CV2019 :)

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$$\left[\begin{array}{ccc|c} 2 & p^2 & p & 1 \\ 2 & p^2 & p^2 & p^2 \\ 2 & p & p & p \end{array}\right] \begin{array}{l} \\ = \text{Row 1 - row 2} \\ = \text{Row 1 - Row 3} \end{array}$$

$$= \left[\begin{array}{ccc|c} 2 & p^2 & p & 1 \\ 0 & 0 & p-p^2 & 1-p^2 \\ 0 & p^2-p & 0 & 1-p \end{array}\right] \quad \text{Exchange row 2 with row 3}$$

$$= \left[\begin{array}{ccc|c} 2 & p^2 & p & 1 \\ 0 & p^2-p & 0 & 1-p \\ 0 & 0 & p-p^2 & 1-p^2 \end{array}\right] = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

unique solution: $1-p^2 \neq 0$, $\underline{p^2 \neq \pm 1}$ X

multiple solution: $1-p^2=0$, $p = \pm 1$ X Problem

no solution: when $p=0$ ✓

(ii) $\left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$x_3 = \alpha$

$x_2 = \beta$

$2x_1 + \alpha + \beta = 1$

$2x_1 = 1 - \alpha - \beta$

$x_1 = \dfrac{1-\alpha-\beta}{2}$ #

(b)(i) $|3A^{-1}| = 3 \times \left(\frac{1}{4}\right)$

$= \cancel{3/4} \quad \frac{3^3}{4} = \frac{27}{4}$

(ii) $|(2A^T)^{-1}| = |(2\times 4)^{-1}| \quad \left(\frac{1}{2}\right)^3\left(\frac{1}{4}\right)$

$= \frac{1}{8} \quad = \frac{1}{32}$

(iii) $|2A^2(B^4)^T - 4A^2(B^3)^T + 6A^2(B^T)^2 + 8A^2B^T - 10A^2|$

$= 2A^2 \boxed{[(B^4)^T - 2(B^3)^T + 3(B^T)^2 + 4B^T - 5]} \to 2$

$= 2^3(4)^2(2)$

$= 256$ #

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(iv) Determinant $= \begin{vmatrix} a_{11} & a_{13} & a_{12} \\ a_{21}-3a_{11} & a_{23}-3a_{13} & a_{22}-3a_{12} \\ a_{21}-2a_{31} & a_{23}-2a_{33} & a_{22}-2a_{32} \end{vmatrix}$

① Exchange colum 2 with column 3 ⇒ Det × −1
② Row 1 × −3 ⇒ Det × −3
③ Row 3 × −2 ⇒ Det × −2

Rmb addition of a multiple of a row to another row won't change value of Det

Det $= 4 \times -1 \times -3 \times -2$
$= -24$ #

2(a) $\left[\begin{array}{ccc|ccc} 2 & 0 & 3 & 1 & 0 & 0 \\ -1 & -2 & 3 & 0 & 1 & 0 \\ 0 & -4 & 3 & 0 & 0 & 1 \end{array}\right]$ Row 1 + (2 × row 2) → Row 2

$= \left[\begin{array}{ccc|ccc} 2 & 0 & 3 & 1 & 0 & 0 \\ 0 & -4 & 9 & 1 & 2 & 0 \\ 0 & -4 & 3 & 0 & 0 & 1 \end{array}\right]$ Row 2 + (Row 3 × −1) → Row 3

$= \left[\begin{array}{ccc|ccc} 2 & 0 & 3 & 1 & 0 & 0 \\ 0 & -4 & 9 & 1 & 2 & 0 \\ 0 & 0 & 6 & 1 & 2 & -1 \end{array}\right]$ Row 3 × $-\frac{1}{2}$ + Row 1 → Row 1

$= \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & \frac{1}{2} & -1 & \frac{1}{2} \\ 0 & -4 & 9 & 1 & 2 & 0 \\ 0 & 0 & 6 & 1 & 2 & -1 \end{array}\right]$ Row 3 × $-\frac{3}{2}$ + Row 2 → Row 2

$= \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & \frac{1}{2} & -1 & \frac{1}{2} \\ 0 & -4 & 0 & -\frac{1}{2} & -1 & \frac{3}{2} \\ 0 & 0 & 6 & 1 & 2 & -1 \end{array}\right]$ row 1 × $\frac{1}{2}$
row 2 × $-\frac{1}{4}$
row 3 × $\frac{1}{6}$

$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{8} & \frac{1}{4} & -\frac{3}{8} \\ 0 & 0 & 1 & \frac{1}{6} & \frac{1}{3} & -\frac{1}{6} \end{array}\right]$

$E^{-1} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{4} & -\frac{3}{8} \\ \frac{1}{6} & \frac{1}{3} & -\frac{1}{6} \end{bmatrix}$

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$$\begin{array}{cc} & \begin{array}{ccc} ① & ② & ③ \end{array} \\ (b)\ F = & \begin{bmatrix} 3 & 1 & 1 \\ 0 & 1 & 0 \\ 6 & -4 & x \end{bmatrix} \end{array} \qquad \lambda_1 = 5$$

(i) $x = 2$ ① is multiple of ③

Det of matrix = product of λ

$0 = 5\lambda_2\lambda_3$ — ⓐ

Sum of main diagonal elements = Sum of E-values

$6 = 5 + \lambda_2 + \lambda_3$

$1 - \lambda_3 = \lambda_2$ — ⓑ

$0 = 5(1-\lambda_3)(\lambda_3)$

$0 = 5(\lambda_3 - \lambda_3^2)$

$0 = 5\lambda_3 - 5\lambda_3^2$

$5\lambda_3^{\cancel{2}} = 5\cancel{\lambda_3}$

$\lambda_3 = 1$

$6 = 5 + \lambda_2 + \lambda_3$

$6 = 5 + \lambda_2 + 1$

$\lambda_2 = 0$ #

(ii) $\lambda_1 = 5$

$$\left(\begin{bmatrix} 3 & 1 & 1 \\ 0 & 1 & 0 \\ 6 & -4 & 2 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & 1 & 1 \\ 0 & -4 & 0 \\ 6 & -4 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & 1 & 1 \\ 0 & -4 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & 1 & 1 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$x_3 = \alpha$

$-4x_2 = 0$

$x_2 = 0$

$-2x_1 + \alpha = 0$

$\alpha = 2x_1$

$x_1 = \frac{1}{2}\alpha$

Ans $\Rightarrow \alpha \begin{bmatrix} \frac{1}{2} \\ 0 \\ 1 \end{bmatrix}$

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(iii) $(3F^2)^T = [3(5)^2]^T$ ← transpose : A & A^T has same E-value

$= 75$

3(a) $y = 3\cos x$ ①

$y = x^2$ ② ① = ②

$3\cos x = x^2$

$\boxed{f(x) = x^2 - 3\cos x = 0}$

(ii) Range of $x = 1 - 2$

$x_r = \dfrac{x_l + x_u}{2}$

First iteration using $x_u = 2$, $x_l = 1$, $x_r = 1.5$

$f(x_l) = f(1) = -1.99954$

$f(x_r) = -0.749$

$f(x_l)(x_r) < 0$, $x_u = x_r$

Second iteration using, $x_u = 1.5$, $x_l = 1$, $x_r = 1.25$

$f(x_l) = f(1) = -1.99954$

$f(x_r) = -1.437$

$f(x_l)(x_r) < 0$, $x_u = x_r$

Third iteration using, $x_u = 1.25$, $x_l = 1$, $x_r = 1.125$

$f(x_l) = f(1) = -1.99954$

$f(x_r) = -1.734$

2nd iteratn

Error $= \dfrac{1.25 - 1.5}{1.5} \times 100\% = 16.67\%$

3rd iteratn

Error $= \dfrac{1.125 - 1.25}{1.25} \times 100\% = 10\%$

(iii) Newton-Raphson $x_0 = 1.50$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$\dfrac{dy}{dx} = 2x + 3\sin x$

x	$f(x_0)$	$f'(x_0)$
1.50	2.0378	5.992
1.15994	0.1473	5.0702
1.13	0.00134	4.9732

$= 1.5 - \dfrac{2.0378}{5.992}$

$= 1.15994$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$= 1.15994 - \dfrac{0.1473}{5.0702}$

$= 1.1309$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$

$= 1.13 - \dfrac{0.00134}{4.9732}$

$= 1.1297$ #

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$$\text{Error} = \frac{1.1309 - 1.15994}{1.15994} \times 100\%$$

$$= 2.5\%$$

$$\text{Error} = \frac{1.1297 - 1.1309}{1.1309}$$

$$= 0.13\%$$

(b)

X(m)	T(°c)	1st	2nd	3rd
0	19.25	0.15	0.0317	-0.003
5	20.00	0.398	-0.005	-0.003
8	21.194	0.363	-0.035	-0.003
12	22.6460	0.118	-0.071	
15	23.000	-0.45		
20	20.75			

(ii)

X(m)	T(°c)	1st	2nd
18	21.194	0.363	-0.035
12	22.646	0.118	
15	23.000		

$$f(x) = 21.194 + 0.363(x - x_0) - 0.035(x - x_0)(x - x_1)$$

$$f(13) = 21.194 + 0.363(13-8) - 0.035(13-8)(13-12)$$

$$= 22.834 \text{ °c}$$

(iii) Fifth degree of polynomial best fits all the data points in Table Q3 as 6 data points are given.

4(a)

	x	q
$\frac{1}{3}$ Simp	0	9.2
	1	7.0
	2	6.5
$\frac{3}{8}$ Simp	2.5	5.7
	3.0	4.6
	3.5	3.2
trap	5	1.8
trap	6	0.2

Area 1 - $\frac{1}{3}$ Simp Rule $= \frac{1}{3}\left[9.2 + 4(7) + 6.5\right]$

$= 14.5667$

Area 2 - $\frac{3}{8}$ Simp Rule $= \frac{1.5}{8}\left[6.5 + 3(5.7 + 4.6) + 3.2\right]$

$= 7.6125$

Area 3 - trap Rule $= 1.5\left(\frac{3.2 + 1.8}{2}\right)$

$= 3.75$

Area 4 - trap Rule $= 1\left(\frac{0.2 + 1.8}{2}\right)$

$= 1$

$\frac{3.5-2}{3}$

Total area $= 14.5667 + 7.6125 + 3.75 + 1 = 26.93$

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	x	$q(x)\,x$
$\frac{1}{3}$ Simp	0	0
	1	7
$\frac{3}{8}$ Simp	2	13
	2.5	14.25
	3.0	13.8
trap	3.5	11.2
	5	9
trap	6	1.2

Area 1 - $\frac{1}{3}$ Simp $= \frac{1}{3}\left[0 + 4(7) + 13\right]$
$= 13.667$

Area 2 - $\frac{3}{8}$ Simp $= \frac{1.5}{8}\left[13 + 3(14.25 + 13.8) + 11.2\right]$
$= 20.316$

Area 3 - trap Rule $= 1.5\left(\frac{9 + 11.2}{2}\right)$
$= 15.15$

Area 4 - trap Rule $= \left(\frac{1.2 + 9}{2}\right)$
$= 5.1$

Total area $= 13.667 + 20.316 + 15.15 + 5.1$
$= 54.233$

(b) $EI\frac{d^2y}{dx^2} = \frac{q_0}{6L}\left(x^3 - 3Lx^2 + 3L^2x - L^3\right)$

Sub in $EI = 10000\ \text{kNm}^2$, $L = 6\text{m}$, $q = 9\ \text{kN/m}$ $h = 3\text{m}$

$\frac{d^2y}{dx^2} = \frac{dz}{dx} = 0$

$z = \frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0.000025\left(x^3 - 18x^2 + 108x - 216\right)$

i	x_0	z_0	z_{i+1}	y_0	y_{i+1}	$\frac{d^2y}{dx^2}$	dy/dx
0	0	0	-0.0162	0	0	-0.0054	0
1	3	-0.0162	-0.01823	0	-0.0486	-0.000675	-0.0162
2	6	-0.01823		-0.0486			

Midpoint

i	x	$\frac{d^2y}{dx^2}$	z_i	$z_{i+0.5}$	dy/dx	y_i	$y_{i+0.5}$	$\left(\frac{d^2y}{dx}\right)_{mid}$ ~~$y_{i+0.5}$~~	y_{i+1}	z_{i+1}
0	0	-0.0054	0	-0.0081	0	0	0	-0.002278	-0.0243	-0.006834
1	3	-0.000675	-0.006834	-0.00784	-0.006834	-0.0243	-0.024551	-0.00008438	-0.04784	-0.007087
2	6		-0.007087			-0.04784				

Ans : 24.3 mm (↓) & 47.8 mm (↓) ☺

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(ii) $E = \begin{bmatrix} 2 & 0 & 3 \\ -1 & -2 & 3 \\ 0 & -4 & 3 \end{bmatrix}$

$= (2\times-2\times3) + (-1\times-4\times3) + (0) - [(2\times3\times-4)]$

$= 24$

$E^{-1} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{4} & -\frac{3}{8} \\ \frac{1}{6} & \frac{1}{3} & -\frac{1}{6} \end{bmatrix}$

$= (\frac{1}{4}\times\frac{1}{4}\times-\frac{1}{6}) + (\frac{1}{8}\times\frac{1}{3}\times\frac{1}{4}) + (\frac{1}{6}\times-\frac{1}{2}\times-\frac{3}{8}) - [(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{6}) + (-\frac{1}{2}\times\frac{1}{8}\times-\frac{1}{6}) + (\frac{1}{4}\times\frac{1}{3}\times-\frac{3}{8})]$

$= \frac{1}{24}$

can also be solved using Rule $|A^{-1}| = \frac{1}{|A|}$

(ii) conditional number

$\|A\| = 24$

$\|A^{-1}\| = \frac{1}{24}$

$[5E] = 25(24)(\frac{1}{24})$

$= 25$

(iii) Rank. $[(E^{-1})^3(E^5)^T]$

$= 3$