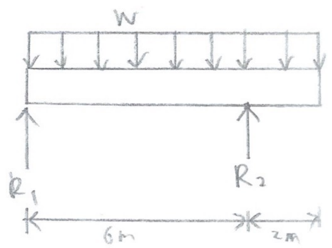
Let $W = 1.35\, g_k + 1.5\, q_k$

$g_k = (\text{Cross section area})(\text{Concrete unit weight})$

$= \left[(500mm)(150mm) + (250mm)(400mm)\right](24\ kN/m^3)$

$= 4.2\ kN/m$

Then, $W = 1.35\,(4.2\,kN/m) + 1.5\,q_k = 5.67 + 1.5\,q_k$



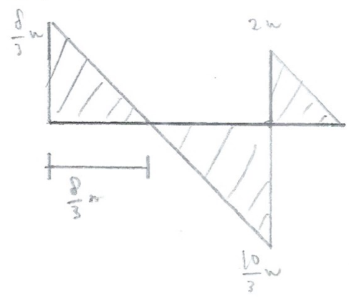
$+\circlearrowright \sum M_{e_2} = 0 \iff 6R_1 - 2(8W) = 0$

$R_1 = \frac{8}{3} W$

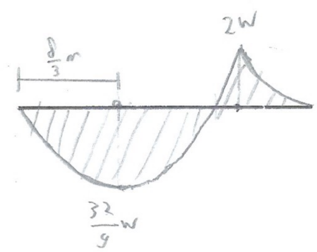
$+\uparrow \sum F_y = 0 \iff R_1 + R_2 - 8W = 0$

$R_2 = \frac{16}{3} W$

SFD



BMD



$\Rightarrow$ For Section 1-1,

$M_{Max} = \frac{32}{9} W\ kN\cdot m$

For Section 2-2,

$M_{Max} = 2W\ kN\cdot m$

Where W is in kN/m.

William

Next, we will find M_{Rd} for section 1-1 and section 2-2.

For section 1-1, ($d' = 30\,mm$, $d = 550 - 40 = 510\,mm$, $A_{sc} = 1006\,mm^2$ (5H16), $A_s = 2414\,mm^2$ (3H32))

Assume the concrete stress block lies only on the flange ($S = 0.8x \leq h_f$)

Check whether compression steel has yielded or not.

$$\frac{d'}{d} = \frac{30\,mm}{510\,mm} = 0.0588 < 0.171 \Rightarrow \text{Compression steel has yielded.}$$

Assume tension steel has yielded.

$$F_{cc} + F_{sc} = F_{st}$$

$$\Leftrightarrow (0.567\,f_{ck})(500\,mm)(0.8x) + (0.87\,f_{yk}) \cdot A_{sc} = (0.87\,f_{yk})\,A_c$$

$$\Leftrightarrow 0.567\,(40\,N/mm^2)(400\,mm)\,x + (0.87)(500\,N/mm^2)(1006\,mm^2) = (0.87)(500\,N/mm^2)(2414\,mm^2)$$

$$\Leftrightarrow \quad x = 67.513\,mm$$

$$\Leftrightarrow \quad 0.8x = S = 54.01\,mm < 150\,mm = h_f \quad \text{(ok!)}$$

$$\Rightarrow \text{Assumption on } S = 0.8x \leq h_f \text{ is correct.}$$

$$\frac{x}{d} = \frac{67.513\,mm}{510\,mm} = 0.132 < 0.617 \Rightarrow \text{Tension steel has yielded}$$
(Assumption ok!)

Taking moment about centroid of tension steel,

$$M_{Rd} = F_{cc}\left(d - \frac{0.8x}{2}\right) + F_{sc}\,(d - d')$$

$$= (0.567)(40\,N/mm^2)(500\,mm)(0.8(67.513\,mm))\left(510 - (0.4)(67.513)\,mm\right)$$

$$+ (0.87)(500\,N/mm^2)(1006\,mm^2)(510 - 30\,mm)$$

$$= 505.88\,kN\cdot m$$

For Section 2-2, ($d' = 40\,mm$, $d = 550 - 30 = 520\,mm$, $A_{sc} = 402\,mm^2$ (2H16), $A_s = 1571\,mm^2$ (5H20))

Assume the concrete stress block does not extend to flange ($S = 0.8x \leq h - h_f$)
(The assumption is obvious but need to be checked later.)

Check whether compression steel has yielded or not.

$$\frac{d'}{d} = \frac{40\,mm}{520\,mm} = 0.0769 < 0.171 \Rightarrow \text{Compression steel has yielded.}$$

Willem

Assume tension steel has yielded.

$$F_{cc} + F_{sc} = F_{st}$$

$$\Leftrightarrow (0.567 f_{ck})(250mm)(0.8x) + (0.87 f_{yk}) A_{sc} = (0.87 f_{yk}) A_{s}$$

$$\Leftrightarrow 0.567 (40 N/mm^2)(200mm)x + 0.87 (500 N/mm^2)(402 mm^2) = 0.87 (500 N/mm^2)(1571 mm^2)$$

$$\Leftrightarrow x = 112.106 \text{ mm}$$

$$\Leftrightarrow 0.8x = s = 89.68 \text{ mm} < 400 \text{ mm} = h - h_f \text{ (Ok!)}$$

$$\Rightarrow \text{Assumption on } s = 0.8x < h-h_f \text{ is correct.}$$

$$\frac{x}{d} = \frac{112.106mm}{520mm} = 0.216 < 0.617 \Rightarrow \text{Tension steel has yielded (Assumption ok!)}$$

Taking moment about centroid of tension steel,

$$M_{Rd} = F_{cc}\left(d - \frac{0.8x}{2}\right) + F_{sc}(d-d')$$

$$= (0.567)(40 N/mm^2)(250mm)(0.8(112.106mm))(520 - (0.4)(112.106) mm)$$

$$+ (0.87)(500 N/mm^2)(402mm^2)(520 - 40mm)$$

$$= 325.56 \text{ kN.m}$$

Then,

At Section 1-1,

$$M_{Rd} \geq M_{Ed} \Leftrightarrow 505.88 \geq \frac{32}{9} w$$

$$\Leftrightarrow w \leq \frac{9}{32}(505.88)$$

$$\Leftrightarrow 5.67 + 1.5q_k \leq \frac{9}{32}(505.88)$$

$$\Leftrightarrow q_k \leq 91.07 \text{ kN/m} \quad ...(1)$$

At Section 2-2,

$$M_{Rd} \geq M_{Ed} \Leftrightarrow 325.56 \geq 2w$$

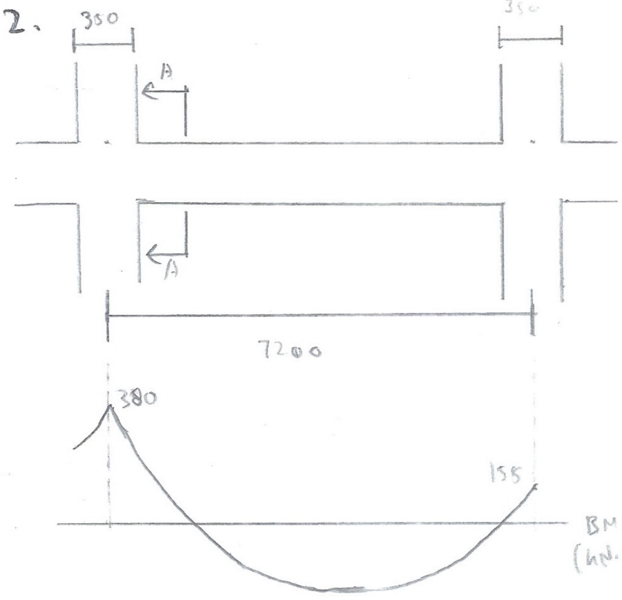
$$\Leftrightarrow w \leq \frac{1}{2}(325.56)$$

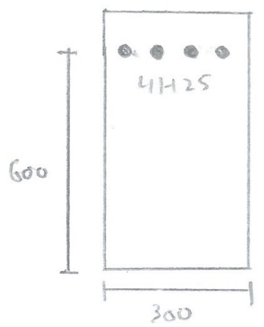
$$\Leftrightarrow 5.67 + 1.5q_k \leq \frac{1}{2}(325.56)$$

$$\Leftrightarrow q_k \leq 104.74 \text{ kN/m} \quad ...(2)$$

From (1) and (2), $q_{k,max} = 91.07$ kN/m //

2.





Section A-A

Note: There is inconsistency in the question. The question asks you to design using 10mm diameter bar. But the second paragraph mentions about T8 (8mm diameter bar). Prof. Li Bing & Prof. Qian Shunzhi said you can use either 10mm or 8mm diameter.

$d = 600mm = 0.6m$; $A_{se} = 1964\ mm^2$ (4H25)

Total UDL = 700 kN $\Rightarrow$ $w = \frac{700kN}{7.2m} = \frac{875}{9}$ kN/m = 97.22 kN/m

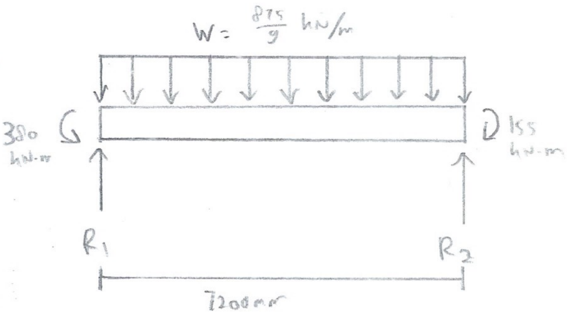
$f_{cu} = 25\ N/mm^2$

$f_{yk} = 500\ N/mm^2$

Unit weight of concrete = 24 kN/m³

Concrete cover = 25mm

Design near support = Design for Zone 1 + 2.

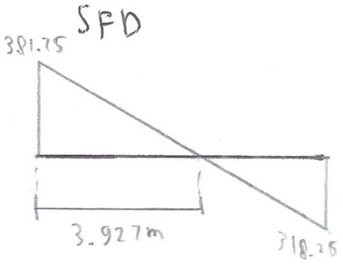


$+\circlearrowright \Sigma M_{@2} = 0 \Leftrightarrow (7.2) R_1 - (7.2w)\left(\frac{7.2}{2}\right) + 155 - 380 = 0$

$\Leftrightarrow R_1 = 381.75$ kN

$+\uparrow \Sigma F_y = 0 \quad \Leftrightarrow \quad R_1 + R_2 - (7.2)w = 0$

$R_2 = 318.25$ kN



At support face,

$V_{Ef} = 381.75 - \left(\frac{350mm}{2}\right)w$

$= 381.75kN - (0.175m)\left(\frac{875}{9} kN/m\right)$

$= 364.74$ kN

At distance 1d away from support face,

$V_{E1d} = 381.75 - \left(\frac{350mm}{2} w\right) - dw$

$= 381.75kN - (0.175m)\left(\frac{875}{9} kN/m\right) - (0.6\ m)\left(\frac{875}{9} kN/m\right)$

$= 306.40$ kN

Wellen

Concrete shear capacity (Material).

$$V_{Rd,c} = 0.12\, k\,(100\, \rho_\ell\, f_{ck})^{\frac{1}{3}}\, b_w d \quad \left(\text{but} \geq (0.035\, k^{\frac{3}{2}} f_{ck}^{\frac{1}{2}})\, b_w d\right)$$

$$= 0.12\left(1+\sqrt{\frac{200}{d}}\right)\left(100\left(\frac{A_{s\ell}}{b_w d}\right)(25\,N/mm^2)\right)^{\frac{1}{3}}(300\,mm)(600\,mm)$$

$$= 0.12\left(1+\sqrt{\frac{200}{600}}\right)\left(100\left(\frac{1964\,mm^2}{(300\,mm)(600\,mm)}\right)(25\,N/mm^2)\right)^{\frac{1}{3}}(180000\,mm^2)$$

$$= 102.56\,kN < 364.74\,kN = V_{Ed} = V_{Ef}$$

Note:

$k = 1+\sqrt{\frac{200}{d}} \leq 2$

$\rho_\ell = \frac{A_{s\ell}}{b_w d} \leq 2\%$

$\Rightarrow$ Stirrup is needed.

Crushing strength of Concrete diagonal strut at section with maximum shear (at face of support)

$$V_{Rd,max} = \frac{0.36\, b_w\, d\,(1 - f_{ck}/250)\, f_{ck}}{\cot\theta + \tan\theta}$$

$$= \frac{0.36\,(300\,mm)(600\,mm)(1-25/250)(25\,N/mm^2)}{\cot\theta + \tan\theta}$$

$$= \frac{1458\,kN}{\cot\theta + \tan\theta}$$

For $\theta = 22^\circ$, $V_{Rd,max} = 506.41\,kN > 364.74\,kN = V_{Ef}$.

So, $\theta = 22^\circ$.

Area of shear link required $\left(\frac{A_{sw}}{s}\right)$ $\left(\frac{A_{sw,min}}{s} = \frac{0.08\, f_{ck}^{0.5}\, b_w}{f_{yk}} = \frac{(0.08)\sqrt{25}\,(300\,mm)}{500} = 0.24\right)$

Use $V_{Ed} = V_{E1D}$

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{0.78\, d\, f_{yk} \cot\theta} = \frac{306.40\,kN}{0.78\,(600\,mm)(500\,N/mm^2)\cot 22^\circ}$$

$$= 0.529\,mm^2/mm > \frac{A_{sw,min}}{s}$$

$\Rightarrow$ Provide T8-175 $\left(\frac{A_{sw}}{s} = 0.575\,mm^2/mm\right)$ or T10-275 $\left(\frac{A_{sw}}{s} = 0.571\,mm^2/mm\right)$ //

Note:
During exam, just choose one (either T8-175 or T10-275).

Next, we will determine the range of Zone 2.

At $\frac{A_{sw,min}}{s}$,

$$\frac{V_{Ed}}{0.78\, d\, f_{yk} \cot\theta} = \frac{0.08\, f_{ck}^{0.5}\, b_w}{f_{yk}}$$

$$\Leftrightarrow \quad V_{Ed} = 0.08\,(25)^{0.5}\,(300\text{ mm})\,(0.78)\,(680\text{ mm})\cot 22°$$

$$= 139.00 \text{ kN}$$

$$V_{Ed} = 381.75 \text{ kN} - x \cdot w$$

$$\Leftrightarrow 139.00\text{ kN} = 381.75\text{ kN} - x\left(\frac{875}{9}\text{ kN/m}\right)$$

$$\Leftrightarrow x = 2.497\text{ m} \text{ (from the support center)}$$

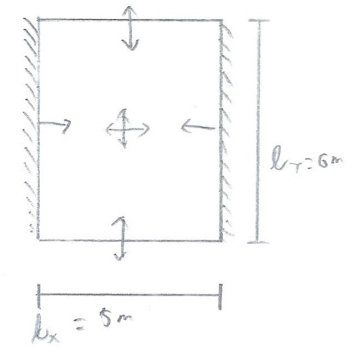
So, Zone 1 + 2 extends from the face of support until $2.497\text{ m} - \frac{350\text{ mm}}{2} = 2.322\text{ m}$ away from the face of support.

Thus, the number of stirrup needed is $1 + \frac{2.322\text{ m}}{\text{stirrup spacing}}$

For T8−175, $1 + \frac{2.322\text{ m}}{175\text{ mm}} = 14.27 \Rightarrow$ number of stirrup needed is 14 bars.

For T10−275, $1 + \frac{2.322\text{ m}}{275\text{ mm}} = 9.44 \Rightarrow$ ———— // ———— 9 bars.

3.



Purpose: Office building.

$h = 190$mm
concrete cover = 30mm
Steel reinforcement diameter = 10mm
} $d_x = 160$ mm, $d_y = 150$mm

$g_k = 6.0$ kN/m²
$q_k = 2.4$ kN/m²
} $n = 1.35\, g_k + 1.5\, q_k = 11.7$ kN/m²

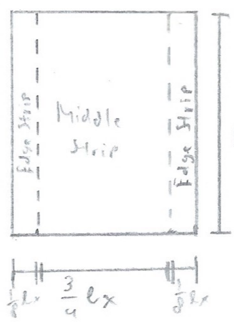
$f_{ck} = 40$ N/mm²

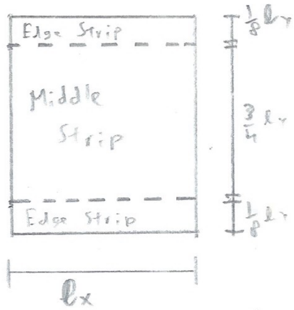
$f_{ctm} = 3.5$ MPa (Concrete mean tensile strenght)

$f_{yk} = 500$ N/mm²

$$\frac{l_y}{l_x} = \frac{6\text{ m}}{5\text{ m}} = 1.2$$

William





For Middle strip and continuous edge strip,

$$M_{sx} = \beta_{sx} \cdot n \cdot l_x^2 \quad , \quad M_{sy} = \beta_{sy} \cdot n \, l_x^2$$

Where β_{sx} and β_{sy} are determined from table.

For discontinuous edge strip, provide minimum reinforcement.

$$A_{s,min} = 0.26 \left(\frac{f_{ctm}}{f_{yk}}\right) b d \qquad (\text{but} \geq 0.0013\,bd)$$

$$= 0.26 \left(\frac{3.5\,MPa}{500\,N/mm^2}\right)(1000\,mm/m)(160\,mm)$$

$$= 291.2\ mm^2/m > 0.0013\,bd$$

$$\left(0.0013\,bd = 0.0013\,(1000\,mm/m)(160\,mm) = 208\ mm^2/m\right)$$

From table of β_{sx} and β_{sy}, for two long edge continuous and $\frac{l_y}{l_x} = 1.2$,

At mid-span, $\beta_{sx} = 0.056$, $\beta_{sy} = 0.034$

At continuous edge, $\beta_{sy} = 0.045$

Location	β_{sx} (β_{sy})	M_{sx} (M_{sy}) (kN·m)	$K = \frac{M}{bd^2 f_{ck}}$	d (mm)	$\frac{z}{d} = 0.5 + \sqrt{0.25 - \frac{K}{1.134}} \leq 0.95$	$A_{s,req} = \frac{M}{0.87 f_{yk} z}$ (mm²)	$A_{s,prov}$ (mm²)
Mid-span X-direction	0.056	16.38	0.0160	160	0.95	247.73 ($< A_{s,min}$)	314 (H10-250)
Mid-span Y-direction	0.034	9.945	0.0111	150	0.95	160.44 ($< A_{s,min}$)	314 (H10-250)
Continuous edge Y-direction	0.045	13.1625	0.0146	150	0.95	212.34 ($< A_{s,min}$)	314 (H10-250)
Discontinuous edge x-direction	N/A	N/A	N/A	160	N/A	$A_{s,min}$ = 291.2	314 (H10-250)

Note: All analysis are based on per 1m width.

If $K < 0.05$, then $\frac{z}{d} = 0.95$

Wellian

Torsional Reinforcement

Provide at every corner within length $\frac{l_x}{5} = \frac{5m}{5} = 1m$ both at top and bottom.

For torsional reinforcement, $A_{s,req} = \frac{3}{8} A_{sx,req} = \frac{3}{8}(247.73\ mm^2/m)(1m)$

$$= 92.90\ mm^2$$

$\Rightarrow$ Provide 2H10 ($157\ mm^2$) for top and bottom in both directions. //

Note: $A_{s,req} = \frac{3}{8} A_{sx,req}$ because all corners has 1 continuous edge and 1 discontinuous edge.

(b) Deflection Control Check

The check is conducted on shorter span as this span direction carries more loads.

$$\rho = \frac{A_{s,req}}{bd} = \frac{A_{sx,req}}{bd} = \frac{247.73\ mm^2}{(1000mm)(160mm)} = 0.155\% \leq 0.35\%.$$

For 2-way slab of end span, from the table,

Basic Allowable $\frac{l}{d} = 39$

$$F1 = 1 - 0.1\left(\frac{b_f}{b_w} - 1\right) = 1 - 0.1(1-1) = 1$$

$$F2 = \frac{7}{\text{Span length}} = \frac{7}{5} = 1.4 \quad (\text{but} \leq 1.0) \Rightarrow F_2 = 1$$

Conservative approach: $F3 = \left(\frac{A_{s,prov}}{A_{s,req}}\right)\delta = \frac{314\ mm^2}{247.73\ mm^2}(1.0) = 1.268 \quad (\leq 1.5\ (OK!))$

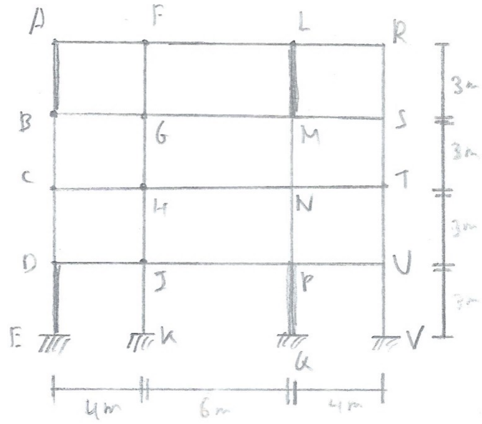
So, Allowable $\frac{l}{d} = \left(\text{Basic Allowable } \frac{l}{d}\right)(F1)(F2)(F3)$

$$= (39)(1)(1)(1.268) = 49.452$$

Actual $\frac{l}{d} = \frac{5m}{160mm} = 31.25 < 49.452 = $ Allowable $\frac{l}{d}$ //

$\Rightarrow$ Deflection Check is adequate. //

4.



Cross-section of all column : 300mm x 300mm

EI column : EI beam = 0.4 : 1

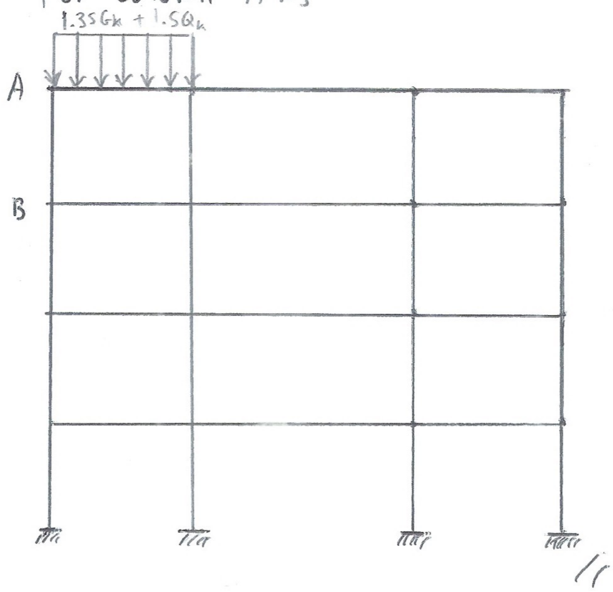
$G_k = 16$ kN/m, $Q_k = 18$ kN/m

$1.35G_k = 21.6$ kN/m

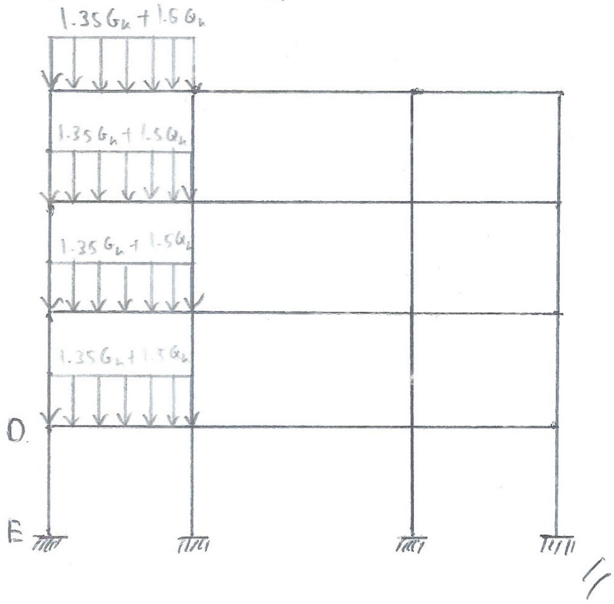
$1.35G_k + 1.5Q_k = 48.6$ kN/m

Concrete unit weight = 25 kN/m³

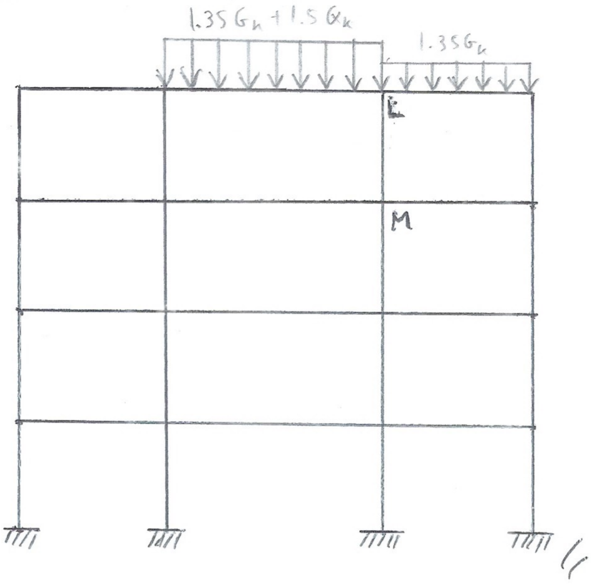
(a) For column AB,



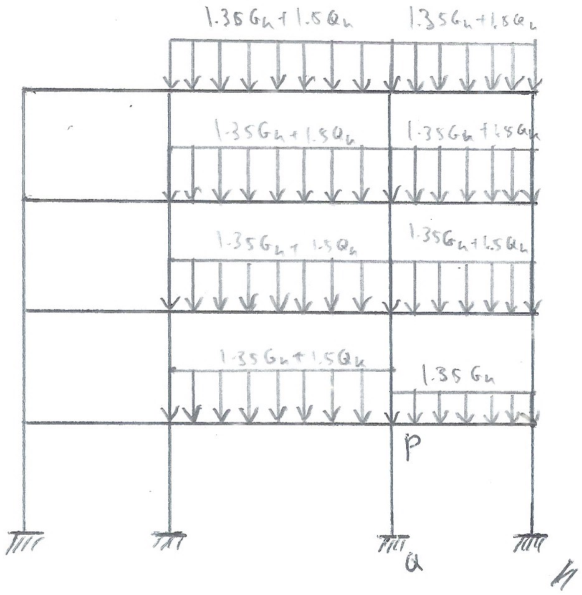
For column DE,



For column LM,



For column PQ,



Between column LM or PQ, Column PQ needs more reinforcement since it receives more axial loads than column LM.

(b) $k_{column} : k_{4\text{-}m\ beam} : k_{6\text{-}m\ beam} = \frac{EI_{column}}{L_{column}} : \frac{EI_{beam}}{L_{4m\ beam}} : \frac{EI_{beam}}{L_{6m\ beam}}$

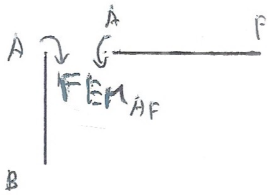
$= \frac{0.4}{3} : \frac{1}{4} : \frac{1}{6}$

$= 8 : 15 : 10$

Let $k_{column} = 8k$, $k_{4\text{-}m\ beam} = 15k$, $k_{6\text{-}m\ beam} = 10k$

For Column AB,

$N_{Ed} = \frac{1}{2}(1.35G_k + 1.5Q_k)(4m) = \frac{1}{2}(48.6\ kN/m)(4m) = 97.2\ kN$



$FEM_{AF} = \frac{WL^2}{12} = \frac{(48.6\ kN/m)(4m)^2}{12} = 64.8\ kN\text{-}m$ (↻ at column AB)

$\Sigma k = k_{column} + \frac{1}{2} k_{4\text{-}m\ beam} = 8k + \frac{1}{2}(15k) = 15.5k$

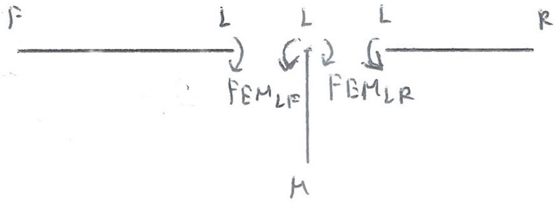
$M_{Ed} = (64.8\ kN\text{-}m)\frac{k_{column}}{\Sigma k} = (64.8\ kN\text{-}m)\frac{8k}{15.5k} = 33.45\ kN\text{-}m$ (↻ at column AB)

$M_{Ed,min} = N_{Ed} \cdot e_{min} = (97.2\ kN) \cdot Max\left\{\frac{h}{30}; 20\right\} = (97.2\ kN)(20mm) = 1.944\ kN\text{-}m < M_{Ed}$ (Ok!)
$(h = 300\ mm)$

For Column LM,

$N_{Ed} = \frac{1}{2}(1.35G_k + 1.5Q_k)(6m) + \frac{1}{2}(1.35G_k)(4m)$

$= (3m)(48.6\ kN/m) + (2m)(21.6\ kN/m) = 189\ kN$



$FEM_{LF} = \frac{WL^2}{12} = \frac{(48.6\ kN/m)(6m)^2}{12} = 145.8\ kN\text{-}m$ (↺ at column LM)

$FEM_{LR} = \frac{WL^2}{12} = \frac{(21.6\ kN/m)(4m)^2}{12} = 28.8\ kN\text{-}m$ (↻ at column LM)

$\Sigma k = k_{column} + \frac{1}{2} k_{4\text{-}m\ beam} + \frac{1}{2} k_{6\text{-}m\ beam}$

$= 8k + \frac{1}{2}(15k) + \frac{1}{2}(10k) = 20.5k$

$M_{Ed} = (145.8 - 28.8\ kN\text{-}m)\frac{k_{column}}{\Sigma k} = (117\ kN\text{-}m)\frac{8k}{20.5k}$

$= 45.66\ kN\text{-}m$ (↺ at column LM)

$M_{Ed} > M_{Ed,min} = 1.944\ kN\text{-}m$ (Ok!)