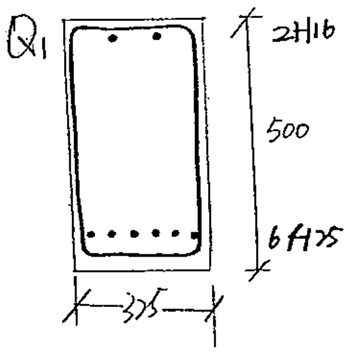
Nanyang Technological University. CV3011 RC Design SEM I 2013-2014.

Q1



As the moment reduction factor $\delta = 0.9$.
the limiting depth of the neutral axis is

$$x = (\delta - 0.4)d = (0.9 - 0.4) \times 445 = 222.5 \text{ mm}$$

$$d = D - \text{cover} = 500 - 10 - 45 = 445 \text{ mm}$$

$$K_{bal} = 0.142.$$

$$K = \frac{M}{bd^2 f_{ck}} = \frac{350}{325 \times 445^2 \times 32} = 0.1699 > K_{bal}$$

Therefore, compression steel is required.

Check: $d'/d = \frac{55}{500} = 0.11 < 0.14$

Therefore $f_{sc} = \cancel{0.87 f_{yk}}$

compression steel:

$$A_s' = \frac{(K - K_{bal}) f_{ck} bd^2}{f_{sc}(d - d')} = \frac{(0.1699 - 0.142) \times 32 \times 325 \times 445^2}{0.87 \times 500 (445 - 55)}$$

$$= 338.7 \text{ mm}^2$$

Provide 2 H16 bars for A_s', area $= 402 \text{ mm}^2$.

Tension steel:

$$A_s = \frac{K_{bal} f_{ck} bd^2}{0.87 f_{yk} z} + A_s' \frac{f_{sc}}{0.87 f_{yk}}, \quad z = d - 0.8x/2$$

$$= 445 - 0.8 \times 222.5/2$$

$$= 356 \text{ mm}$$

$$= \frac{0.142 \times 32 \times 325 \times 445^2}{0.87 \times 500 \times 356} + 338.7$$

$$= 2227.14 \text{ mm}^2$$

Provide 6 H25 bars for A_s, area $= 2946 \text{ mm}^2$

$$P = 100 \frac{A_s}{bd} = 100 \times \frac{2946}{325 \times 445} = 2.04\% \begin{matrix} > (\text{Min } 0.13\%) \text{ OK!} \\ < (\text{Max } 4\%) \text{ OK!} \end{matrix}$$

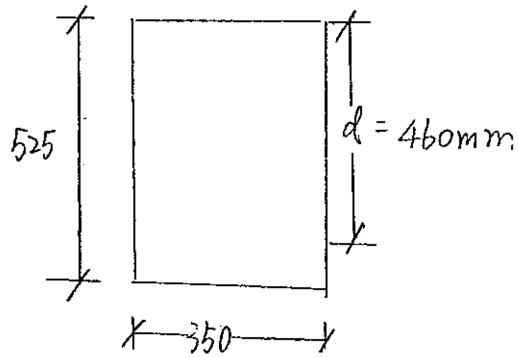
2. Ultimate Limit state.

DL : 58 KN/m × 1.35 = 78.3 KN/m

LL : 68 KN/m × 1.5 = 102 KN/m

Total Ultimate load on beam = (78.3+102)×6.8

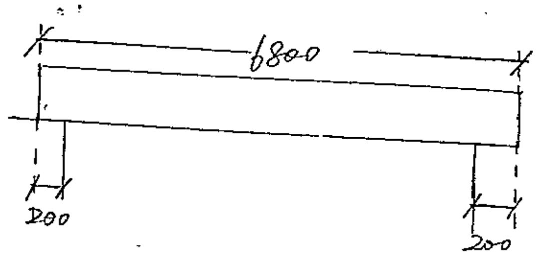
= 1226.04 KN



Shear at the Support face

$V_{Ef} = 1226.04/2 - (78.3+102)\times 0.2$

$= 576.96 KN$



Shear at distance 1d from face of Support.

$V_{Ed} = 1226.04/2 - (78.3+102)\times(0.2+0.46) = 494.022 KN$

check crushing strength $V_{Rd,max}$ of the concrete diagonal strut at the face of Support V_{Ef}.

For $\theta=22^\circ$ $V_{Rd\,max(22)} = 0.124 b_w d (1-f_{ck}/250) f_{ck}$

$= 0.124 \times 350 \times 460 (1-25/250) \cdot 25$

$= 449.2 KN \quad (< V_{Ef})$

For $\theta=45^\circ$ $V_{Rd\,max(45)} = 0.18 b_w d (1-f_{ck}/250) f_{ck}$

$= 0.18 \times 350 \times 460 (1-25/250) 25$

$= 652.05 KN \quad (> V_{Ef})$.

Determine Angle θ.

For $22^\circ < \theta < 45^\circ$ $\theta = 0.5 \sin^{-1}\left\{\frac{V_{Ef}}{V_{Rd\,max45}}\right\} = 0.5 \sin^{-1}\left\{\frac{576.96}{652.05}\right\} = 31^\circ$

$\cot\theta = 1.657$

Stirrup Design.

$$\frac{A_{sw2}}{S_2} = \frac{V_{Ed}}{0.78 d f_{yk} \cot\theta} = \frac{494.022 \times 10^3}{0.78 \times 460 \times 500 \times 1.657} = 1.662$$

From Table A.4, use ϕ 13 mm stirrups at 150 mm spacing. $A_{sw}/s,prov = 1.770$.

minimum stirrups. $V_2 = 0.0624 \,(f_{ck}^{0.5} \cdot b_w d) \cot\theta = 0.0624 \,[(\sqrt{25}) \times 350 \times 460] \times 1.65$

$= 83.5$ KN

$$\frac{A_{sw3}}{S_3} = \frac{V_2}{0.78 d f_{yk} \cot\theta} = \frac{83.5}{0.78 \times 460 \times 500 \times 1.657} = 0.281$$

From Table, using ϕ 13 mm stirrups at 400 mm spacing. $A_s/s, prov = 0.664$

The provided shear resistance of stirrups is =

$$V_2 = \frac{A_{sw}}{s} \times 0.78 d f_{yk} \cot\theta = 0.664 \times 0.78 \times 460 \times 500 \times 1.657 = 197.4 \text{ KN}$$

Extent of shear stirrups.

$$X_2 = (R - V_2)/W = (1226.04/2 - 197.4)/(178.3 + 102) = 2.31 \text{ m.}$$

measured from the face of support.

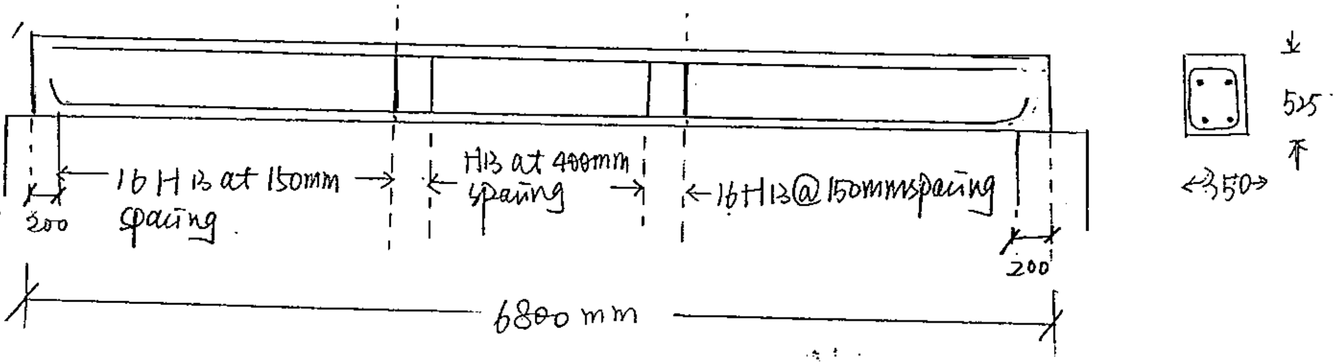
$$X = 2.31 - 0.2 = 2.11 \text{ m.}$$

Therefore, the number of H13 stirrups at 150mm spacing required at each end of the beam is $(1 + \frac{X}{S}) = 16$ spaced at a distance of $(16-1) \times 150 = 2250$ mm.

Additional longitudinal reinforcement.

$$F_{td} = 0.5 V_{Ed} \cot\theta = 0.5 \times 494.02 \times 1.657 = 409.3 \text{ KN.}$$

This additional longitudinal tensile force is provided for by extending the curtailment point of the mid-span longitudinal reinforcement.



[Question 2. Reinforcement Details.]

END of Question 2

Question 3.

(a). Load. $DL = 25 \times 0.22 = 5.5\ KN/m^2$

$LL = 4 KN/m^2$

→ Design Load:

$W_L = 1.35 \times 5.5 KN/m^2 + 4KN/m^2 \times 1.5$

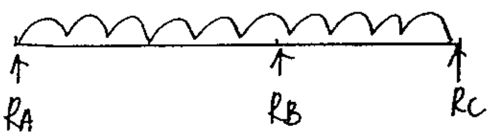
$= 13.425\ KN/m^2$

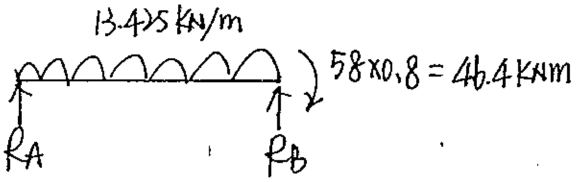
Single load case analysis is applied and take a 1m wide strip.

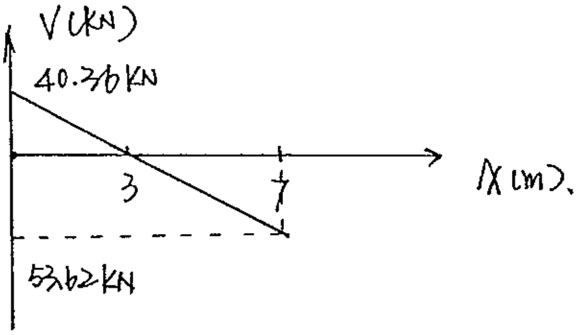
Moment at the interior support is 58KN/m

According to EC2, 20% of this moment should be redistributed.

Flexual Design.







Taking moment about A.

$13.425 \times 7 \times 3.5 + 46.4 = R_B \cdot 7$.

$R_B = 53.62\ KN$.

$R_A = 13.425 \times 7 - 53.62 = 40.36\ KN$

$X_A = 40.36 \div W_L = 40.36 \div 13.425 = 3m$.

$M_{max} = 40.36 \times \frac{1}{2} \times 3 = 60.7\ KNm$.

Given that $d = 182$ mm (effective depth).

$$K = \frac{M}{f_{ck} b d^2} = \frac{60.7 \times 10^6}{25 \times 1000 \times 182^2} = 0.073$$

$$Z = d\left\{0.5 + \sqrt{0.25 - K/1.134}\right\} = d\left\{0.5 + \sqrt{0.25 - \frac{0.073}{1.134}}\right\} = 0.931d < 0.95d$$

$$= 0.931 \times 182mm = 169.41\ mm.$$

$$A_s = M/0.87 f_{yk} \cdot z = 60.7 \times 10^6 / 0.87 \times 500 \times 169.41 = 824\ mm^2/m.$$

Therefore, we use H13 at 150mm spacing. $A_{s,prov} = 885\ mm^2/m$.

(b) Deflection Check.

$\rho_0 = 10^{-3} \cdot \sqrt{f_{ck}} = 0.005$, $\rho = A_{s,req}/bd = 824/1000 \times 182 = 0.453\% < \rho_0$

$= 0.5\%$

$K = 1.3$ for Simply Supported slabs.

The basic span/depth ratio :

$$\frac{l}{d} = K\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_0}{\rho} + 3.2\sqrt{f_{ck}}\left(\frac{\rho_0}{\rho} - 1\right)^{3/2}\right]$$

$$= 1.3\left[11 + 1.5\sqrt{25}\frac{0.005}{0.00453} + 3.2\sqrt{25}\left(\frac{0.005}{0.00453} - 1\right)^{3/2}\right]$$

$$= 25.76$$

Allowable $l/d = 25.76 \times A_{s,prov}/A_{s,req} = 25.76 \times 885/824 = 27.6635$

Actual $l/d = 7000/182 = 38.5 > 27.66$

Not OK. Add more tension reinforcement.

$38.5 = MF \times 25.76 \Rightarrow MF = 1.495$

$1.495 \times 824 = 1232\ mm^2 \rightarrow$ Provide H16 at 150 spacing.

$A_{s,prov} = 1340\ mm^2/m$

check again

Allowable $l/d = 25.76 \times 1340/824 = 41.9$

Actual $l/d = 7000/182 = 38.5 < 41.9 \rightarrow ok.$

Use. H16 – 150 (H16mm at 150mm spacing).

[END OF QUESTION 3].

Question 4.

(a). $e_{min} = \max(h/30 ; 20mm) = \max(\frac{400}{30} ; 20) = 20mm$

Minimum moment $= Ne_{min} = 2250 \times 0.02 = 45 KNm$

Use $M = 304 KNm$ for design.

Using H32 bars.

$d = 400 - 60 = 340mm$

$d/h = 340/400 = 0.85$

Longitudinal reinforcement

$N/bhf_{ck} = 10^3 \times 2250 / 300 \times 400 \times 32 = 0.586$

$M/bh^2 f_{ck} = 10^6 \times 304 / 300 \times 400^2 \times 32 = 0.198$

From design chart $A_s f_{yk} / bhf_{ck} = 0.7$

$A_s = 0.7 \times 300 \times 400 \times 32 / 500 = 5376 mm^2$

provide. 8 H32 bars. $A_{s,prov} = 6437 mm^2$.

Q4 (b). $A_s = A_s' = 2414 \ mm^2$

Consider 3 modes of failure.

1. Pure axial compression. $M=0$.

$N_0 = 0.567 f_{ck} A_{col} + 0.87 f_{yk} A_{sc}$

$= 0.567 \times 32 \times (300 \times 400) + 0.87 \times 500 \times 4828 = 6455 KN$

$\rightarrow$ point (0, 6455).

2. Balanced Condition of failure.

By definition, $\varepsilon_{cu} = 0.0035$

$\varepsilon_y = 0.87 \times 500 / 2 \times 10^5 = 0.00217$

$$X_{bal} = \frac{d}{1 + \frac{\varepsilon_y}{0.0035}} = 340 \times \frac{0.0035}{0.00567} = 210 mm$$

check if the compression steel has yielded.

$$\varepsilon_{sc} = 0.0035 \left(\frac{X - d'}{X}\right) = 0.0035 \times \frac{210 - 60}{210} = 0.0025 > \varepsilon_y$$

$\rightarrow$ yield

Calculate the internal forces corresponding to $X_{bal} = 210 mm$.

$F_{s1} = F_{s2} = 0.87 f_{yk} \times \frac{1}{2} A_s = 0.87 \times 500 \times \frac{1}{2} \times 4828 = 1050.09 KN$

$F_c = 0.567 f_{ck} \times 300 \times 0.8 X_b$

$= 0.567 \times 32 \times 300 \times 0.8 \times 210 = 914.5 KN$

$N = N_{bal} = F_c + F_{s1} - F_{s2} = 914.5 KN$

Taking moment about the centroidal axis:

$M_{bal} = F_c (200 - 0.4 \times 210) + (F_{s1} + F_{s2})(340 - 200)$

$= 914.5 \times (200 - 0.4 \times 210) + 2 \times 1050.09 \times (340 - 200)$

$= 400 KNm$

So, $(M_{bal}, N_{bal}) = (400, 914.5)$.

3. Tension failure.

Assume $x = 150mm$, (x should be less than x_{bal}).

$$\varepsilon_{s1} = \frac{150-60}{150} \times 0.0035 = 0.0021 \longrightarrow \text{yielded.}$$

$$F_{s1} = F_{s2} = 0.87 f_{yk} \times 2414/1000 = 0.87 \times 500 \times 2414/10^3 = 1050.1 \ KN$$

$$F_c = 0.567 \times 32 \times 300 \times (0.8 \times 150)/10^3 = 653.184 \ KN$$

Calculate (N, M), $N = F_c + F_{s1} - F_{s2} = 653 \cdot KN$ ...

Taking moment about the centroidal axis.

$$M = F_c(200 - 0.4 \times 150) + (F_{s1} + F_{s2}) \times (340-200).$$

$$= 653.184\ (200 - 150 \times 0.4) + (2 \times 1050.09) \times (340-200).$$

$$= 385.5 \ KNm.$$

Point 3 (385.5 , 653.2).

4. Pure Bending. (N=0).

Equilibrium Equation (at the ultimate limit state).

$$\Sigma F = 0. \quad F_c + F_{s1} = F_{s2}$$

Taking moment about the centre line of tension steel.

$$M = F_c(d - 0.4x) + F_{s1}(d - d')$$

Assume the compression steel has <u>not</u> yielded.

$$\varepsilon_{s1} = \frac{x-60}{x} \times 0.0035$$

$$f_{s1} = \varepsilon_{s1} E_s = \frac{x-60}{x} \cdot 0.0035 \cdot 200 \times 10^3 = \frac{x-60}{x} \cdot 700$$

$$\Sigma F = 0. \quad 0.567 \times 32 \times 300 \times (0.8x) + f_{s1} \times 2414 = 0.87 f_{yk} \times 2414$$

$$x = 95.8mm.$$

$f_{s1} = 261.6$. N/mm^2 $\quad F_c = 0.567 \times 32 \times 300 \times 0.8 \times 95.8 = 417.17 KN$

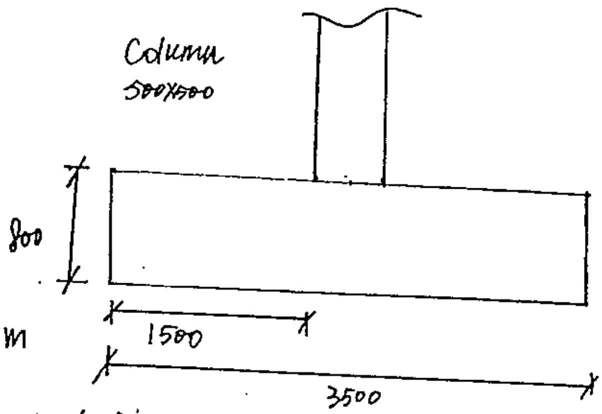
$F_{sc} = 261.6 \times 2414 = 631.5 \ KN$

Taking moment about tension steel.

$$M = F_c(d - 0.4x) + F_{s1}(d - 60) = 417.17 \times (340 - 0.4 \times 95.8) + 631.5\ (340-60) = 302.66 \ KNm$$

Point 4 (303, 0).

Question 5.



$$P_1 = \frac{N+W}{A} + \frac{6M}{BL^2} = \frac{1500}{3.5^2} + 0.8\times 25 + \frac{6\times 400}{3.5^3}$$

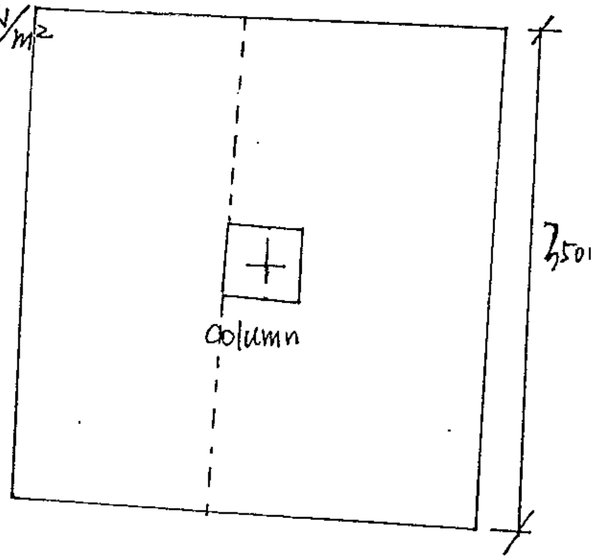
$$= 198.43\ KN/m^2 < 250\ kpa.$$

$\rightarrow$ ok.

Reinforcement Design. $N_u = 1800KN, M_u = 480KNm$

Design Pressure

$$\frac{N_u}{L^2} \pm \frac{6M_u}{L^3} = \frac{1800}{3.5^2} \pm \frac{6\times 480}{3.5^3} = \begin{cases} 214.11\ KN/m^2 \\ 79.77\ KN/m^2 \end{cases}$$



$$P_0 = 79.77 + \frac{2000}{3500}(214.11 - 79.77) = 156.54\ KN$$

$$d = 800 - 60 - \frac{1}{2}\times 10 = 730\ mm$$

$$M = \frac{1}{2}\times 1.5\times 156.54\times 3.5\times\left(\frac{1.5}{3}\right)$$

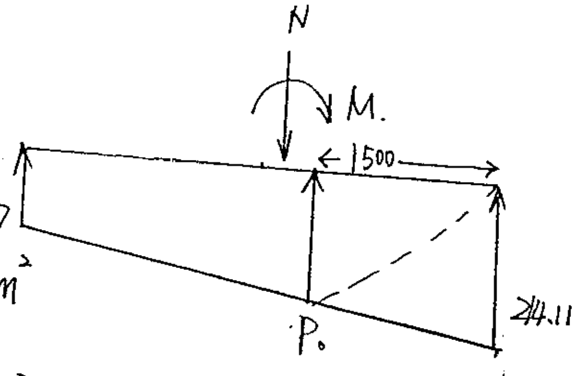
$$+ \frac{1}{2}\times 1.5\times 214.11\times 3.5\times\left(\frac{2}{3}\times 1.5\right)$$

$$= 767.5\ KNm$$

$$K = \frac{M}{f_{ck}bd^2} = \frac{767.5\times 10^6}{30\times 3500\times 730^2} = 0.0137$$

$$z = d\left\{0.5 + \sqrt{0.25 - \frac{K}{1.134}}\right\} = 0.988d > 0.95d$$

$$\rightarrow z = 0.95d = 0.95\times 730 = 693.5\ mm$$



$$A_{s,req} = \frac{M}{0.87 f_{yk} z} = \frac{767.5\times 10^6}{0.87\times 500\times 693.5} = 2544.15\ mm^2$$

$\therefore$ provide 10H20 ($3143\ mm^2$) in each direction.

$$\text{check bar spacing} = \frac{3500 - 60\times 2 - 20}{9} = 373.33 < \max(3\times 800; 400).\ O.K.$$

check shear resistance.

Max. shear at the column perimeter.

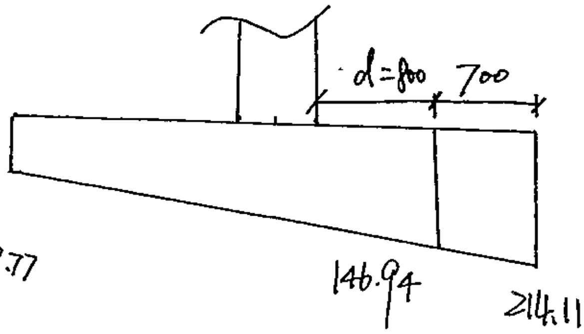
$$d_m = 730 - 10 = 720\ mm$$

Maxim shear resistance $V_{Rd,max}$.

$$V_{Rd,max} = 0.5\,ud\left[0.6\left(1-\frac{f_{ck}}{250}\right)\right]\frac{f_{ck}}{1.5} = 0.5\,(4\times 500)\times 720\left[0.6\left(1-\frac{30}{250}\right)\right]\frac{30}{1.5}\times 10^{-3}$$

$$= 7603.2\ kN > 1800\,kN$$

Vertical shear at 1.0d from the column face.



Design shear (V_{Ed}). $= \dfrac{146.94 + 214.11}{2} \times 3.5 \times 0.7$

$= 442.23\,kN.$

$V_{Ed} = 214.11 - (214.11 - 79.77) \times \dfrac{700}{3500}$

$= 146.94\,kN$

The reinforcement ratio $\rho = \dfrac{3143}{3500\times 730} \times 100\%$

$= 0.123\ \% < 2\%$

The shear resistance of the concrete without shear reinforcement, $v_{Rd,c}$:

$$v_{Rd,c} = \frac{0.18}{\gamma_c} k\,(100\rho_1 f_{ck})^{1/3} = 0.12 \times \left(1+\sqrt{\frac{200}{730}}\right) \times (0.123 \times 30)^{1/3} = 0.28.$$

$$v_{min} = 0.035\,k^{3/2} f_{ck}^{1/2} = 0.035\left(1+\sqrt{\frac{200}{730}}\right)^{3/2} \times \sqrt{30} = 0.254$$

$v_{Rd,c} > v_{min}$.

$$V_{Rd,c} = v_{Rd,c} \times bd = 0.2825 \times 3500 \times \cancel{800}\,730 \times 10^{-3} = 721.77\,kN > V_{Ed} = 442.23\,kN$$

(END OF QUESTION 5).

Good Luck! ☺