

$$(a) i. \begin{bmatrix} 1 & 1 & 1 & 1 \\ p & 1 & 2 & p+2 \\ -1 & 1 & 1-p & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1-p & 1 \\ p & 1 & 2 & p+2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2-p & 2 \\ p & 1 & 2 & p+2 \end{bmatrix} \xrightarrow{\text{Row(1)} \leftrightarrow \text{Row(2)}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2-p & 2 \\ 0 & 1-p & 2-p & 2 \end{bmatrix} \xrightarrow{p \times \text{Row(1)} - \text{Row(3)}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2-p & 2 \\ 0 & 1-p & 2-p & 2 \end{bmatrix}$$

↓

Unique solution

$$1+p \neq 0 \\ p \neq -1 //$$

$$\Leftarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2-p & 2 \\ 0 & 1+p & 0 & 0 \end{bmatrix} \xrightarrow{\text{Row(2)} - \text{Row(3)}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2-p & 2 \\ 0 & 1+p & 0 & 0 \end{bmatrix}$$

Multiple solution

$$1+p = 0 \\ p = -1 //$$

All of the values is either unique solution or no solution.

$$ii. \begin{bmatrix} -1 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 0 & 2 & 3 \\ 0 & 4 & 6 \end{bmatrix} \xrightarrow{\begin{matrix} \text{Row(1)} + \text{Row(2)} \\ \text{Row(1)} + \text{Row(3)} \end{matrix}} \begin{bmatrix} -1 & 1 & 2 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{2 \times \text{Row(2)} - \text{Row(3)}} \begin{bmatrix} -1 & 1 & 2 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Let } x_3 = \alpha$$

$$2x_2 + 3\alpha = 1$$

$$2x_2 = 1 - 3\alpha$$

$$x_2 = \frac{1}{2} - \frac{3}{2}\alpha$$

$$-x_1 + \frac{1}{2} - \frac{3}{2}\alpha + 2\alpha = 1$$

$$-x_1 + \frac{1}{2} + \frac{\alpha}{2} = 1$$

$$-x_1 = -\frac{\alpha}{2} + \frac{1}{2}$$

$$x_1 = -\frac{1}{2} + \frac{\alpha}{2}$$

$$x = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} \frac{1}{2} \\ -\frac{3}{2} \\ 1 \end{pmatrix} //$$

$$(b.) H^2 = \begin{bmatrix} a & a & 0 & 0 \\ a & a & 0 & 0 \\ 0 & 0 & 8 & 6 \\ 0 & 0 & 6 & -8 \end{bmatrix} \begin{bmatrix} a & a & 0 & 0 \\ a & a & 0 & 0 \\ 0 & 0 & 8 & 6 \\ 0 & 0 & 6 & -8 \end{bmatrix} = \begin{bmatrix} 2a^2 & 2a^2 & 0 & 0 \\ 2a^2 & 2a^2 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}$$

$$H^3 = \begin{bmatrix} 2c^2 & 2a^2 & 0 & 0 \\ 2a^2 & 2c^2 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix} \begin{bmatrix} a & 0 & 0 & 0 \\ a & a & 0 & 0 \\ 0 & 0 & 8 & 6 \\ 0 & 0 & 6 & -8 \end{bmatrix} = \begin{bmatrix} 4a^3 & 4a^3 & 0 & 0 \\ 4a^3 & 4a^3 & 0 & 0 \\ 0 & 0 & 800 & 600 \\ 0 & 0 & 600 & -800 \end{bmatrix}$$

$$H^4 = \begin{bmatrix} 4a^3 & 4a^3 & 0 & 0 \\ 4a^3 & 4a^3 & 0 & 0 \\ 0 & 0 & 800 & 600 \\ 0 & 0 & 600 & -800 \end{bmatrix} \begin{bmatrix} a & a & 0 & 0 \\ a & a & 0 & 0 \\ 0 & 0 & 8 & 6 \\ 0 & 0 & 6 & -8 \end{bmatrix} = \begin{bmatrix} 8a^4 & 8a^4 & 0 & 0 \\ 8a^4 & 8a^4 & 0 & 0 \\ 0 & 0 & 10^4 & 10^4 \\ 0 & 0 & 10^4 & 10^4 \end{bmatrix}$$

(for an even number)

$$H^n = \begin{bmatrix} 2^{n-1} a^n & 2^{n-1} a^n & 0 & 0 \\ 2^{n-1} a^n & 2^{n-1} a^n & 0 & 0 \\ 0 & 0 & 10^n & 10^n \\ 0 & 0 & 10^n & 10^n \end{bmatrix} \Rightarrow$$

$$H^{18} = \begin{bmatrix} 2^{17} a^{18} & 2^{17} a^{18} & 0 & 0 \\ 2^{17} a^{18} & 2^{17} a^{18} & 0 & 0 \\ 0 & 0 & 10^{18} & 10^{18} \\ 0 & 0 & 10^{18} & 10^{18} \end{bmatrix} //$$

c: $E = BCD$

$$B^{-1}E = B^{-1} \cdot BCD$$

$$B^{-1}E = CD$$

$$B^{-1}ED^{-1} = C$$

$$B^{-1}ED^{-1}C^{-1} = I$$

$$C^{-1} = BE^{-1}D //$$

②(a) $G^{-1} = \frac{1}{\det(G)} F$ $\det(G^{-1}) = \frac{\det(F)}{(\det(G))^4}$

$$\det(I) = \det(G) \cdot \det(G^{-1})$$

$$1 = \det(G) \cdot \frac{\det(F)}{(\det(G))^4}$$

$$\det(F) = \det(G)^3 //$$

$$\det(F) = (-3)^3 = -27 //$$

(b.) i Let $Q = \begin{bmatrix} 0 & 1 & 2 \\ 0 & -2 & 1 \\ 1 & 7 & 0 \end{bmatrix}$ $D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

$$\det(Q) = (1 \times 1 \times 1) - (2 \times 2 \times 1) = 5$$

$$Q^{-1} = \frac{1}{5} \begin{bmatrix} -7 & 14 & 5 \\ 1 & -2 & 0 \\ 2 & -1 & 0 \end{bmatrix} = \begin{bmatrix} -\frac{7}{5} & \frac{14}{5} & 1 \\ \frac{1}{5} & -\frac{2}{5} & 0 \\ \frac{2}{5} & -\frac{1}{5} & 0 \end{bmatrix}$$

$$A = QDQ^{-1} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & -2 & 1 \\ 1 & 7 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} -\frac{7}{5} & \frac{14}{5} & 1 \\ \frac{1}{5} & -\frac{2}{5} & 0 \\ \frac{2}{5} & -\frac{1}{5} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 14 \\ 0 & -4 & 7 \\ -1 & 14 & 0 \end{bmatrix} \begin{bmatrix} -\frac{7}{5} & \frac{14}{5} & 1 \\ \frac{1}{5} & -\frac{2}{5} & 0 \\ \frac{2}{5} & -\frac{1}{5} & 0 \end{bmatrix}$$

$$(b) i. A = \begin{bmatrix} 6 & 2 & 0 \\ 2 & 3 & 0 \\ 4 & -8 & -1 \end{bmatrix} //$$

$$ii. \det(A) = (6 \times 3 \times -1) - (2 \times 2 \times -1) \\ = -18 + 4 = -14 //$$

$$\text{trace of } A = 6 + 3 - 1 = 8 //$$

$$iii. \text{Eigenvalue of the matrix } (A^T - 4I) = -1 - 4 = -5, \text{ (from } \lambda_1)$$

$$iv. \text{Eigenvalue of the matrix } (2A - 7A^{-1})^3 = (2 \times 2 - 7 \times \frac{1}{2})^{-3} \\ = (4 - 3.5)^{-3} = 8 \text{ (from } \lambda_2)$$

$$\text{Eigenvector} = \begin{bmatrix} 1 \\ -2 \\ 7 \end{bmatrix} //$$

3) (a)

t (hr)	0	5	10	13	16	19	24
	combine			combine		combine	

$$i. \int_0^t Q(t) dt = \frac{5}{3}(36 + 4 \cdot 24 + 48) + \frac{3 \cdot 3}{8}(48 + 3 \cdot 65 + 3 \cdot 72 + 65) + \frac{5}{2}(65 + 36) \text{ m}^3 \\ = \left(\frac{916}{3} + \frac{1179}{2} + \frac{505}{2} \right) \text{ m}^3 = \frac{3442}{3} \text{ m}^3 //$$

$$ii. \int_0^t Q(t) c(t) dt = \frac{5}{3}(36 + 4 \cdot (24 \cdot 1.8) + (48 \cdot 2)) + \frac{3 \cdot 3}{8}((48 \cdot 2) + 3(65 \cdot 4) + 3(72 \cdot 5.2) + (65 \cdot 5)) \\ + \frac{5}{2}((65 \cdot 5) + (36 \cdot 1.2)) \text{ g} \\ = \left(\frac{2588}{5} + \frac{104589}{40} + \frac{1841}{2} \right) \text{ g} = 4052.825 \text{ gr} //$$

$$iii. \frac{\int_0^t Q(t) c(t) dt}{\int_0^t Q(t) dt} = \frac{4052.825 \times 3}{3442} \text{ gr/m}^3 = 3.53 \text{ gr/m}^3 //$$

(b) i. There are 4 data points are required for 3rd order Newton's interpolating Polynomial.
I would use data point of $t = 15s, 22s, 27s$, and $30s //$

$$ii. f(t) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$$

$$\text{For } t=20s \\ b_0 = 793$$

$$x_0 = 22s \quad x_1 = 15s \quad x_2 = 27s \quad x_3 = 30s$$

b_1	b_2	b_3
44.42	0.85	0.024
48.67	1.04	
64.33		

$$f(x) = 793 + 44.42(x-22) \\ + 0.85(x-22)(x-15) \\ + 0.024(x-22)(x-15)(x-27)$$

$$f(20) = 793 + 44.42(20-22) + 0.85(20-22)(20-15) + 0.024(20-22)(20-15)(20-21) \\ = 697.34 \text{ m/s}$$

For $t=25s$ $x_0=27s$ $x_1=22s$ $x_2=30s$ $x_4=15s$

$$b_0 = 1066$$

b_1	b_2	b_3
54.6	1.217	0.025
58.25	0.921	
51.8		

$$f(x) = 1066 + 54.6(x-27) + 1.217(x-27)(x-22) + 0.025(x-27)(x-22)(x-30)$$

$$f(25) = 1066 + 54.6(25-27) + 1.217(25-27)(25-22) + 0.025(25-27)(25-22)(25-30) \\ = 950.25 \text{ m/s}$$

iii. $O(\Delta t^2) = \text{centered}$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1}))}{2h_i} = \frac{2154 - 1259}{10} = 89.5 \text{ m/s}^2$$

(4) (a) Let $\frac{dv}{dt} = z \Rightarrow \frac{dz}{dt} + 0.6z + 40v = 4\sin(\pi t) \Rightarrow \frac{dz}{dt} = -0.6z - 40v + 4\sin(\pi t)$

$t=0s$ $v=0.1m$ $z=0m/s$ $\frac{dv}{dt}=0m/s$ $\frac{dz}{dt} = -40 \cdot 0.1 = -4 \text{ m/s}^2$

$t=0.1s$ $\frac{dv}{dt} = 0 - 4 \cdot 0.1 = -0.4 \text{ m/s}$ $v = 0.1 - 0.4 \cdot 0.1 = 0.06 \text{ m}$ $\frac{dz}{dt} = -0.6 \cdot -0.4 - 40 \cdot 0.06 + 4\sin(0.1\pi) = -2.36 \text{ m/s}^2$

$t=0.1s$ $v = 0.1 + \frac{0-0.4}{2} \cdot 0.1 = 0.08 \text{ m}$ $z = 0 + \frac{-4-2.36}{2} \cdot 0.1 = -0.318 \text{ m/s}$

$\frac{dz}{dt} = -0.6 \cdot -0.318 - 40 \cdot 0.08 + 4\sin(0.1\pi) = -1.773 \text{ m/s}^2$

$t=0.2s$ $\frac{dv}{dt} = -0.318 - 1.773 \cdot 0.1 = -0.4953 \text{ m/s}$

$t=0.2s$ $v = 0.098 + \frac{-0.318-0.4953}{2} \cdot 0.1 = 0.0573 \text{ m}$

$$\textcircled{4} (b) \lambda = \frac{0.8 \cdot 0.1}{2^2} = 0.02.$$

$$\begin{aligned} T_2^{0.1s} &= 0.02 \cdot 50 + (1-0.04) \cdot 20 + 0.02 \cdot 20 \\ &= 20.6^\circ \text{C} \end{aligned}$$

$$\begin{aligned} T_8^{0.1s} &= 0.02 \cdot 20 + (1-0.04) \cdot 20 + 0.02 \cdot 100 \\ &= 21.6^\circ \text{C} \end{aligned}$$

$$T_4^{0.1s} = T_6^{0.1s} = 20^\circ \text{C}.$$

$$\begin{aligned} T_2^{0.2s} &= 0.02 \cdot 50 + (1-0.04) \cdot 20.6 + 0.02 \cdot 20 \\ &= 21.176^\circ \text{C} // \end{aligned}$$

$$\begin{aligned} T_8^{0.2s} &= 0.02 \cdot 20 + (1-0.04) \cdot 21.6 + 0.02 \cdot 100 \\ &= 23.136^\circ \text{C} // \end{aligned}$$

Good luck!!! You can do it!!!