

- 1a) hydraulically smooth: $e < 5v$
 transitional rough: $5v < e < 14.8v$
 fully rough: $e > 14.8v$

$$(b) \quad Z = \rho l^2 \left(\frac{du}{dy} \right)^2 \quad \text{--- (1)}$$

$$l = ky \quad \text{--- (2)}$$

Subs (2) into (1).

$$Z = \rho k^2 y^2 \left(\frac{du}{dy} \right)^2$$

$$\sqrt{\frac{Z}{\rho k^2 y^2}} = \frac{du}{dy}$$

$$du = \frac{1}{k} \sqrt{\frac{Z}{\rho}} \cdot \frac{dy}{y}$$

$$\int du = \frac{1}{k} M_* \int \frac{dy}{y}$$

$$\text{, where } M_* = \sqrt{\frac{Z_0}{\rho}}, \quad k = 0.4$$

$$u = 2.5 M_* \ln y + C$$

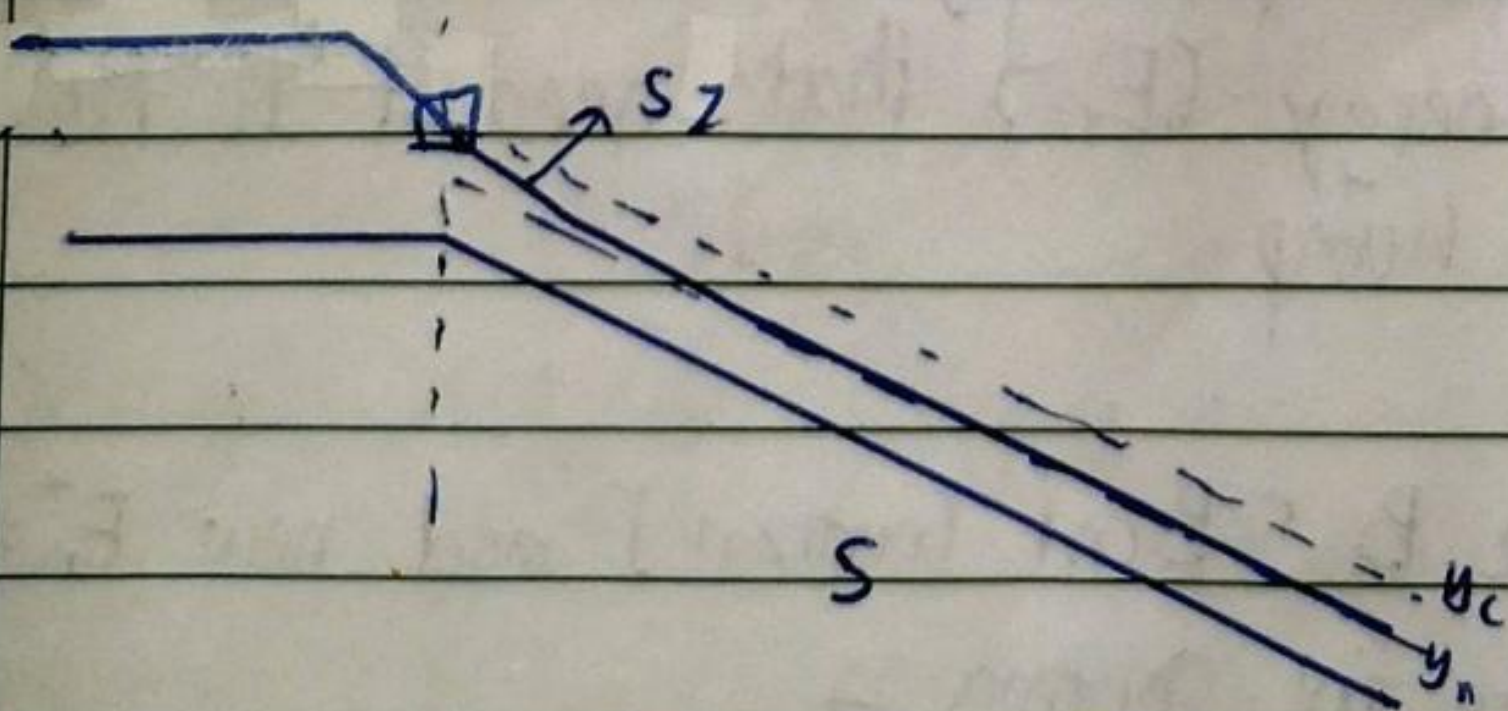
- (c) $E = 2m$ (measured from bed lv to upstream section)

$$E = \frac{3y_c}{2}$$

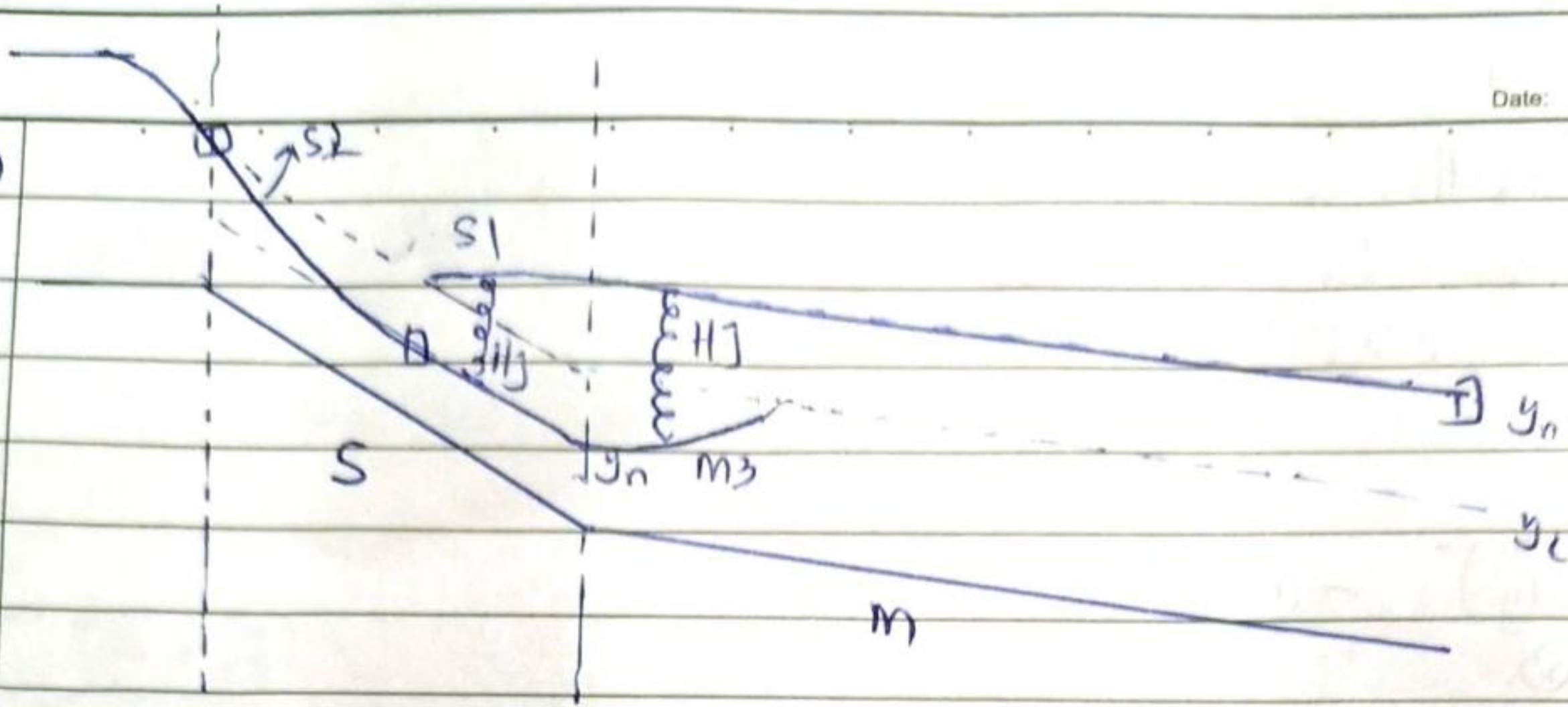
$$y_c = \frac{4}{3} m$$

$$y_c = \sqrt[3]{\frac{q^2}{g}}$$

$$q = 4.82 \text{ m}^2/\text{s}$$



(ii)



(2a)

$$R_h = \frac{A}{P_w}$$

$$P = 2y + b$$

$$A = by$$

$$b = \frac{A}{y}$$

$$P = 2y + \frac{A}{y}$$

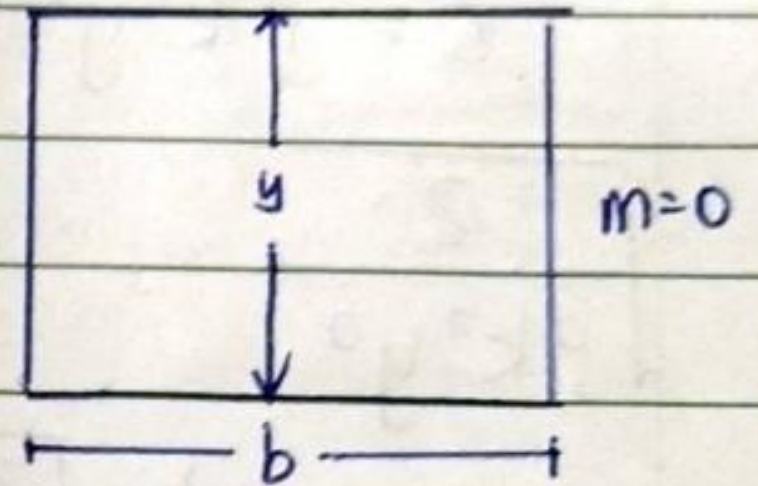
$$\frac{dP}{dy} = 2 - \frac{A}{y^2} = 0 \quad (\text{for min } P_w)$$

$$\frac{A}{y^2} = 2$$

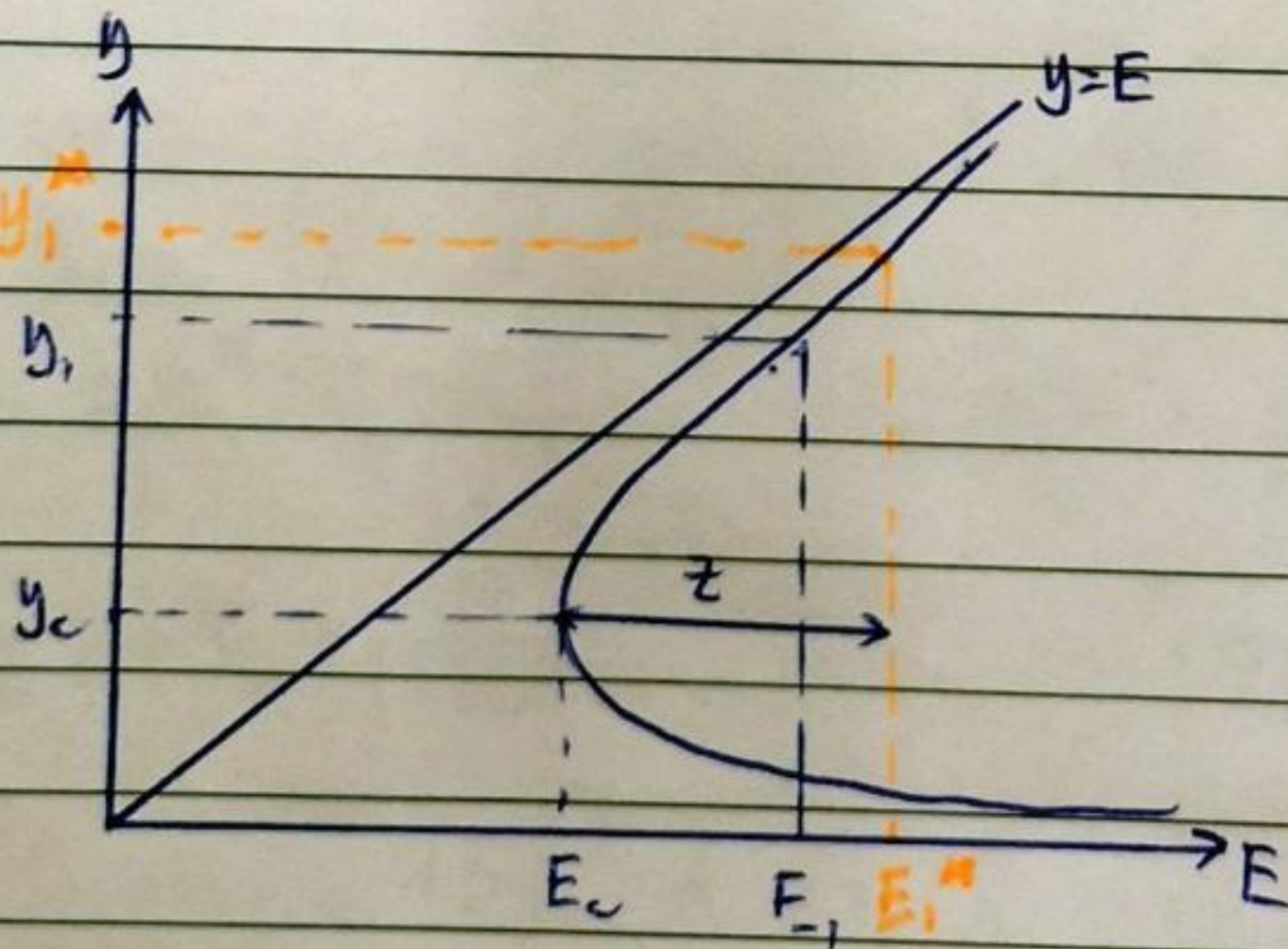
$$A = 2y^3$$

$$by = 2y^3$$

$$b = 2y^2$$

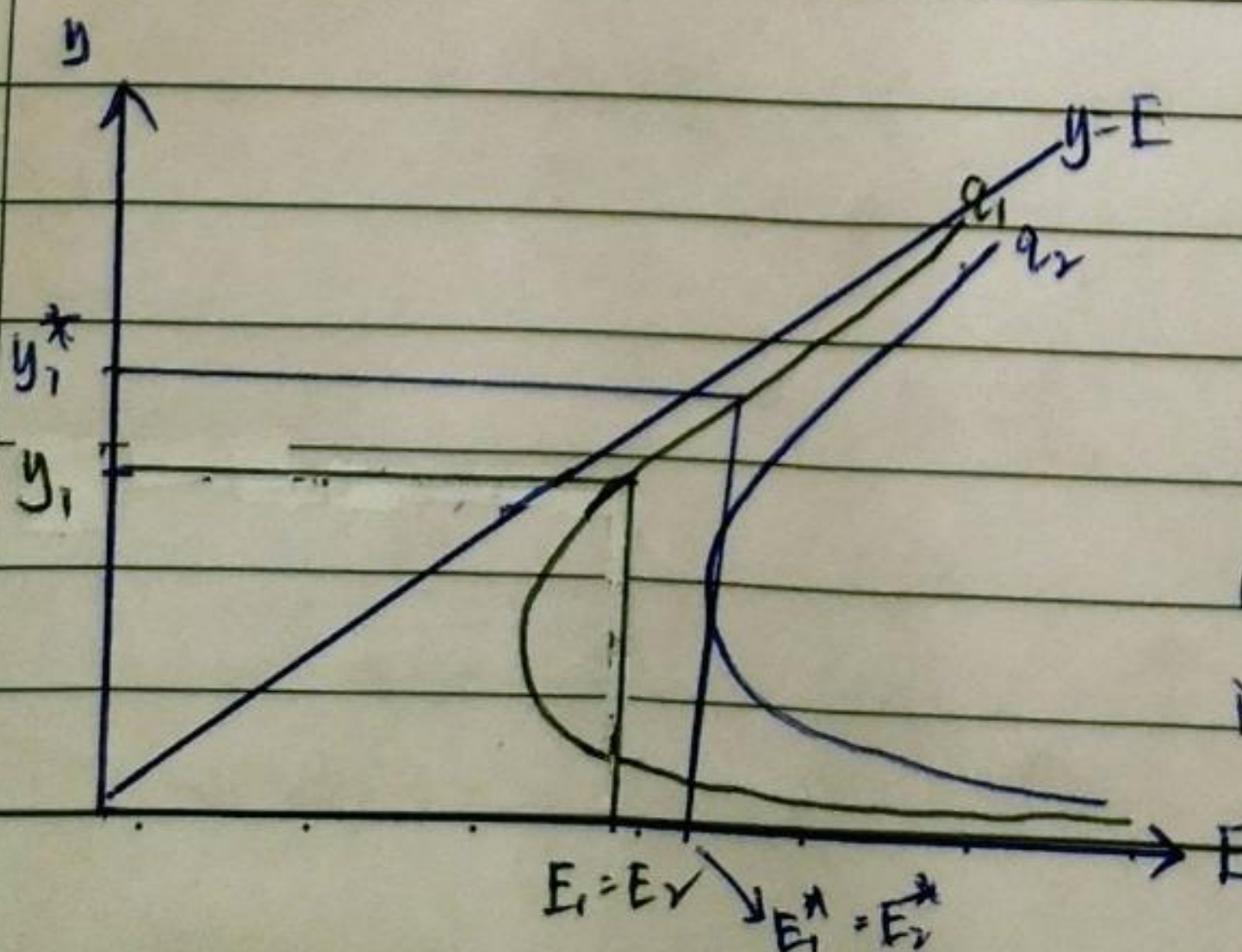


(b)(i)



In this condition, choking and ponding both occur since $\Delta z > \Delta z_{crit}$.

Since E_1 is too small for water to flow through the hump with a height of Δz thus, the flow increases its upstream flow depth (ponding) to reach the minimum Energy (E_c) that needed to flow through the hump.



Since $B < B_{crit}$ (critical) and new $E_1^* = E_2^*$ occur at Section 2

y_1 becomes new y_1^* , $y_2 = y_{c2}$ for q_2 and choking occur at Section 2, Since y_1 increases its depth to y_1^* , ponding occurs at Section 1.

$$(ii) \quad F_1 = F_2 = \rho Q (V_2 - V_1)$$

$$\rho g h_c A_1 + \frac{\rho Q^2}{A_1} = \rho g h_c A_2 + \frac{\rho Q^2}{A_2}$$

$$\text{From (3a)} \quad A = by + my^2$$

$$h_c = \frac{\frac{my^3}{3} + \frac{by^2}{2}}{by + my^2}$$

$$g \left(\frac{\frac{my_1^3}{3} + \frac{by_1^2}{2}}{by_1 + my_1^2} \right) A_1 + \frac{Q^2}{by_1 + my_1^2} = g \left(\frac{\frac{my_2^3}{3} + \frac{by_2^2}{2}}{by_2 + my_2^2} \right) A_2 + \frac{Q^2}{by_2 + my_2^2}$$

$$9.81 \left(\frac{\frac{2(0.4)^3}{3} + \frac{6(0.4)^2}{2}}{6(0.4) + 2(0.4)^2} \right) A_1 + \frac{40^2}{6(0.4) + 2(0.4)^2} = 9.81 \left(\frac{\frac{2y_2^3}{3} + \frac{6y_2^2}{2}}{2y_2 + 6y_2^2} \right) A_2 + \frac{40^2}{2y_2 + 6y_2^2}$$

$$5.127 + 588.24 = 6.54y_2^3 + 29.43y_2^2 + \frac{1600}{2y_2 + 6y_2^2}$$

$$593.367 = 6.54y_2^3 + 29.43y_2^2 + \frac{1600}{2y_2 + 6y_2^2}$$

by trial and error,
 $y \approx 3.5m$.

Yes the answer computed in part (i) and part (ii) should be the same since equation from part (i) is derived from part (ii)

$$(c) \quad m=0, \quad b=6m$$

$$F_x = \frac{by^2}{2} + \frac{Q^2}{gyb}$$

$$\frac{6y_1^2}{2} + \frac{40^2}{9.81y_1(6)} = \frac{6y_2^2}{2} + \frac{40^2}{9.81(6y_2)}$$

$$3(0.4)^2 + \frac{40^2}{9.81(6)(0.4)} = 3y_2^2 + \frac{40^2}{9.81(6y_2)}$$

$$-68.44y_2 + 3y_2^2 + 27.18 = 0$$

$$y_2 = 4.56m$$

$$\frac{y_2}{y_1} = \frac{4.56}{0.4} = 11.4$$

checking:

$$F_{r1} = \frac{V_1}{\sqrt{gy_1}}$$

$$V_1 = 16.67$$

$$= 8.42$$

$$\frac{y_2}{y_1} = \frac{1}{2} \left(\sqrt{1 + 8(8.42)^2} - 1 \right)$$

$$= 11.4$$

Ans: St1] eqn can be used since St1] eqn is derived based on the condition that the channel is rectangle.

4a) In S_2 profile,
At downstream limit, $y \rightarrow y_n \therefore S_0 \rightarrow S_f, \frac{dy}{dx} \rightarrow 0 \therefore y$ asymptotes to y_n

In S_3 profile.
At downstream limit, $y \rightarrow y_n \therefore S_0 \rightarrow S_f, \frac{dy}{dx} \rightarrow 0 \therefore y$ asymptotes to y_n .

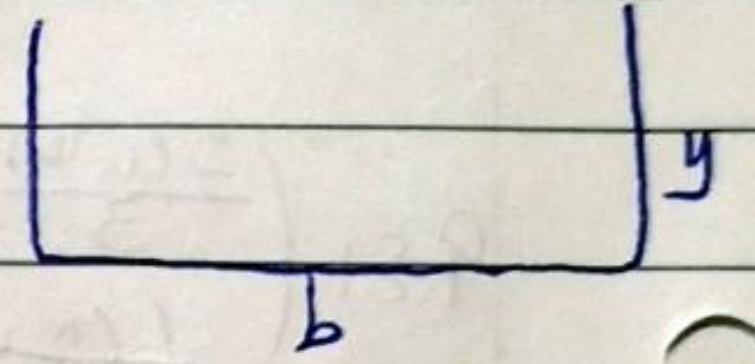
4b) i) $B = 5\text{m}$ $Q = 17\text{m}^3/\text{s}$ $S_0 = 0.001$, $n = 0.024$. $q = 3.4\text{m}^2/\text{s}$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = 1.06\text{m}$$

$$Q = \frac{1}{n} A R_h^{2/3} S_0^{1/2}$$

$$R_h = \frac{A}{P_w}$$

$$= \frac{by}{2y+b}$$

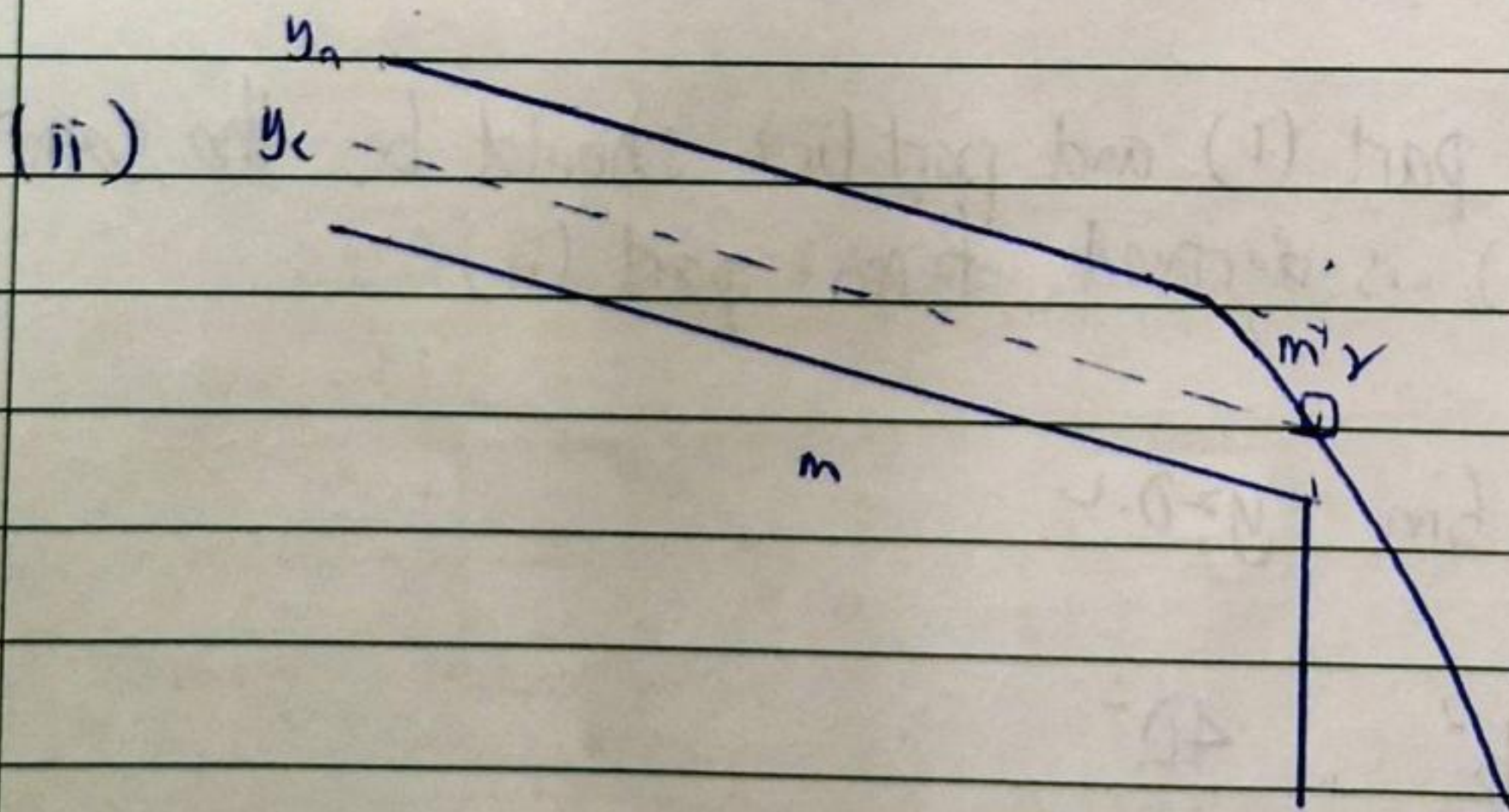


$$17 = \frac{1}{0.024} (5y) \left(\frac{5y}{2y+5} \right)^{2/3} (0.001)^{1/2}$$

by trial and error,

$$y_n = 2.3\text{m}$$

\therefore mild slope $\therefore y_n > y_c$



$$(biii) y_1 = 0.99(2.3) \quad V_1 = 1.49 \text{ m/s.}$$

$$= 2.277$$

$$R_1 = \frac{2.277 \times 5}{5 \times 2.277 + 5} = 1.19$$

$$y_2 = y_c = 1.06 \text{ m.}$$

$$R_2 = \frac{1.06 \times 5}{2 \times 1.06 + 5} = 0.744 \quad V_2 = 3.2 \text{ m/s.}$$

$$\bar{R} = 0.967 \quad \bar{V} = 2.345 \text{ m/s}$$

$$S_f = \frac{n^2 V^2}{R^{4/3}} = 3.31 \times 10^{-3}$$

$$\Delta E = \left[2.277 + \frac{1.49^2}{2(9.81)} \right] - \left[1.06 + \frac{3.2^2}{2(9.81)} \right]$$

$$= 0.808 \text{ m.}$$

$$\Delta X = \frac{\Delta E}{S_0 - S_f}$$

$$= \frac{0.808}{0.001 - 3.31 \times 10^{-3}}$$

$$= 349.42 \text{ m (from free fall)}$$

