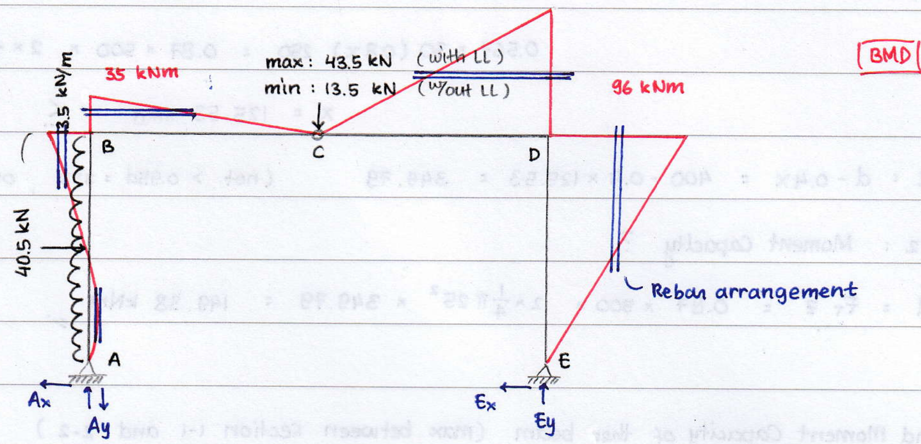
CV3011 Reinforced Concrete Design PYP AY 16/17

1. a.



#1. Whole Frame FBD.

$\Sigma M_A = 0$ [Max case] $\Sigma F_y = 0$

$40.5 \times 1.5 + 43.5 \times 3 = 6 \times E_y$ $\quad 43.5 = 31.875 + A_y(\uparrow)$

$\therefore E_y = 31.875$ kN $\quad \therefore A_y(\uparrow) = 11.625$ kN

$\Sigma M_A = 0$ [Min case] $\Sigma F_y = 0$

$40.5 \times 1.5 + 13.5 \times 3 = 6 \times E_y$ $\quad 13.5 + A_y(\downarrow) = 16.875$

$E_y = 16.875$ kN $\quad \therefore A_y(\downarrow) = 3.375$ kN

b. #2. Member CDE $\quad \Sigma F_x = 0$

$\Sigma M_C = 0$ $\quad 40.5 = 31.875 + A_x$

$3 \times 31.875 = 3 E_x$ $\quad \therefore A_x = 8.625$ kN

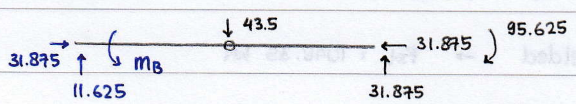
$\therefore E_x = 31.875$ kN

#3. Member DE

$\Sigma M_D = 0$

$M_D = 3 \times 31.875 = 95.625$ kNm (↶)

#4. Member BCD



$\Sigma M_B = 0$

$M_B + 31.875 \times 6 = 43.5 \times 3 + 95.625$

$M_B = 34.875$ kNm (↶)

c. Check 1 : Ductile Failure Mode ?

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I also keep my answers for 3-4 former years, you may ask if you want.

" Do your best, prepare for the worst
— then trust God to bring victory. "

Proverbs 21 : 31 (MSG)

Assume Tension steel has yielded , $F_c = F_T$

$$0.567\,f_{ck}\,(0.8x)\,b = 0.87 f_{yk} A_s$$

$$0.567 \times 30\,(0.8x)\,250 = 0.87 \times 500 \times 2 \times \frac{1}{4}\pi 25^2$$

$$x = 125.53 \text{ mm} \leq 0.617d = 246.8 \text{ mm}$$ ductile!

$z = d - 0.4x = 400 - 0.4 \times 125.53 = 349.79$ (not $> 0.95d = 380$, ok!)

Check 2 : Moment Capacity ?

$M = F_T z = 0.87 \times 500 \times 2 \times \frac{1}{4}\pi 25^2 \times 349.79 = 149.38$ kNm $> M_{max} = 96$ kNm ok!

2. a. #1. Find Moment Capacity of the beam (max between section 1-1 and 2-2)

⇒ For Section 1-1 (Sagging)

Check 1 : Ductile Failure Mode

Assume NA lies below the flange i.e. $x > h_f$

→ $F_{cf} = 0.567\,f_{ck}\,(b - b_w)\,h_f = 0.567 \times 30\,(500 - 250) \times 150 = 637.875$ kN

→ $F_{cw} = 0.567\,f_{ck}\,b_w\,(0.8x) = 0.567 \times 30 \times 250\,(0.8x) = 3402x$ kN

$d'/d = 30/510 = 0.059 < 0.171$ → comp. steel has yielded.

→ $F_{sc} = 0.87 f_{yk} A_{sc} = 0.87 \times 500 \times 1010 = 439.35$ kN

Assume Tension steel has yielded

→ $F_{st} = 0.87 f_{yk} A_s = 0.87 \times 500 \times 2410 = 1048.35$ kN

⇔ By Eqm : $F_{cf} + F_{cw} + F_{sc} = F_{st}$

$$637.875 + 3.402x + 439.35 = 1048.35$$

$x = -8.49$ mm ⇒ assump. of NA is Incorrect !!

↳ Now try NA lies on the flange i.e. $x < h_f$

→ $F_c = 0.567\,f_{ck}\,b\,(0.8x) = 0.567 \times 30 \times 500 \times 0.8x = 6.804x$ kN

→ [comp. steel has yielded] $F_{sc} = 439.35$ kN

Assume Tension steel has yielded → $F_{st} = 1048.35$ kN

⇔ By Eqm : $6.804x + 439.35 = 1048.35$

$x = 89.51$ mm $< 0.617d = 314.67$ mm Ten. steel yields ok! (Ductile)

Check 2 : Moment Capacity

$M = F_{st}\,(d - 0.4x) = 1048.35\,(510 - 0.4 \times 89.51) = 497.12$ kNm

⇒ For Section 2-2 (Hogging)

Check 1 : Ductile Failure Mode

$d'/d = 40/520 = 0.077 \quad < 0.171 \rightarrow$ comp. steel has yielded.

By Eqm : $F_c + F_{sc} = F_{st}$

$0.567 f_{ck} b_w (0.8x) + 0.567 f_{yk} A_{sc} = 0.567 f_{yk} A_s$

$0.567 \times 30 \times 250 (0.8x) + 0.567 \times 500 \times 402 = 0.567 \times 500 \times 1570$

$x = 97.33$ mm $\quad < 0.617d = 320.84$ mm $\quad$ OK!

Check 2 : Moment Capacity

$M = F_{st}(d - 0.4x) = 0.567 \times 500 \times 1570 (520 - 0.4 \times 97.33) = 214.12$ kNm

$\therefore M_{max}$ (critical) is at Section 1-1 $= 497.12$ kNm

#2. Max udl q_k

$\rightarrow$ self-weight $= (0.5 \times 0.15 + 0.4 \times 0.25) \times 24 = 4.2$ kN/m

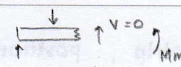
$\rightarrow w = 1.35 \times 4.2 + 1.5 q_k = 5.67 + 1.5 q_k$ kN/m

$\rightarrow$ Load $= (5.67 + 1.5 q_k) \times 8 = 45.36 + 12 q_k$ kN

$\Sigma M_2 = 0$

$6 A_y = 2(45.36 + 12 q_k)$

$A_y = 15.12 + 4 q_k$ kN

Where is the position of M_{max} ? where $V = 0$  $\Sigma F_y = 0$

$15.12 + 4q_k = (5.67 + 1.5 q_k) \times x$

$x = \dfrac{15.12 + 4q_k}{5.67 + 1.5q_k} = \dfrac{8}{3}$

Thus, $\Sigma M_1 = 0$

$(15.12 + 4q_k)\frac{8}{3} = (5.67 + 1.5 q_k) \frac{1}{2} \times \left(\frac{8}{3}\right)^2 + 497.12$

$\therefore q_k = 89.43$ kN/m

b. At support face, $V_{Ef} = A_y = 15.12 + 4 \times 89.43 = 372.84$ kN

$V_{Rd,max(22)} = 0.124\, b_w d (1 - f_{ck}/250) f_{ck} = 0.124 \times 250 \times 510 (1 - 30/250) \times 30 = 417.384$ kN

Since $V_{Rd,max(22)} > V_{Ef}$, let $\theta = 22°$ and $\cot\theta = 2.5$

For stirrup spacing $s = 250$ and $A_{sw} = 2 \times \frac{1}{4}\pi 8^2 = 101$ mm^2,

$V_{Rd,s} = A_{sw}/s\, (0.78d) f_{yk} \cot\theta = 101/250 \times 0.78 \times 510 \times 500 \times 2.5 = 199.96$ kN

3. a. #1. Slab Division and Moments

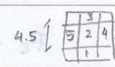
Design load $n = 1.35 \times 6.5 + 1.5 \times 2.23 = 12.12$ kN/m^2

Corner with two adjacent discontinuous edges $\Rightarrow$ case 4 with $l_y/l_x = 1$ (square)

$\hookrightarrow \beta_{sx} = 0.036$; $\beta_{sx}' = 0.047$

Assume use $\phi 10$ bar , $d_x = 190 - 30 - \frac{10}{2} = 155$ mm $d_y = 155 - 10 = 145$ mm

#2. Bending Reinforcement

→ Middle strip , position 1,2,4 width = 4.5 m 

- Midspan (2) : $M_{sx} = \beta_{sx}\, n\, l_x^2 = 0.036 \times 12.12 \times 6^2 = 15.7$ kNm/m

$K = M / bd^2 f_{ck} = 15.7 / 1000 \times 155^2 \times 35 = 0.019$

$z = (0.5 + \sqrt{0.25 - K/1.134})d = (0.5 + \sqrt{0.25 - 0.019/1.134}) \times 155 = 155$ mm

but $z \leq 0.95d = 147$ mm ⇒ $z = 147$ mm

$A_{s,req} = M / 0.87 f_{yk} z = 15.7 / 0.87 \times 500 \times 147 = 245$ mm^2

#Check $A_{s,min} = 0.17\%\, bd = 0.17\% \times 1000 \times 155 = 258$ mm^2 # ~ more precise : $0.26 \frac{f_{ctm}}{f_{yk}}$ $\simeq 0.17\%$

$S_{max} = 2h = 2 \times 190 = 380 \leq 250$ mm so $S_{max} = 250$ mm #.

∴ $A_{s,prov} = 314$ mm^2 (H10 - 250)

- At support (1,4) : $M_{sx}' = \beta_{sx}'\, n\, l_x^2 = 0.047 \times 12.12 \times 6^2 = 20.5$ kNm/m

$K = 20.5 / 1000 \times 155^2 \times 35 = 0.024$

$z = (0.5 + \sqrt{0.25 - 0.024/1.134})\, 155 = 155$ mm → take $z = 147$ mm .

$A_{s,req} = 20.5 / 0.87 \times 500 \times 147 = 320$ mm^2

∴ $A_{s,prov} = 393$ mm^2 (H10 - 200)

→ Middle strip , position 3,5 (discontinuous edge) → Top steel

Top steel = $0.25\, A_{sx} = 0.25 \times 245 = 61$ mm^2 but ≥ the min. reo = 258 mm^2 .

$S_{max} = 3h = 3 \times 190 = 570 \leq 400$ mm so $S_{max} = 400$ mm

↳ $A_{s,prov} = 262$ mm^2 (H10 - 300)

→ Edge Strip

Provide min. reo (258 mm^2) → $A_{s,prov} = 262$ mm^2 (H10 - 300)

#3. Torsion Reinforcement

(of length equals to $l_x/5$ (= 1.2 m) are needed at Corner X and Y)

→ Corner X : $A_s = 3/4\, A_{sx} = 0.75 \times 245 = 184$ mm^2/m width

Total area of steel = $184 \times 1.2 = 220$ mm^2 ⇒ 4H10 (314 mm^2)

→ Corner Y : $A_s = 3/8\, A_{sx} = 92$ mm^2/m width

Total area of steel = $92 \times 1.2 = 110$ mm^2 ⇒ 2H10 (157 mm^2)

8. #4. Deflection Check

Take the smallest d → $d_y = 145$ mm (conservative)

$\rho_{(req)} = A_{s,req}/bd \times 100\% = 245 / 1000 \times 145 \times 100\% = 0.17\% < 0.35\%$

↳ basic $l/d = 39$ (End span, Table 5.8 IStructE)

↳ Allowable $l/d = 39 \times A_{s,prov} / A_{s,req} = 39 \times 314/245 = 50$

Actual $l/d = 6000/145 = 41.4 < 50$ ok!

#5. Crack Control

$$\sigma_s = \frac{f_{yk}}{\gamma_{ms}} \times \frac{\psi_2 Q_k + G_k}{1.5 Q_k + 1.35 G_k} \times \frac{A_{s,req}}{A_{s,prov}} \times \frac{1}{\delta}$$

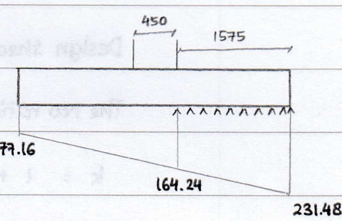
$$= \frac{500}{1.15} \times \frac{\times 2.23 + 6.5}{1.5 \times 2.23 + 1.35 \times 6.5} \times \frac{245}{314} \times \frac{1}{1} = \quad \text{MPa}$$

✓ Bar $\phi \leq 32$ mm

OK.

✓ S_{max} is considered

4. a. #1. Check Base Area (SLS)



$$P_1 = \frac{N+W}{A} + \frac{6M}{BL^2} \leq 230 \ ?$$

$$\frac{1600}{3.6^2} + 0.8 \times 25 + \frac{6 \times 500}{3.6^3} \ ... \ 230$$

$207.76 < 230$ ok!

Middle third rule : $L > 6 \times M/N = 6 \times 500/1600 = 1.875$ ok!

$\therefore$ $3.6 \times 3.6 \times 0.8$ size is okay.

#2. Design for Bending Reinforcement

$$\text{Design Pressure} = \frac{N_u}{L^2} \pm \frac{6M_u}{L^3} = \frac{2000}{3.6^2} \pm \frac{6 \times 600}{3.6^3} = \begin{cases} 231.48 \text{ kPa} \\ 77.16 \text{ kPa} \end{cases}$$

$d = 800 - 70 - 8 = 722$ mm (Assume using $\phi 16$)

Find eqn for Pressure $P(x)$: $\dfrac{y - 77.16}{231.48 - 77.16} = \dfrac{x - 0}{3600 - 0}$

$P(x) = 0.043x + 77.16$

↳ So, $P(1575 + 450) = 0.043 \times 2025 + 77.16 = 164.24$ kPa

$M = 164.24 \times 1.575 \times 3.6 \times (1.575/2) + \frac{1}{2} \times 1.575 \ (231.48 - 164.24) \times 3.6 \times (\frac{2}{3} \times 1.575)$

$= 933.51$ kNm

$K = M/f_{ck}bd^2 = 933.51 \times 10^6 / 30 \times 3600 \times 722^2 = 0.017$

$z = 0.95d = 0.95 \times 722 = 685.9$ mm → actually once $k \leq 0.05$, use $z = 0.95d$

$A_{s,req} = M/0.87 f_{yk} z = 933.51 \times 10^6 / 0.87 \times 500 \times 685.9 = 3129 \text{ mm}^2$

Check $A_{s,min} = 0.0015 \times 3600 \times 722 = 3899 \text{ mm}^2$ ✓

↳ Provide 20H16 ($A_{s,prov} = 4021 \text{ mm}^2$)

Check bar spacing $= (3600 - 70 \times 2 - 16) / 20 = 172$ mm $\leq$ min (2h ; 250) ok!

b. #3. Check Shear Resistance (ULS)

$d_x = 800 - 70 - 8 = 722$ mm $= d$
$d_m = 722 - 8 = 714$ mm
$d_y = 714 - 8 = 706$ mm

→ Max shear at Col. perimeter

$V_{Rd,max} = 0.5 u_0 d_m (0.6(1 - f_{ck}/250)) f_{ck}/1.5$

$= 0.5 (4 \times 450) 714 \times 0.6 (1 - \frac{30}{250}) \frac{30}{1.5}$

$= 6786$ kN $> V_{Ed} = N_u = 2000$ kN ok!

→ Vertical Shear at 1.0d from col. face (one-way shear → use d_x)

$P(2025 + 722) = 0.043 \times 2747 + 77.16 = 195.28$ kPa

Design shear $V_{Ed} = \frac{1}{2}(195.28 + 231.48)(1.575 - 0.722) \times 3.6 = 655$ kN

The reo ratio $\rho_x = 4021 / 3600 \times 722 \times 100\% = 0.155\% < 2\%$

$k = 1 + \sqrt{200/d} = 1 + \sqrt{200/722} = 1.53$

<=> $v_{Rd,c} = \frac{0.18}{\gamma_c} k (100 \rho_x f_{ck})^{1/3} = 0.12 \times 1.53 (0.155 \times 30)^{1/3} = 0.306$

But $v_{min} = 0.035 k^{3/2} f_{ck}^{1/2} = 0.035 \times 1.53^{3/2} \times 30^{1/2} = 0.363$ ✓

$V_{Rd,c} = v_{Rd,c} \times bd = 0.363 \times 3600 \times 722 = 943$ kN $> V_{Ed} = 655$ kN

→ Punching shear at 2.0d from col. face (two-way shear → use dm)

Critical perimeter u_1 = column perimeter $+ 4\pi d_m = 4 \times 450 + 4\pi \times 714 = 10772$ mm

Area within critical perimeter $= (c + 4d_m)^2 - (4 - \pi)(2d_m)^2 = (450 + 4 \times 714)^2 - (4 - \pi)(2 \times 714)^2$

$= 9179185 \text{ mm}^2$

Punching force $V_{Ed} = \frac{231.48 + 77.16}{2} \times (3.6^2 - 9.18) = 583$ kN

Average reo. ratio $\rho_l = 4021 / 3600 \times 714 \times 100\% = 0.156\% < 2\%$

$k = 1 + \sqrt{200/d_m} = 1 + \sqrt{200/714} = 1.53$

<=> $v_{Rd,c} = \frac{0.18}{1.5} \times 1.53 (0.156 \times 30)^{1/3} = 0.307$

But $v_{min} = 0.035 \times 1.53^{3/2} \times 30^{1/2} = 0.363$ ✓

$V_{Rd,c} = v_{Rd,c} \times u_1 d_m = 0.363 \times 10772 \times 714 = 2792$ kN $> V_{Ed} = 583$ kN.

∴ All Shear Resistance is ADEQUATE.