NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2010-2011

CV3201 – REINFORCED CONCRETE DESIGN

December 2010 **Time Allowed: 2½ hours**

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FOUR (4)** pages.
2. Answer **ALL FOUR (4)** questions.
3. All questions carry equal marks.
4. This paper is an Open Book Examination.

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1. A rectangular beam is 400 mm wide by 600 mm overall depth. The beam is continuous through 3 spans as shown in Figure Q1. It supports the following uniformly distributed loads.

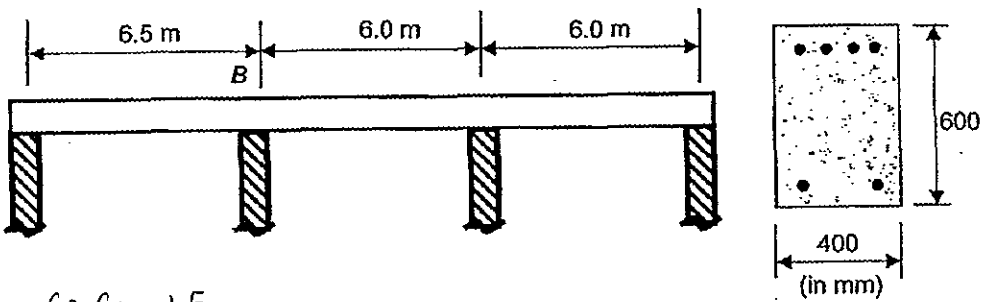
   Dead load g_k = 55 kN/m, including self weight
   Live load q_k = 50 kN/m

   The materials to be used are grade 30 concrete and grade 460 steel reinforcement bars. Cover to reinforcement bars is 40mm. Use Φ12 stirrups.

   (a) Use coefficient method to determine the critical design moments over support ***B*** (assuming which are the results of elastic analysis). Hence, design the steel bars required to resist the moment at ***B***. Apply a moment re-distribution factor β_b=0.9. Sketch the cross sections showing the reinforcement arrangement.
   (11 marks)

   (b) Repeat part (a) but applying a moment re-distribution factor β_b=0.7. Sketch the cross sections showing the reinforcement arrangement.
   (10 marks)

   (c) Explain why the moment re-distribution factor β_b is not allowed to be less than 0.7.
   (4 marks)



Factor of safety =1.5

$\frac{f_{ck}}{1.5} = 0.67 f_{ck}$

If redistribution < 70% it will fall below allowable factor of safety

Figure Q1

2. A simply supported beam having rectangular box section as shown in Figure Q2 spans 12 m (clear span). The supports are 500 mm wide. It supports the following uniformly distributed loads.

Dead load g_k = 22 kN/m (in addition to its self weight)
Live load q_k = 28 kN/m

Use concrete having cylindrical strength of 32 N/mm² and grade 460 steel for all reinforcement bars. Cover to all steel reinforcement bars is 35 mm. Use Φ10 stirrups.

(a) Design the steel bars required to resist the maximum bending moment in the beam.

(10 marks)

(b) Hence, design the shear stirrup reinforcements. Sketch the longitudinal section showing the stirrup arrangement in different zones. Assume same steel ratio throughout the beam. Use f_{yv}=250 N/mm².

(11 marks)

(c) Given a sufficient number of standard concrete cube sample tests which yield a mean strength of 55 N/mm² with a standard deviation of 6.1 N/mm². Determine the allowable compressive strength of concrete in bending having a safety factor γ_m=1.5, and the allowable yield strength of grade 460 steel reinforcement bars having a safety factor γ_m=1.05.

(4 marks)

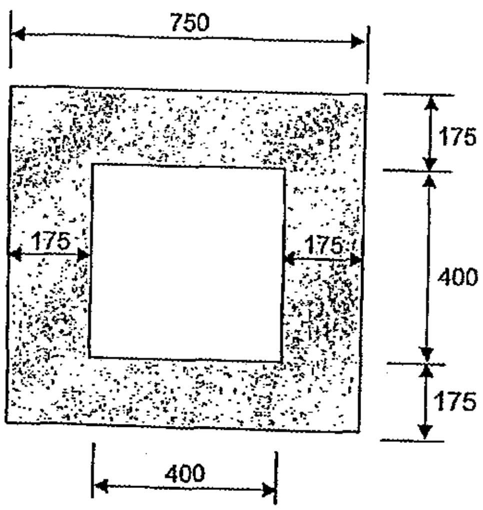


Figure Q2

(All dimensions are in mm)

two way

3. Figure Q3 shows the structural plan of a beam-slab floor system. The thickness of the slab is 160 mm and it is reinforced with 10 mm bottom bars at 200 mm spacing in both X and Y directions in the sagging moment zones and similar reinforcement at the top of the slab in the hogging moment zones or above the beams.

(a) For f_{cu} = 30 N/mm^2 and f_y = 460 N/mm^2, determine the moment of resistance per meter width of the slab in bending, assuming the mean effective depth is 120 mm.

(6 marks)

(b) Assuming that the loading from the floor finishes is 1 kN/m^2, the unit weight of concrete is 24 kN/m^3 and using the result from part (a), determine the uniform characteristic imposed load per unit area that the slab can carry in bending.

(14 marks)

(c) What is the function of corner reinforcement in slab design? Show on a sketch the locations where corner reinforcements are needed for two typical slab panels. Do not calculate the area of reinforcement.

(5 marks)

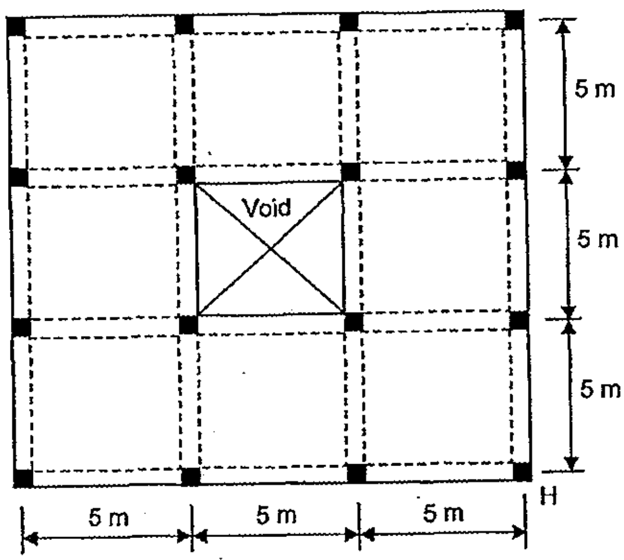


Figure Q3

4. (a) A square, short, and braced column in a multi-storey building supports an approximately symmetrical arrangement of beams. The ultimate (factored) axial load that the column has to carry is 2000 kN. Determine a suitable cross section for the column with 2% longitudinal reinforcement.
f_{cu} = 40 N/mm^2; f_y = 460 N/mm^2.

Next, assume the column has a square cross section of 315 mm × 315 mm and is reinforced with four 25 mm longitudinal bars at d/h = 0.85. Determine the maximum ultimate axial load that the column can resist if the load is applied at an eccentricity of 100 mm along one of the principal axes. (The column design chart shown in Figure Q4 is available for use).

(18 marks)

(b) Show that the axial load capacity of a symmetrical reinforced rectangular column section at balanced condition is $N_{bal} = 0.25 f_{cu} b d$.

(7 marks)

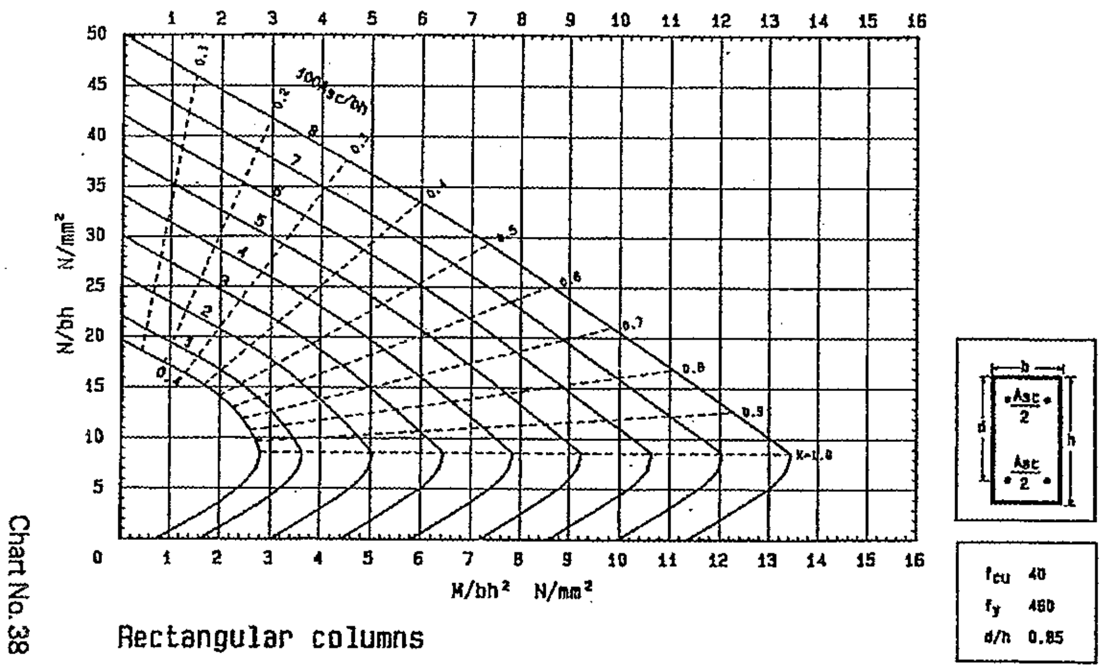


Figure Q4

END OF PAPER

01. $b = 400$ mm $g_k = 55$ kN/m

$h = 600$ mm $q_k = 50$ kN/m.

Grade 30 concrete, cover 40 mm, steel reinforcement 460 N/mm².

Stirup $\phi = 12$ mm.

(a). By using coefficient method :

$M_B = -0.11Fl$ < 1st interior support >.

Design load $w = 1.4g_k + 1.6q_k$ $F = wl$

$= 1.4(55) + 1.6(50)$ $= (157)(6.5)$

$= 157$ kN/m. $= 1020.5$ kN

$M_B = -0.11(1020.5)(6.5) = -730$ kN·m

Applying $\beta_b = 0.9 \Rightarrow M_B = 657$ kN·m < hogging >.

$k = \frac{M}{f_{cu} \cdot b \cdot d^2} = \frac{657 \cdot 10^6}{30 \cdot (400)(532)^2} = 0.1934 > 0.156 \rightarrow$ need to provide doubly reinforcement.

* $d = 600 - 40 - 12 - 16$ < assuming main tension rebars $\phi = 32$ mm >.

$= 532$ mm

$\frac{d'}{d} = \frac{60}{532} = 0.113 \leq 0.215 \rightarrow$ compression steel bar yields.

$d' = 40 + 12 + 8$

$= 60$ mm < assume compression rebar $\phi = 16$ mm >

$M = k' f_{cu} \cdot b \cdot d^2 + 0.87 \cdot A_s' (d - d') \cdot f_y$

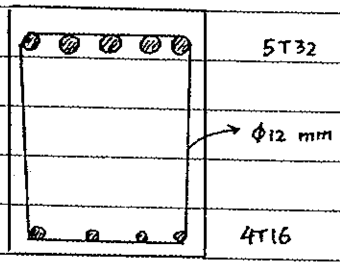
$657 \cdot 10^6 = 0.156(30)(400)(532)^2 + 0.87 \cdot A_s' (532 - 60) \cdot (460)$.

$A_s' = 673$ mm². (provide 4T16 − 804 mm²).

$A_s = \frac{k' \cdot f_{cu} \cdot b \cdot d^2}{0.87 \cdot f_y \cdot z} + A_s'$

$= \frac{0.156 \cdot 30 \cdot 400 \cdot 532^2}{0.87 \cdot 460 \cdot 0.775 \cdot 532} + 673 = 3884$ mm² (provide 5T32 − 4021 mm²

Cross section :



(b). Applying $\beta_b = 0.7 \Rightarrow M_B = \underline{512}$ kN·m < hogging >.

$$K = \frac{512 \cdot 10^6}{30 \cdot 400 \cdot (532)^2} = 0.151 > 0.105 \rightarrow \text{provide doubly reinforcement}$$

$$M = k' f_{cu} \cdot bd^2 + 0.87 \cdot f_y \cdot A_s' \cdot (d - d').$$

$$512 \cdot 10^6 = 0.105 (30)(400)(532)^2 + 0.87 \cdot 460 \cdot A_s' (532 - 60).$$

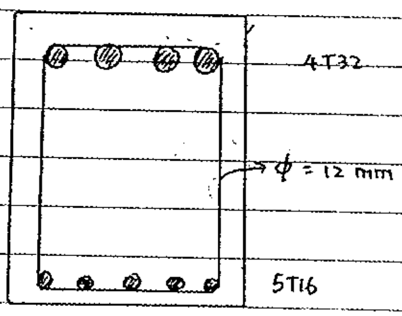
$A_s' = \underline{823}$ mm^2 (provide 5T16 - 1005 mm^2).

$$A_s = \frac{k' \cdot f_{cu} \cdot bd^2}{0.87 \cdot f_y \cdot z} + A_s'$$

$$= \frac{0.105 \cdot (30)(400)(532)^2}{0.87 \cdot 460 \cdot 0.865 \cdot 532} + 823$$

$= \underline{2760}$ mm^2 (provide 4T32 - 3217 mm^2).

Cross section :



(c). Re-distribution less than 70% is not allowed to ensure that adequate compression region on concrete instead of a very thin layer of compression zone. In practice, a very thin layer of compression zone is not feasible and by applying less than 70% redistribution, $x < 0.3d$.

02. Dead load = 22 kN/m. Cover = 35 mm.

Live load = 28 kN/m.

Strirup ϕ = 10 mm.

$f_c = 0.8 f_{cu}$

$f_{cu} = \frac{32}{0.8} = \underline{40}$ N/mm^2

(a) Design load $= 1.4DL + 1.6LL$

$= 1.4(22) + 1.6(28)$

$= \underline{75.6}$ kN/m

$d = 750 - 35 - 10 - 20$

$= \underline{685}$ mm

$M_{max} = \frac{1}{8} w \cdot l^2 = \frac{1}{8}(75.6)(12)^2 = \underline{1360.8}$ kN·m

$M_f = 0.45 f_{cu} \cdot b \cdot h_f \cdot (d - h_f/2)$

$= 0.45 \cdot 40 \cdot 750 \cdot 175 \, (685 - 175/2)$

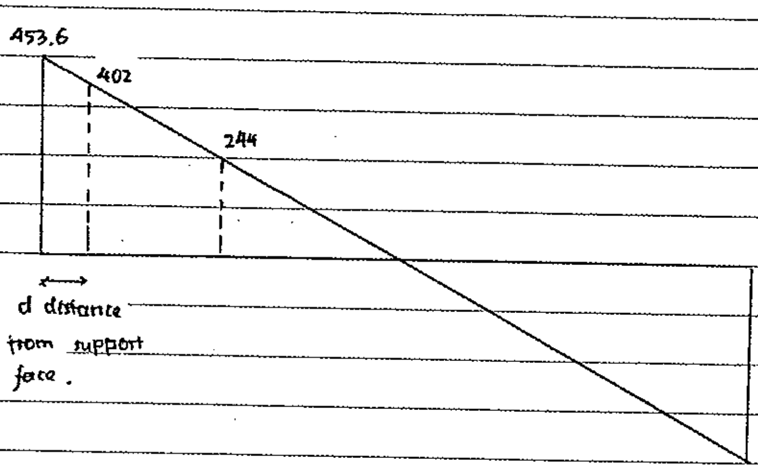
$= \underline{1412}$ kN·m $\rightarrow$ neutral axis in the flange.

$k = \frac{1360.8 \cdot 10^6}{40 \cdot 750 \cdot 685^2} = 0.0967$.

$z = d\left[0.5 + \sqrt{0.25 - \frac{0.0967}{0.9}}\right] = 0.877d = 601$ mm

$A_s = \frac{1360.8 \cdot 10^6}{0.87 \cdot 460 \cdot 601} = \underline{5656}$ mm^2 (provide 5T40 - 6283 mm^2).

(b).



$$v_c = \frac{0.79}{1.25} \cdot \left(\frac{100 \cdot 6283}{(175 \cdot 2) \cdot 685}\right)^{\frac{1}{3}} \left(\frac{40}{25}\right)^{\frac{1}{3}} = 1.019 \ N/mm^2.$$

$$V_c = \frac{1.019 \cdot (175 \cdot 2) \cdot 685}{10^3} = \underline{\underline{244}} \ kN.$$

$$v = \frac{402 \cdot 10^3}{(175 \cdot 2) \cdot 685} = 1.677 \ N/mm^2.$$

$$A_{sv} = \underline{\underline{314}} \ mm^2.$$

$$\frac{A_{sv}}{s_v} = \frac{b_v (v - v_c)}{0.87 \cdot f_{yv}} \Rightarrow \frac{314}{s_v} = \frac{(175 \cdot 2)(1.677 - 1.019)}{0.87 \cdot 250}$$

$$s_v = 297 \ mm$$

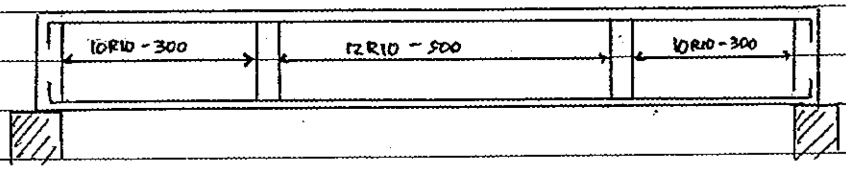
$$s_v \approx \underline{\underline{300}} \ mm.$$

$$\frac{A_{sv}}{s_v} = \frac{0.4 \, b_v}{0.87 \cdot f_{yv}} \Rightarrow \frac{314}{s_v} = \frac{0.4 \, (175 \cdot 2)}{0.87 \cdot 250}$$

$$s_v = 487.8 \ mm$$

$$s_v \approx \underline{\underline{500}} \ mm$$

Longitudinal section :



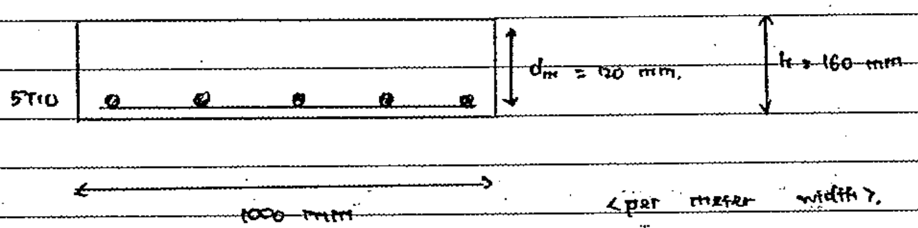
(c). $f_m = 55 \ N/mm^2$,
$\sigma = 6.1 \ N/mm^2$.

$f_k = f_m - 1.64\sigma$
$= 55 - 1.64 \times 6.1$
$= \underline{\underline{45}} \ N/mm^2$

- Allowable concrete strength $= \dfrac{f_k}{\gamma_m} = \dfrac{45}{1.5} = \underline{\underline{30}} \ N/mm^2$

- Allowable yield strength of steel $= \dfrac{f_y}{\gamma_m} = \dfrac{460}{1.05} = \underline{\underline{438}} \ N/mm^2$

Q3. (a).



$C = T$

$0.45 f_{cu} \cdot 0.9x \cdot b = 0.87 \cdot f_y \cdot A_s$.

$x = \dfrac{0.87 \cdot 460 \cdot (393)}{0.405 \cdot 30 \cdot 1000} = \underline{\underline{12.94}}$ mm

$M = Tz$
$= 0.87 \cdot f_y \cdot A_s \cdot (d - 0.45x)$.
$= 0.87 \ (460)(393)(120 - 0.45 \cdot 12.94)$
$= \underline{\underline{17.96}}$ kN-m

(b). From table 3.1.5, consider arrangement case 4 & 5.

$\frac{l_y}{l_x} = 1.0 \Rightarrow$	$\beta_{sx} = 0.047$	$\beta_{sy} = 0.045$	case 4.
	$\beta_{sx} = 0.036$	$\beta_{sy} = 0.034$
	$\beta_{sx} = 0.046$	$\beta_{sy} = -$	case 5.
	$\beta_{sx} = 0.034$	$\beta_{sy} = 0.034$

The largest β value $= 0.047$.

$M_{sx} = \beta_{sx} \cdot n \cdot l_x^2$.

$17.96 = 0.047 \cdot n \cdot 5^2$.

$n = 15.29$ kN / m².

slab weight $= 0.16 \times 24 = 3.84$ kN / m².

SDL (finishes) $= 1$ kN / m² +

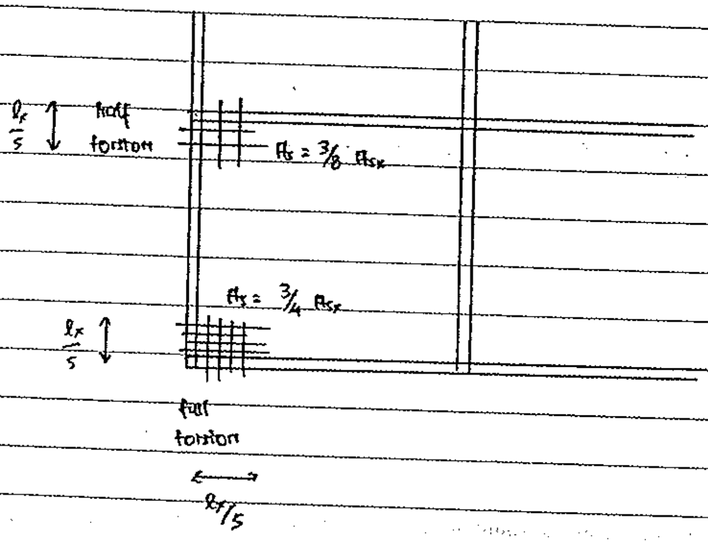
Total DL $= 4.84$ kN / m².

$n = 1.4DL + 1.6IL$

$15.29 = 1.4(4.84) + 1.6IL$

$IL = \underline{\underline{5.32}}$ kN / m².

(c). The corner reinforcement is required to resist torsion where one or both edges of the slab are discontinuous.



Yes, U can!

Q4. (a). Column supporting symmetrical arrangement :

$N = 0.35 f_{cu} \cdot A_c + 0.67 f_y \cdot A_{sc}$

$2000 \cdot 10^3 = 0.35(40) \cdot A_c + 0.67 \cdot 460 \cdot 0.02 A_c.$

$A_c = 99187 \ mm^2.$

Cross section = $\boxed{315 \ mm \times 315 \ mm.}$

$A_{sc} = 4T25 = 1963 \ mm^2.$

$\frac{100 A_{sc}}{bh} = \frac{100 \cdot 1963}{315 \cdot 315} = 1.98 \approx 2.$

eccentricity $e = 100 \ mm = 0.1 \ m.$

$M = N \cdot e = 0.1 N.$

$\frac{M}{bh^2} = \frac{0.1 N}{bh^2} = \frac{0.1}{h} \cdot \frac{N}{bh}.$

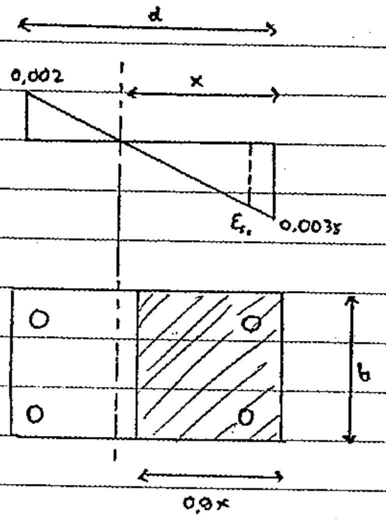
$\frac{0.315}{0.1} \cdot \frac{M}{bh^2} = \frac{N}{bh} \Rightarrow \frac{N}{bh} = 3.15 \cdot \frac{M}{bh^2}.$

By trial & error from the chart :

$\frac{N}{bh} = 13$

$N = 13 \cdot 315^2 = 1289925 \ N = 1290 \ kN$

(b).



$$x_{bal} = \frac{0.0035}{0.0055} \times d = 0.636\,d$$

Assume compression steel has yielded : $F_{s_1} = F_{s_2}$.

$$\Rightarrow \quad N_{bal} = F_c + \cancel{F_{s_1}} - \cancel{F_{s_2}}$$
$$= 0.45 \cdot f_{cu} \cdot b \cdot 0.9\,x$$
$$= 0.45 \cdot (0.9)(0.636) \cdot f_{cu} \cdot b \cdot d$$
$$= 0.257\, f_{cu} \cdot b \cdot d$$

$$N_{bal} \approx 0.25\, f_{cu} \cdot b \cdot d \quad //$$