NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2009-2010

CV2002 – Computational Methods

November - December 2009 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **TWO (2)** pages.
2. Answer **ALL FOUR (4)** questions.
3. All questions carry equal marks.
4. Candidates may bring into the examination hall one sheet of A4 size paper containing any reference material.

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1. (a) Solve the following set of equations:

$$x_1+2x_2-x_3=6$$
$$3x_1+8x_2+9x_3=10$$
$$2x_1-x_2+2x_3=-2$$

(12 marks)

(b) Given that $A=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$, calculate A^{-1}.

(6 marks)

(c) Prove that $(AB)^{-1}=B^{-1}A^{-1}$ if both A and B are invertible.

(7 marks)

2. (a) Given $A=\begin{bmatrix}2 & -4 & 1\\ 1 & 3 & -1\\ 6 & 0 & 1\end{bmatrix}$, calculate $|5A^{-1}|$.

(7 marks)

(b) Let $A=\begin{bmatrix}0 & -2 & 1\\ 1 & 3 & -1\\ 0 & 0 & 1\end{bmatrix}$, find its eigenvalues and eigenvectors.

(18 marks)

3. The deflection curve of a simple beam subjected to uniformly distributed load (UDL) q (Figure Q3) is given as

$$v = \frac{qx}{24EI}\left(L^3 - 2Lx^2 + x^3\right)$$

where the deflection v is measured downward and the axis x of the beam is measured to the right. The length of the beam L is 10 m, the intensity of the UDL q is 100 kN/m, the modulus of elasticity of the material E is 200 GPa (2×10^{11} N/m^2), and the second moment of area of the cross section I is 0.005 m^2.

Determine the segment of the beam, in terms of the range of x values, where the deflection is equal to or larger than 10 mm using:

(a) Graphical method.

(7 marks)

(b) A bracketing method (either the bisection method or the false-position method).

(8 marks)

(c) An open method (either the Newton-Raphson method or the secant method).

(10 marks)

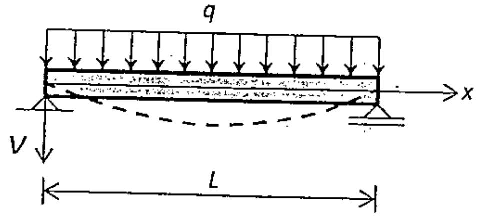

Figure Q3

4. The population growth of a species of animals in an island is modelled as

$$\frac{dp}{dt} = G(p_{max} - p)p$$

where G is a population-dependent growth rate (= 10^{-5} per animal-year), p_{max} is the maximum sustainable population for that particular species in the island (= 20,000 animals), p is the population and t is the time.

Estimate the population two years from the present time if the current population is 10,000 animals, using a step size of 1 year. Use the following methods:

(a) Standard Euler method (8 marks)

(b) Heun's method (8 marks)

(c) Midpoint method (9 marks)

END OF PAPER

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1 (a) $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 6 \\ 3 & 8 & 9 & 10 \\ 2 & -1 & 2 & -2 \end{array}\right] \xrightarrow[\text{2row1-row3} \to \text{row3}]{\text{row2-3row1} \to \text{row2}} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 6 \\ 0 & 2 & 12 & -8 \\ 0 & 5 & -4 & 14 \end{array}\right] \xrightarrow[\frac{1}{2}\text{row2} \to \text{row2}]{\frac{5}{2}\text{row2-row3} \to \text{row3}}$

$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 6 \\ 0 & 1 & 6 & -4 \\ 0 & 0 & 34 & -34 \end{array}\right]$ $\therefore x_3 = -1$ $x_2 + 6x_3 = -4 \Rightarrow x_2 = 2$

$x_1 + 2x_2 - x_3 = 6 \Rightarrow x_1 = 1$

$\therefore x = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix} \xrightarrow{\text{row1-row2} \to \text{row2}} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & -1 \end{bmatrix} \xrightarrow[\frac{1}{2}\text{row2} \to \text{row2}]{\text{row1} - \frac{1}{2}\text{row2} \to \text{row1}} \begin{bmatrix} 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

(c) $(AB)(AB)^{-1} = I$ $A^{-1}(AB)(AB)^{-1} = A^{-1}I$ $B\,(AB)^{-1} = A^{-1}I = A^{-1}$

$B^{-1}B\,(AB)^{-1} = B^{-1}A^{-1}$ $(AB)^{-1} = B^{-1}A^{-1}$

2 (a) $|A| = 16$ $|5A^{-1}| = 5^3|A^{-1}| = 125\frac{1}{|A|} = \frac{125}{16}$

(b) $A - \lambda I = \begin{bmatrix} -\lambda & -2 & 1 \\ 1 & 3-\lambda & -1 \\ 0 & 0 & 1-\lambda \end{bmatrix}$ $|A - \lambda I| = -\lambda(3-\lambda)(1-\lambda) + 2(1-\lambda) = 0$

$\lambda_1 = 1$ $\lambda_2 = 2$

when $\lambda_1 = 1$ $A - \lambda I = \begin{bmatrix} -1 & -2 & 1 \\ 1 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ $x = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\alpha + \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}\beta$

when $\lambda_2 = 2$ $A - \lambda I = \begin{bmatrix} -2 & -2 & 1 \\ 1 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$ $x = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}\alpha$

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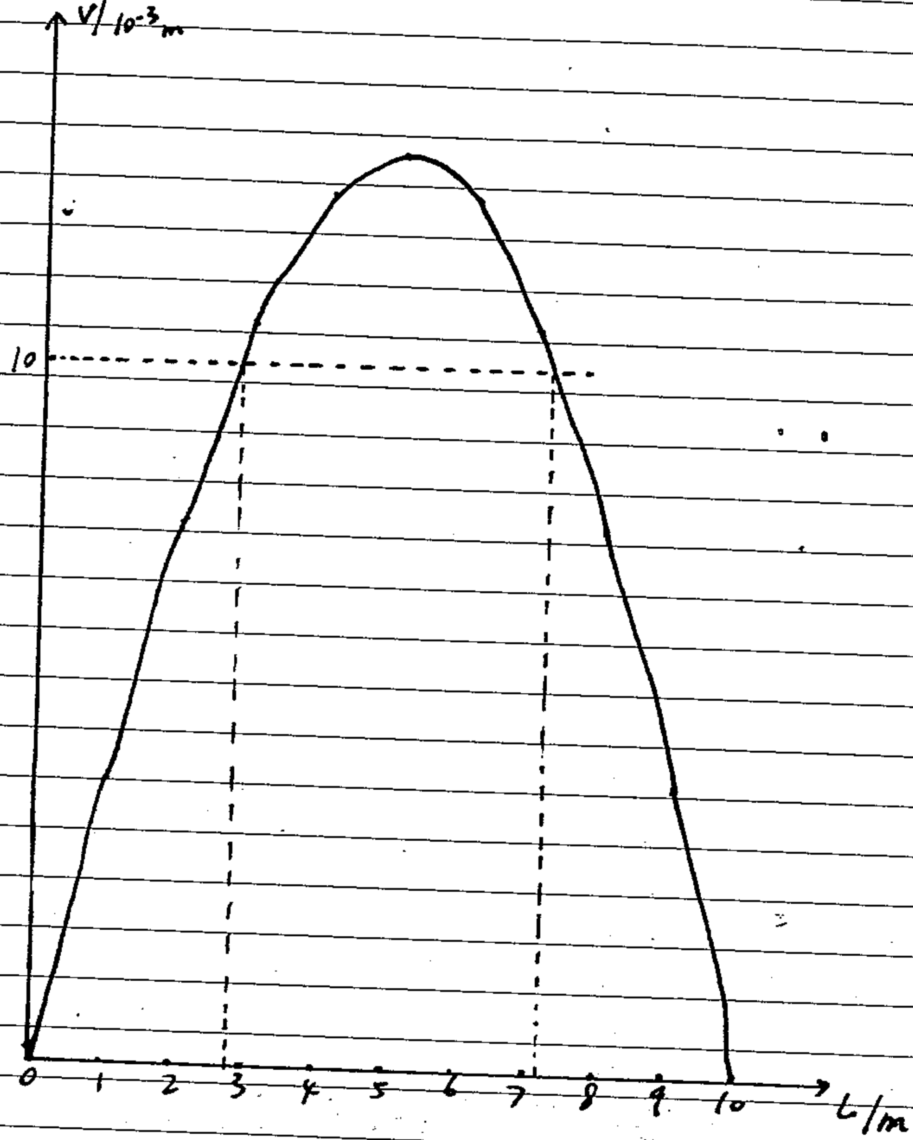
3 (a) $V = \frac{1}{240000}x^4 - \frac{1}{12000}x^3 + \frac{x}{240}$



$2.8 \le x \le 7.2$

(b) $f(x) = \frac{1}{240000}x^4 - \frac{1}{12000}x^3 + \frac{x}{240} - \frac{1}{100} = 0$

$g(x) = x^4 - 20x^3 + 1000x - 2400 = 0$

False-Position: ① $x_l = 2.7$ $\quad x_u = 2.9$ $\quad g(x_l) = -40.5159$ $\quad g(x_u) = 82.9481$

$x_r = x_u - \frac{g(x_u)(x_l - x_u)}{g(x_l) - g(x_u)} = 2.766$ $\quad g(2.766) = 1.294$ $\quad \therefore x_u = 2.766 \quad x_l = 2.7$

$x_r = 2.764$ $\quad g(2.764) = 0.04256$ $\quad \therefore x_u = 2.764 \quad x_l = 2.7$

$x_r = 2.764$

$\therefore x_r = 2.76$

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② $x_l = 7.1 \quad x_u = 7.3 \quad g(7.1) = 82.9481 \quad g(7.3) = -40.5159$

$x_r = 7.234 \quad g(7.234) = 1.294 \quad \therefore x_u = 7.3 \quad x_l = 7.234$

$x_r = 7.236 \quad g(7.236) = 0.04256 \quad \therefore x_u = 7.3 \quad x_l = 7.236$

$x_r = 7.236$

$\therefore x_r = 7.24$

$2.76 \le x \le 7.24$

(c) Newton-Raphson Method. $\left(x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}\right)$

$g'(x) = 4x^3 - 60x^2 + 1000$

① $x_1 = 2.8$

$x_2 = x_1 - \frac{g(x_1)}{g'(x_1)} = 2.764$

$x_3 = x_2 - \frac{g(x_2)}{g'(x_2)} = 2.764$

$x_r = 2.76$

② $x_1 = 7.2$

$x_2 = x_1 - \frac{g(x_1)}{g'(x_1)} = 7.236$

$x_3 = x_2 - \frac{g(x_2)}{g'(x_2)} = 7.236$

$x_r = 7.24$

$2.76 \le x \le 7.24$

4 $\frac{dP}{dt} = 0.2P - 10^{-5}P^2$

(a) $P_1 = P_0 + \frac{dP}{dt} h = 10000 + (0.2 \times 10000 - 10^{-5} \times 10000^2) \times 1 = 11000$

$P_2 = P_1 + \frac{dP}{dt} h = 11000 + (0.2 \times 11000 - 10^{-5} \times 11000^2) \times 1 = 11990$

(b) ① $\left(\frac{dP}{dt}\right)_0 = 1000$

The predictor point: $P_1^0 = P_0 + \left(\frac{dP}{dt}\right)_0 h = 11000$

The slope at the predictor point: $\left(\frac{dP}{dt}\right)_1^0 = 0.2 \times 11000 - 10^{-5} \times 11000^2 = 990$

$P_1 = P_0 + \frac{\left(\frac{dP}{dt}\right)_0 + \left(\frac{dP}{dt}\right)_1^0}{2} h = 10995$

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② $(\frac{dP}{dt})_1 = 0.2\times 10995 - 10^{-5}\times 10995^2 = 990.09975$

The predictor point: $P_2^0 = P_1 + (\frac{dP}{dt})_1 h = 11985.09975$

The slope at the predictor point: $(\frac{dP}{dt})_2^0 = 0.2\times 11985.09975 - 10^{-5}\times 11985.09975^2$

$= 960.59$

$P_2 = P_1 + \frac{(\frac{dP}{dt})_1 + (\frac{dP}{dt})_2^0}{2} h = 11970$

(c) ① $P_{\frac{1}{2}} = P_0 + (\frac{dP}{dt})_0 \frac{h}{2} = 10000 + (0.2\times 10000 - 10^{-5}\times 10000^2)\times\frac{1}{2} = 10500$

The slope at the mid-point: $(\frac{dP}{dt})_{\frac{1}{2}} = 0.2\times 10500 - 10^{-5}\times 10500^2 = 997.5$

$P_1 = P_0 + (\frac{dP}{dt})_{\frac{1}{2}} h = 10997.5$

② $P_{\frac{3}{2}} = P_1 + (\frac{dP}{dt})_1 \frac{h}{2} = 10997.5 + (0.2\times 10997.5 - 10^{-5}\times 10997.5^2)\times\frac{1}{2} = 11492.5$

The slope at the mid-point: $(\frac{dP}{dt})_{\frac{3}{2}} = 0.2\times 11492.5 - 10^{-5}\times 11492.5^2 = 977.7$

$P_2 = P_1 + (\frac{dP}{dt})_{\frac{3}{2}} h = 11975$