S2 2014/15

CV2019

Matrix Algebra & Computational Method.

Q1. (a) $[A|b] = \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2q \\ 3 & 7 & 6 & 5 \\ 1 & 5 & p & 3q \end{array}\right] \xrightarrow{R_3 - R_1 \to R_3} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2q \\ 3 & 7 & 6 & 5 \\ 0 & 2 & p-2 & q \end{array}\right] \xrightarrow{3R_1 - R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2q \\ 0 & 2 & 0 & 6q-5 \\ 0 & 2 & p-2 & q \end{array}\right]$

$\xrightarrow{R_3 - R_2 \to R_3} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2q \\ 0 & 2 & 0 & 6q-5 \\ 0 & 0 & p-2 & 5-5q \end{array}\right]$

i.)

unique solution : $p-2 \neq 0$

$p \neq 2$

No Solution : $p = 0 \quad q \neq 0$

$p = 2 \quad q \neq 1$

multiple solution : $p = 0 \quad q = 0$

$p = 2 \quad q = 1$

ii.) Multiple solution : $\left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

let $x_3 = \alpha$.

$2x_2 = 1$

$x_2 = \frac{1}{2}$

$x_1 + 3(\frac{1}{2}) + 2\alpha = 2$

$x = \frac{1}{2} - 2\alpha$

$$X = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

(b) (i) $[C|I] = \left[\begin{array}{ccc|ccc} 6 & 0 & 0 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 & 1 & 0 \\ 2 & 6 & 4 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 6 & 0 & 0 & 1 & 0 & 0 \\ 0 & 6 & 0 & -1 & 3 & 0 \\ 0 & 18 & 12 & -1 & 0 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 6 & 0 & 0 & 1 & 0 & 0 \\ 0 & 6 & 0 & -1 & 3 & 0 \\ 0 & 0 & 4 & 2/3 & -3 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/6 & 0 & 0 \\ 0 & 1 & 0 & -1/6 & 1/2 & 0 \\ 0 & 0 & 1 & 1/6 & -3/4 & 1/4 \end{array}\right]$

$$C^{-1} = \begin{bmatrix} 1/6 & 0 & 0 \\ -1/6 & \frac{1}{2} & 0 \\ 1/6 & -3/4 & 1/4 \end{bmatrix}$$

D^{-1} is a special case of diagonal matrix $=$ $D^{-1} = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$

Yes, U Can!

(ii) $[C - 2D] = \begin{bmatrix} 6-6 & 0 & 0 \\ 2 & 2-2 & 0 \\ 2 & 6 & 4-8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 2 & 6 & -4 \end{bmatrix}$

2 independent rows.
Rank : 2

(iii) $(2C + 4D) = XD \rightarrow (2C + 4D)D^{-1} = X$

$$\begin{bmatrix} 24 & 0 & 0 \\ 4 & 8 & 0 \\ 4 & -12 & 24 \end{bmatrix} \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 4/3 & 8 & 0 \\ 4/3 & 12 & 6 \end{bmatrix} = X$$

(iv) $\left| \frac{1}{2} (C^{-1})^T (D^2)^{-1} C^2 (D^T)^3 \right| = \left(\frac{1}{2}\right)^3 \times |C^{-1}| \times |(D^2)^{-1}| \times |C^2| \times |(D^T)^3|$

All matrices are 3x3

$$= \frac{1}{8} \times \frac{1}{\cancel{48}} \times \frac{1}{\cancel{12^2}} \times 48^{\cancel{2}} \times 12^{\cancel{3}}$$

$$= 72$$

Q2. (a) i) $|-6B^2(3B^T)^{-1}| = |-6B^2(3B)^{-1}| = (-6)^3 \times |B|^2 \times \frac{1}{3^3} \times \frac{1}{|B|}$

$$= -32$$

ii) $\begin{bmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \rightarrow \begin{bmatrix} b_1 & b_2 & 5b_3 \\ a_1 & a_2 & 5a_3 \\ c_1 & c_2 & 5c_3 \end{bmatrix} \rightarrow \begin{bmatrix} 3b_1+2a_1 & 3b_2+2a_2 & 15b_3+10a_3 \\ a_1 & a_2 & 5a_3 \\ c_1 & c_2 & 5c_3 \end{bmatrix}$

interchange row $\times(-1)$ | column multiplication $\times(5)$ | $3R_1 + 2R_2 \rightarrow R_1$ $\times(3)$

$$= (-1) \times (5) \times (3) \times (4) = -60$$

(b) $|F^{-1}| = 4$

$f(-6) + 2(6 - f^2) = 4$

$-6f + \cancel{8} - 2f^2 = \cancel{4}$

$f^2 + 3f - 4 = 0$

$(f+4)(f-1) = 0$

$f = 1$

$(f \geq 0)$

$F^{-1} = \begin{bmatrix} 3 & 1 & 3 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

suppose that λ' is e-values of F^{-1}

Yes, U Can

$$\begin{vmatrix} 3-\lambda' & 1 & 3 \\ 1 & 2-\lambda' & 0 \\ 1 & 0 & 2-\lambda' \end{vmatrix} = 0$$

$(2-\lambda)(2-\lambda)$

$-3(2-\lambda') + (2-\lambda')\left[(3-\lambda')(2-\lambda') - 1\right]$

$-6 + 3\lambda' + (2-\lambda)\left[\lambda'^2 - 5\lambda' + 5\right]$

$-6 + 3\lambda' + (-\lambda'^3 + 7\lambda'^2 - 15\lambda' + 10)$

$-\lambda^3 + 7\lambda^2 - 12\lambda + 4 = 0$

$\lambda_1' = 4.562$

$\lambda_2' = 2$

$\lambda_3' = 0.438$

Eigenvalues of F are reciprocal of F^{-1}

$\lambda_1 \times \lambda_2 + \lambda_1 \times \lambda_3 + \lambda_2 \times \lambda_3$

$= \frac{1}{4.562} \times \frac{1}{2} + \frac{1}{4.562 \times 0.438} + \frac{1}{2 \times 0.438}$

$= 1.752$

(c) i)
$$\begin{bmatrix} 1-\mu_1 & a & 2 \\ -1 & 2-\mu_1 & 1 \\ 4 & -1 & -1-\mu_1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Row 2 $\rightarrow$ $-1 + y = 0$

$y = 1$

Row 1 $\rightarrow$ $1 - \mu_1 + 2y = 0$

$\mu_1 = 3$

ii). For $\mu_2 = -2$, eigenvalues of A^{-1} corresponding to $\mu_2 = -\frac{1}{2}$

For matrix $\left[-4A^{-1} + 6I\right]^T$

eigenvalue correspond to $\mu_2 \rightarrow (-4)(-\frac{1}{2}) + 6 = 8$

Q3. (a) $X_{r_1} = \frac{0.7 + 0.8}{2} = 0.75$

1st Iteration : $f(X_r) = 0.317$ $f(X_L) \times f(X_r) < 0$ $X_U = X_{r_1} = 0.75$

$f(X_L) = -0.151$

2nd Iteration : $X_{r_2} = \frac{0.7 + 0.75}{2} = 0.725$

$f(X_r) = 0.084$ $f(X_L) \times f(X_r) < 0$ $X_U = X_{r_2} = 0.725$

$f(X_L) = -0.151$

$\varepsilon = \left|\frac{0.725 - 0.75}{0.75}\right| = 3.33\%$

3rd - Iteration $X_{r_3} = \frac{0.7 + 0.725}{2} = 0.7125$ $\varepsilon = \left| \frac{0.7125 - 0.725}{0.725} \right| = 1.72\%$

$f(X_r) = -0.033$ $f(X_L) \times f(X_r) > 0$

$f(X_L) = -0.151$ $X_L = X_{r_3} = 0.7125$

4th - Iteration $X_{r_4} = \frac{0.7125 + 0.725}{2} = 0.719$ $\varepsilon = \left| \frac{0.719 - 0.7125}{0.7125} \right| = 0.91\%$

$X_r \approx 0.719$

(b). Take 4-9, because point 6 & 7 lie within this interval, thus giving us more correct approximation.

	X_i	$f(X_i)$	1st	2nd	3rd
x_0	4	60	20	0.333	-0.112
x_1	5	80	21.33	-0.266	
x_2	8	144	20		
x_3	9	164			

$$f(x) = 60 + 20(x-4) + 0.333(x-4)(x-5) - 0.112(x-4)(x-5)(x-8)$$

$f(6) = 101.1$

$f(7) = 122.7$

(c). $$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times f(x_1) + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times f(x_3)$$

Input $x = 6.5$, $x_0 = 4$, $x_1 = 5$, $x_2 = 8$, $x_3 = 9$

$f(6.5) = 112$

Yes, U Can!

Q4. (a)

x	q	
0	5	Trap
1	11.5	
1.5	9.6	Simpson 1/3
2	9	
3	11	Simpson 3/8
4	9.3	
5	10.5	
5.5	8.5	Simpson 1/3
6	9.5	
7	7	Trap
7.5	7.5	Simpson 1/3
8	3	

$\left(1-\frac{x}{L}\right)$	R_A	$\int q(x)\cdot\left(\frac{L}{2}-x\right)$	$q(x)\cdot\left(\frac{L}{2}-x\right)$
1	7.53	27.25	20
7/8	8.0	24.75	34.5
13/16			24.
3/4	16.97	15.19	18
5/8			11
1/2			0
3/8	2.82	-13.42	-10.5
5/16			-12.75
1/4	1.63	-20	-19
1/8	0.46	-23	-21
1/16			-26.25
0			-12.
Σ	37.41	10.77	

$M = 149.64 - 10.77$

$= 138.87$ kN.m.

(b). $P(t) = 10.\sin(\pi t)$

$$\frac{d^2u}{dt^2} + 0.2\pi\frac{du}{dt} + 4\pi^2 u = 0.4\pi^2.\sin(\pi t)$$

$U(0) = 0.1$

$U'(0) = 0$

t	U(t)	$U' = z$	z'	$z'_{1/2+i}$	$z_{1/2+i}$
0	0.1	0	-3.95	-3.813	-0.197
0.1	0.0803	-0.381	-2.91		-0.526
0.2	0.0277				

$z_{1/2} = z_0 + z'(x_0, y_0)\times\frac{h}{2}$

$U_{(0.1)} = 0.1 + (-0.197)\times 0.1$

$= 0.0803$

$z_{(0.1)} = 0 - 3.813\times 0.1$

$= -0.381$

$U_{(0.2)} = 0.0803 + (-0.526)\times 0.1$

$= 0.0277$

U at $x = 0.1 \rightarrow 0.080$ m

U at $x = 0.2 \rightarrow 0.028$ m.

ALL THE BEST !

Kevin Jamiandy