U1220554K

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CV2019 2013-14 PYP Solution.

Q1: (a)(i). if $P=0$, the matrix $B=\begin{bmatrix} 2 & 1 & -1 \\ 0 & 0 & 2q \\ 4 & 2 & -(p+q) \end{bmatrix}$

by Gauss Elimination: $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 0 & 2q \\ 0 & 0 & q \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix}$

As $qx_3=-1$, $2qx_3=-3$

The equation has no solution

(ii). if $P=1$, the matrix $A=\begin{bmatrix} 2 & 1 & -1 & 1 \\ 0 & -3 & 1+2q & -3 \\ 4 & 3 & -2-q & 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 1 & -1 & 1 \\ 0 & -3 & 1+2q & -3 \\ 0 & 1 & -q & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 1 & -1 & 1 \\ 0 & -3 & 1+2q & -3 \\ 0 & 0 & 1-q & 0 \end{bmatrix}$

if $q=1$ the system has multiple solution

if $q\neq1$, the system has unique solution: $[0,1,0]^T$

(iii) if $q=1$ $\begin{bmatrix} 2 & 1 & -1 & 1 \\ 0 & -3 & 3 & -3 \end{bmatrix}$

let $x_3=\alpha$ $\quad -3x_2+3\alpha=-3 \quad x_2=\alpha+1, \quad 2x_1+\alpha+1-\alpha=1 \quad x_1=0$

The solution $[0, \alpha+1, \alpha]^T$

(iv). if $P=1$ $q=3$, for $(A^8-A^7)x=0$. $A^8(A-I)x=0$.

one possible solution could be $x=0$.

(b). (i) $|B|=2 \quad |B^2(-2B)^{-1}| = |B^2| \cdot \frac{1}{|-2B|} = |B|^2 \cdot \frac{-1}{2^3 B} = -\frac{1}{4}$

(ii) $|3(B^{-1})^T| = 3^3|B^{-1}| = 27/2 = 13.5$

(iii). By observation: the matrix has ①. exchange of row

② row 3 multiple by 4 ③ column 2 multiple by 2.

$|B'| = 2\times(-1)\times2\times4 = -16$

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Q2. (a). (i). $\det[G] = -g(-2g^2-g^2) - g(4g^2+g^2) = -2g^3$.

$$[C_{jk}] = \begin{bmatrix} g^2 & 3g^2 & 5g^2 \\ -g^2 & -g^2 & -g^2 \\ 0 & -2g^2 & -2g^2 \end{bmatrix} \qquad [C_{jk}]^T = \begin{bmatrix} g^2 & -g^2 & 0 \\ 3g^2 & -g^2 & -2g^2 \\ 5g^2 & -g^2 & -2g^2 \end{bmatrix}$$

$$G^{-1} = \frac{1}{\det(G)}[C_{jk}]^T = \begin{bmatrix} -\frac{1}{2g} & \frac{1}{2g} & 0 \\ -\frac{1.5}{g} & \frac{1}{2g} & \frac{1}{g} \\ -\frac{2.5}{g} & \frac{1}{2g} & \frac{1}{g} \end{bmatrix}$$

(ii). As $\det[G] = -2g^3 \neq 0$ $\det[G^{-1}] = \frac{-1}{2g^3} \neq 0$.

$\det[(G^{-1})\cdot(G^T)^5] \neq 0$. Rank $= 3$.

(iii). $\|3G\| = (0 + 9g^2 + 9g^2 + 36g^2 + 9g^2 + 9g^2 + 9g^2 + 36g^2 + 9g^2)^{\frac{1}{2}}$
$= 11.22g$.

(b): (i). As $\lambda_1 = -2$ $\det[F - I\lambda_1] = 0$

$$F - I\lambda_1 = \begin{bmatrix} 3 & f & 0 \\ -1 & 1 & 1 \\ 3 & f & 2-f \end{bmatrix}$$

i.e. $3(2-f-f) - f(f-2-3) = 0$.
$f^2 + f - 6 = 0$
as $f > 0$ $f = 2$.

$$F = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 3 & 2 & -2 \end{bmatrix} \qquad \det[F] = 2.$$

$$\begin{cases} -2\lambda_2\lambda_3 = 2 \\ \lambda_2 + \lambda_3 - 2 = -2 \end{cases} \qquad \begin{cases} \lambda_2 = 1 \\ \lambda_3 = -1. \end{cases}$$

(ii). To find the eigenvector: $\begin{bmatrix} 3 & 2 & 0 \\ -1 & 1 & 1 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = 0$

let $X_2 = \alpha$. $3X_1 + 2\alpha = 0$ $X_1 = -\frac{2}{3}\alpha$ $\frac{2}{3}\alpha + \alpha + X_3 = 0$ $X_3 = -\frac{5}{3}\alpha$.

The eigenvector $= [-\frac{2}{3}\alpha, \alpha, -\frac{5}{3}\alpha]^T$

(iii). for matrix $(5I + 3F^T - F^2)^{-1}$

let's find the corresponding ~~eigenvool~~ eigenvalue of $\lambda_2 = 1$.

$\lambda_2' = (3\cdot 1 - 1^2 + 5)^{-1} = \frac{1}{7}$

Eigen values

I : 1

Refer back for working

A^T : same as A

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Q3 (a). (i) Using bisection method:

let $g(x) = f'(x) = 3x + \sin x - e^x$.

To find minimum value of $f(x)$, $g(x) = 0$.

$x = 0.5$ $g(x) = 0.331$ $x = 0.25$ $g(x) = -0.287$.

① . Try $x_1 = \frac{0.5+0.25}{2} = 0.375$ $g(x_1) = 0.0363$

As $g(0.375) \cdot g(0.25) < 0$.

② . Try $x_2 = \frac{0.375+0.25}{2} = \frac{5}{16}$ $g(x_2) = -0.122$.

As $g(x_2) \cdot g(x_1) < 0$.

③ . Try $x_3 = \frac{0.375+\frac{5}{16}}{2} = \frac{11}{32}$ $g(x_3) = -0.042$.

(ii). Use Newton-Raphson method.

$x_0 = 0.$ $g(x) = 3x + \sin x - e^x = -1$

$g'(x) = 3 + \cos x - e^x = 3$

① . $x_1 = x_0 - g(x)/g'(x) = 0 - \frac{-1}{3} = \frac{1}{3}$

② . $g(\frac{1}{3}) = 3x + \sin x - e^x = -0.068$.

$g'(\frac{1}{3}) = 3 + \cos x - e^x = 2.55$

$x_2 = \frac{1}{3} + \frac{0.068}{2.55} = 0.36$

③ . $g(0.36) = -1.06 \times 10^{-3}$ $g'(0.36) = 2.506$.

$x_3 = 0.36 + \frac{-1.06\times10^{-3}}{2.506} = 0.3604$

(b). (i). $Z = \frac{V_0 - \bar{V}}{S_V} = \frac{120-86}{20} = 1.7$

Z	f(Z)	1st	2nd	3rd
1.4	0.0818	-0.135	0.10125	-0.0525
1.6	0.0548	-0.0945	0.075	
1.8	0.0359	-0.072		
1.9	0.0287			

$f(z) = 0.0818 - 0.135(x-1.4) + 0.10125(x-1.4)(x-1.6)$

$- 0.0525(x-1.4)(x-1.6)(x-1.8) = \cdots$

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(ii). To find $f_{(z=1.7)}$.

Use:

Z	$f_{(z)}$	1st	2nd	3rd
1.6	0.0548	−0.0945	0.075	−0.0525
1.8	0.0359	−0.072	0.0855	
1.9	0.0287	−0.1062		
1.4	0.0818			

$$f_{(z)} = 0.0548 - 0.0945(X-1.6) + 0.075(X-1.6)(X-1.8) - 0.0525(X-1.6)(X-1.8)(X-1.9)$$

when $X=1.7$

$$f_{(1.7)} = 0.0548 - 0.00945 - 0.00075 - 1.05\times10^{-4}$$
$$= 0.0445$$

(iii) $f_{(z)}=0.04$. 3 data point is needed.

Z	$f_{(z)}$	1st	2nd
1.8	0.0359	−0.072	0.075.
1.9	0.0287	−0.087	
1.6	0.0548		

$$f_{(z)} = 0.0359 - 0.072(X-1.8) + 0.075(X-1.8)(X-1.9)$$
$$= 0.075X^2 - 0.3495X + 0.422.$$

$\cancel{0.04=}$ $f_{(z)} = 0.04$.

$\Rightarrow$. $X^2 - 4.66X + 5.07 = 0$ $X = 1.74785$

As $1.74785 = \dfrac{V_D - 86}{20}$ $\Rightarrow$ $V_D = 120.96$ km/h.

Q4 (a). By using proper trapezoidal and Simpson's method

The ~~slog~~ distance $= \frac{1}{2}\times4.5 + \frac{5+4.5}{2}\times(2-1) + \frac{1.5}{3}(5+6\times4+8)$
$+ 0.5\times\frac{8+7.5}{2} + \frac{3}{8}\times1\times(7.5+3\times7+3\times6.5+8)$
$+ 1.5\times\frac{8+7}{2}$
$= 61.625$ m.

No.: Q4 b).

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Assume $z = \frac{dy}{dx}$

$\Rightarrow \frac{dz}{dx} = \frac{1}{4000}(64x - 12x^2 - x^3)$

First $z(0) = -0.03$

$z(0.5) = z_0 + \frac{dz}{dx}\cdot 0.5 = -0.03$

$\rightarrow z(1) = z_0 + \frac{dz}{dx}\cdot 1 = -0.03$

$y_1 = y_0 + z_{0.5}\cdot h = -0.03$

$z(1.5) = z_1 + \frac{dz}{dx}\cdot 0.5 = -0.03 + 0.01275(0.5)$
$= -0.023625$

$z(2) = z_1 + \frac{dz}{dx}\cdot 1 = -0.03 + 0.01275$
$= -0.01725$

$y_2 = y_1 + z_{1.5}\cdot h = -0.03 - 0.023625$
$= -0.053625$

$z(2.5) = z(2) + \frac{dz}{dx}\cdot 0.5 = -0.01725 + 0.018\times 0.5$
$= -0.00825$

$z(3) = z(2) + \frac{dz}{dx}\cdot 1 = -0.01725 + 0.018$
$= 0.00075$

$y_3 = y_2 + z_{(2.5)}\cdot h = -0.053625 - 0.00825$
$= -0.061875$

$z(3.5) = z(3) + \frac{dz}{dx}\cdot 1 = 0.00075 + 0.01425\times 0.5$
$= 0.007875$

$y_4 = y_3 + z_{3.5}\cdot h = -0.061875 + 0.007875$
$= \underline{\underline{-0.054}}$

Note: $0.01275 = \frac{1}{4000}(64 - 12 - 1)$

$0.018 = \frac{1}{4000}(64\times 2 - 12\cdot 2^2 - 2^3)$

$0.01425 = \frac{1}{4000}(64\times 3 - 12\cdot 3^2 - 3^3)$

second $z(0) = 0$

$z(0.5) = z_0 + \frac{dz}{dx}\cdot 0.5 = 0$

$z(1) = z_0 + \frac{dz}{dx}\cdot 1 = 0$

$y(1) = y_0 + z_{(0.5)}\cdot h = 0$

$z(1.5) = z_1 + \frac{dz}{dx}\cdot 0.5 = 0 + 0.01275\cdot 0.5$
$= 0.006375$

$y_2 = y_1 + z_{1.5}\cdot h = 0.006375$

$z(2) = z(1) + \frac{dz}{dx}\cdot 1 = 0 + 0.01275 = 0.01275$

$z(2.5) = z(2) + \frac{dz}{dx}\cdot 0.5 = 0.01275 + 0.018\times 0.5$
$= 0.02175$

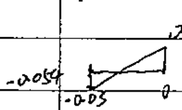
$y_3 = y_2 + z_{2.5}\cdot h = 0.006375 + 0.02175$
$= 0.028125$

$z(3) = z(2) + \frac{dz}{dx}\cdot 1 = 0.01275 + 0.018$
$= 0.03075$

$z(3.5) = z(3) + \frac{dz}{dx}\cdot 0.5 = 0.03075 + 0.01425\cdot 0.5$
$= 0.037875$

$y_4 = y_3 + z_{(3.5)}h = 0.028125 + 0.037875$
$= \underline{\underline{0.066}}$

By shooting Method.



$\frac{0-x}{0.066} = \frac{0-(-0.03)}{0.066-(-0.054)}$

$\Rightarrow x = -0.0165$

Third Try $z = -0.0165$

$z(0.5) = z_0 + \frac{dz}{dx}\cdot 0.5 = -0.0165$

$z(1) = z_0 + \frac{dz}{dx} = -0.0165$

$y_{(1)} = y_0 + z_{(0.5)}\cdot h = -0.0165$

$z(1.5) = z_1 + \frac{dz}{dx}\cdot 0.5 = -0.0165 + 0.01275\cdot 0.5$
$= -0.010125$

$y_{(2)} = y_1 + z_{(1.5)}\cdot h = -0.0165 - 0.010125$
$= -0.026625$

$z(2) = z(1) + \frac{dz}{dx}\cdot 1 = -0.00375$

$z(2.5) = z(2) + \frac{dz}{dx}\cdot 0.5 = 0.00525$

$y_3 = y_2 + z_{(2.5)}\cdot h = -0.021375$

$z(3) = z(2) + \frac{dz}{dx}\cdot 1 = 0.01425$

$z(3.5) = z(3) + \frac{dz}{dx}\cdot 0.5 = 0.021375$

$y(4) = y_{(3)} + z(3.5)\cdot h = 0$ #.

consist with ~~boundary~~ boundary condition, $y_4 = 0$.

$$Fx = \lambda x$$

$$F^T x = \lambda x$$

$$3F^T x = 3\lambda x$$

$$(3F^T - F^2)x = (3\lambda - \lambda^2)x$$

$$(5I + 3F^T - F^2)x = (5 + 3\lambda - \lambda^2)x$$

$$(5I + 3F^T - F^2)^{-1}x = \left(\frac{1}{5+3\lambda-\lambda^2}\right)x$$

For $\lambda_1 = -2$, $\lambda_1' = \dfrac{1}{5+3(-2)-(-2)^2} = -\dfrac{1}{5}$

For $\lambda_2 = 1$, $\lambda_2' = \dfrac{1}{5+3(1)-(1)^2} = \dfrac{1}{7}$

For $\lambda_3 = -1$, $\lambda_3' = \dfrac{1}{5+3(-1)-(-1)^2} = 1$