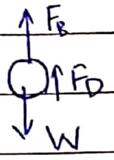
1(a)



V_s = volume

$$W = mg = \rho_s V_s g = \rho_s \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 g = \frac{\pi \rho_s D^3 g}{6}$$

$$F_B = \rho_w g V_s = \rho_w g \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = \frac{\pi \rho_w g D^3}{6}$$

$$F_D = C_D \frac{\rho_w V^2}{2} A = C_D \frac{\rho_w V^2}{2}\left(\frac{\pi}{4} D^2\right) = \frac{C_D \rho_w V^2 D^2 \pi}{8}$$

Equilibrium: $W = F_D + F_B$

$$\Rightarrow \frac{\pi \rho_s D^3 g}{6} = \frac{C_D \rho_w V^2 D^2 \pi}{8} + \frac{\pi \rho_w g D^3}{6}$$

$$\Rightarrow \frac{C_D \rho_w}{8} V^2 = \frac{gD}{6}(\rho_s - \rho_w)$$

$$\Rightarrow V = \sqrt{\frac{4gD}{3C_D}\left(\frac{\rho_s - \rho_w}{\rho_w}\right)}$$

With $D = 5mm$, $\rho_s = 2650\ kg/m^3$, $\nu = 1\times10^{-6}\ m^2/s$, $\rho_w = 1000\ kg/m^3$:

$$Re = \frac{VD}{\nu} = \frac{0.005V}{10^{-6}} = \cancel{5V(10^{-9})}\ 5000V$$

- Try $C_D = 0.4$ $\Rightarrow V = \sqrt{\frac{4(9.81)(0.005)}{3(0.4)}\left(\frac{2650-1000}{1000}\right)} = \cancel{0.921}\ 0.519\ m/s$

$\Rightarrow Re = 5000 \times \cancel{0.921}\ 0.519 = \cancel{4605}\ 2595$

~~With $C_D = 0.4$~~, from C_D versus Re diagram, $Re = 2595 \Rightarrow C_D = 0.39$

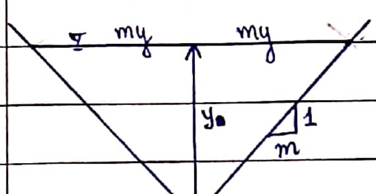
- Try $C_D = 0.39 \Rightarrow V = 0.526\ m/s$

$\Rightarrow Re = 5000 \times 0.526 = 2630$

From C_D vs Re diagram, $Re = 2630 \Rightarrow C_D = 0.39$

$\therefore V = 0.526\ m/s$

(b)



$$\left.\begin{aligned} A &= 2\times\frac{1}{2}y\times my = my^2 \\ P &= 2y\sqrt{1+m^2}\end{aligned}\right\} \Rightarrow R_h = \frac{A}{P} = \frac{my^2}{2y\sqrt{1+m^2}}$$

$$Q = \frac{1}{n} A R_h^{2/3} S_o^{1/2} = \frac{1}{n} A \left(\frac{A}{P}\right)^{2/3} S_o^{1/2}$$

$$= \frac{1}{n} \times \frac{A^{5/3}}{P^{2/3}} S_o^{1/2}$$

To maximize Q, P must be minimum.

For constant A, $m = \frac{A}{y^2}$

No.:

$$\Rightarrow P = 2y\sqrt{1+\left(\frac{A}{y^2}\right)^2} = 2y\sqrt{1+\frac{A^2}{y^4}}$$

$$\frac{dP}{dy} = 2\sqrt{1+\frac{A^2}{y^4}} + 2y\,\frac{-\frac{4A^2y^3}{y^8}}{2\sqrt{1+\frac{A^2}{y^4}}}$$

$$= 2\sqrt{1+\frac{A^2}{y^4}} - \frac{4A^2}{y^4\sqrt{1+\frac{A^2}{y^4}}}$$

To get P_{min}, $\frac{dP}{dy} = 0 \Rightarrow 2\sqrt{1+\frac{A^2}{y^4}} = \frac{4A^2}{y^4\sqrt{1+\frac{A^2}{y^4}}}$

$$\Rightarrow y^4\left(1+\frac{A^2}{y^4}\right) = 2A^2$$

$$\Rightarrow y^4 = A^2 \Rightarrow y = \sqrt{A} \Rightarrow A = y^2 \Rightarrow m = 1$$

Hence, $R_h = \frac{my^2}{2y\sqrt{1+m^2}} = \frac{y^2}{2\sqrt{2}\,y} = 0.354y$

(c) Manning's equation: $V = \frac{1}{n} R_h^{2/3} S_o^{1/2}$

$$\Rightarrow n = \frac{R_h^{2/3} S_o^{1/2}}{V} = R_h^{1/6} \cdot \frac{\sqrt{R_h S_o}}{V} \quad (1)$$

Since $u_* = \sqrt{gR_hS_o} \Rightarrow \sqrt{R_hS_o} = \frac{u_*}{\sqrt{g}}$ (2)

Since $\tau_o = \frac{\rho f V^2}{8} \Rightarrow V = \sqrt{\frac{8\tau_o}{\rho f}} = \sqrt{\frac{8\rho u_*^2}{\rho f}} = u_*\sqrt{\frac{8}{f}}$ (3)

Subst (2) and (3) into (1):

$$n = R_h^{1/6} \cdot \frac{u_*/\sqrt{g}}{u_*\sqrt{\frac{8}{f}}}$$

$$\therefore n = \sqrt{\frac{f}{8g}}\, R_h^{1/6}$$

NO.:

2.

(a) 4 characteristics of the specific energy E vs flow depth y curve.
- 2 limits: the curve has 2 asymptotes $y = 0$ and $y = E$.
- There is a minimum point between the 2 limits, which is E_{min} or E_c. $E < E_c$ ~~??~~ are physically impossible
- At E_{min} or E_c, flow is at critical flow condition and flow depth under this condition is called critical depth, y_c
- For any E, there are 2 alternate depths - one higher than y_c (subcritical flow regime) and the other lower than y_c (supercritical flow regime)

(b) (i) $q = V y_o = 1.3 \times 0.5 = 0.65\ m^2/s$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{0.65^2}{9.81}} = 0.351\ m$$

Since $y_o > y_c$, the bed slope is mild slope.

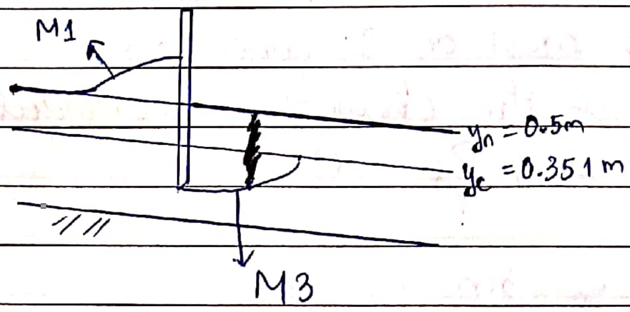
(ii) ~~$V_2 = \frac{q}{y_2}$~~ $\quad E_1 = y_1 + \frac{q^2}{2gy_1^2}$

$$E_2 = y_2 + \frac{q^2}{2gy_2^2} = 0.1 + \frac{0.65^2}{2(9.81)(0.1^2)} = 2.253\ m$$

Since $E_1 = E_2$, we have equation: $y_1 + \frac{0.65^2}{2(9.81)y_1^2} = 2.253$

$\Rightarrow y_1 = 2.249\ m$

(iii)



(c) (i) $q_1 = \frac{Q}{B_1} = \frac{10}{10} = 1\ m^2/s \quad \Rightarrow y_{c_1} = \sqrt[3]{\frac{q_1^2}{g}} = 0.467$

$$E_1 = y_1 + \frac{q_1^2}{2gy_1^2} = 0.52 + \frac{1^2}{2(9.81)(0.52^2)} = 0.708\ m$$

No.:

Depression ⇒ $E_1 = E_2 - \Delta Z \Rightarrow E_2 = 0.708 + 0.4 = 1.108\ m$

⇒ $\cancel{y_2^*}$ $q_2 = \frac{Q}{b_2} = \frac{10}{5} = 2\ m^2/s \Rightarrow y_{c_2} = 0.742 m$

$\Rightarrow E_{c_2} = \frac{3}{2} y_{c_2} = 1.112\ m$

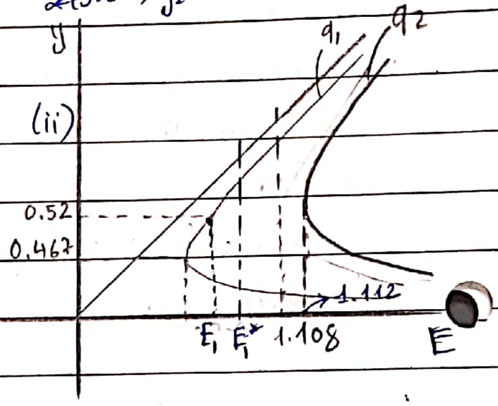
$\cancel{E_2 = y_2 + \frac{q_2^2}{2gy_2^2}}$ $\cancel{\Rightarrow 1.108 = y_2 + \frac{2^2}{2(9.81)y_2^2} = 1.108}$

⇒ ~~y_2 =~~ Since $E_2 < E_{c_2}$, ~~that~~ ponding occurs ⇒ $y_2 = y_{c_2} = 0.742 m$

$E_1^* = 1.112 - 0.4 = 0.712 m$

⇒ $y_1^* = 0.532\ m$

(ii)



(iii) When b_2 is smaller, q_2 is larger. Hence, y_{c_2} and E_{c_2} is larger. Therefore, flow at section 1 will have to adjust more (i.e. increase more) for the choking condition.

3.

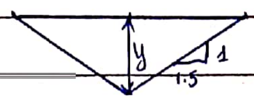
(a) A simple hydraulic jump occurs in a gradually varied flow. It is an abrupt change of flow depths from super-critical to subcritical flow. The values of momentum functions of sub- and supercritical ~~flows~~ flow depths at the jump must be the same. The two flow depths are called conjugate depths and their ratio can be calculated as : $\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1+8F_1^2} - 1\right)$

The above equation is based on 2 conditions:

(1) the cross section of the channel is rectangular

(2) the external forces ; $P_f = 0$

(b) (i)



$y_n = 3m$

$A = 1.5 y_n^2 = 1.5 \times 3^2 = 13.5\ m^2$

$P = 2(3)\sqrt{1^2 + 1.5^2} = 10.82\ m$

$R = \frac{A}{P} = 1.248\ m$

$Q = \frac{1}{n} A R^{2/3} S_o^{1/2} \Rightarrow S_o = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.03 \times 30}{13.5 \times 1.248^{2/3}}\right)^2 = 3.308 \times 10^{-3}$

(ii) Critical flow condition: $\frac{Q^2 B_c}{g A_c^3} = 1$

$B_c = 3y_c$

$A_c = 1.5 y_c^2$

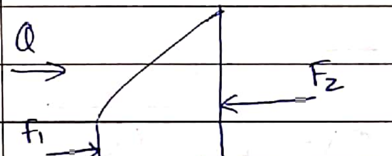
$\Rightarrow \frac{30^2 (3y_c)}{9.81 (1.5 y_c^2)^3} = 1 \Rightarrow y_c = 2.411 \text{ m}$

Since $y_c < y_n$, the slope is Mild.

(iii) $y_2 = y_n = 3$ m

$A_2 = 13.5 \text{ m}^2 \Rightarrow V_2 = \frac{Q}{A_2} = \frac{30}{13.5} = 2.22$ m/s

$A_1 = 1.5 y_1^2 \Rightarrow V_1 = \frac{Q}{A_1} = \frac{30}{1.5 y_1^2} = \frac{20}{y_1^2}$ (m/s)



$F_1 = \rho g A_1 h_1 = \rho g (1.5 y_1^2) \frac{y_1}{3} = \rho g \frac{y_1^3}{2}$

$F_2 = \rho g A h_2 = \rho g (13.5) \frac{3}{3} = 13.5 \rho g$

Momentum equation: $F_1 - F_2 = \rho Q (V_2 - V_1)$

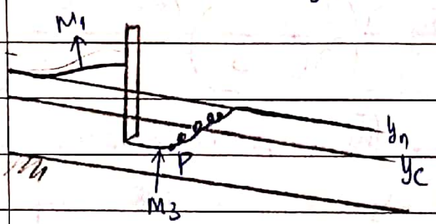
$\Rightarrow \rho g \frac{y_1^3}{2} - \rho g (13.5) = \rho (30)\left(2.22 - \frac{20}{y_1^2}\right)$

$\Rightarrow 9.81 \left(\frac{y_1^3}{2}\right) - 9.81 (13.5) = 30 \left(2.22 - \frac{20}{y_1^2}\right)$

$\Rightarrow y_1 = 1.907$ m

The hydraulic jump is not SHJ because the cross section is triangular, not rectangular.

(iv) Put a sluice gate in the middle of the river, with the opening smaller than y_c



(v) If blocks are added (i.e., $P_f \neq 0$), such that the conjugate depth is less than that calculate in part (iii), i.e., $y_1 < 1.907$ so the location of HJ must occur at a lowerdepth of M3 profile.

No.:

Therefore that location must be upstream of point P, or nearer to the sluice gate.

4.

(a) Energy equation : $E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2} = y + \frac{Q^2}{2gB^2y^2}$

(with $y = \frac{A}{B}$ is hydraulic flow depth, B = top width of the section)

$\Rightarrow \frac{dE}{dy} = 1 - \frac{Q^2}{2gB^2} \times \frac{2y}{y^4} = 1 - \frac{Q^2}{gB^2y^3} = 1 - \frac{Q^2}{gA^2y} = 1 - \frac{V^2}{gy}$

$= 1 - Fr^2$

We have $\frac{dE}{dx} = \frac{dy}{dx} \times \frac{dE}{dy}$

Bernoulli's eq : $H = z + \frac{V^2}{2g} + \frac{P}{\gamma} = z + \frac{V^2}{2g} + y$ (since $y \approx \frac{P}{\gamma}$)

$\Rightarrow H = z + E \Rightarrow \frac{dH}{dx} = \frac{dz}{dx} + \frac{dE}{dx} \Rightarrow -S_f - (-S_o) = \frac{dE}{dx}$

$\Rightarrow \frac{dE}{dx} = S_o - S_f$

Hence, $S_o - S_f = (1 - Fr^2)\frac{dy}{dx} \rightarrow \frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$

(b)(i) $q = \frac{Q}{b} = \frac{50}{5} = 10 \text{ m}^2/\text{s} \Rightarrow y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{10^2}{9.81}} = 2.17 \text{ m}$

- Reach 1 has horizontal slope $\Rightarrow y_n$ does not exist
- Reach 2 : $S_{o2} = 0.01$; $A_2 = by_{n2} = 5y_{n2}$; $P_2 = 2y_{n2} + b = 2y_{n2} + 5$

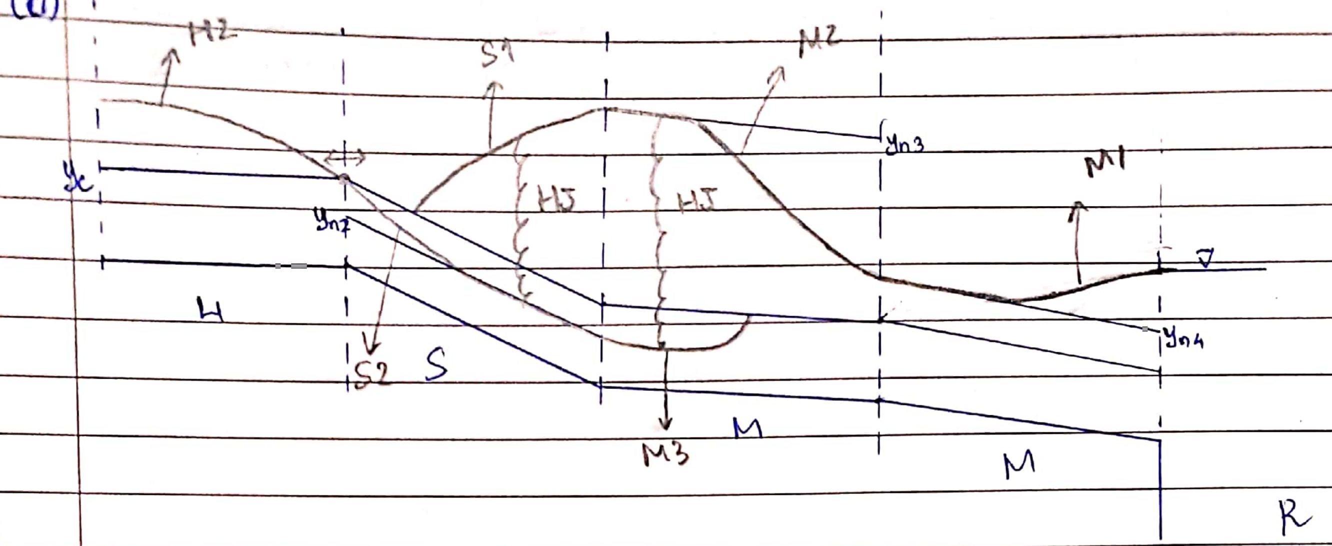
  $Q = \frac{1}{n} A_2 R_2^{2/3} S_{o2}^{1/2} \Rightarrow 50 = \frac{1}{0.02}(5y_{n2})\left(\frac{5y_{n2}}{2y_{n2}+5}\right)^{2/3} 0.01^{1/2}$

  $\Rightarrow y_{n2} = 1.90\text{m}$. Since $y_{n2} < y_c$, the slope is steep
- Reach 3 : $y_{n3} = 6.\cancel{7}06$ m . Since $y_{n3} > y_c$, the slope is mild
- Reach 4 : $y_{n4} = 2.90$ m. Since $y_{n4} > y_c$, the slope is mild

(ii) flow depth at the ~~ex~~ exit $= 1266 - 1261.2 = 4.8$ m

No.:

(ii)



(iii) Assume that the jump will form in reach 2.

$y_1 = y_{n2} = 1.90m$

$V_1 = \frac{q}{y_1} = \frac{10}{1.90} = 5.26 \ m/s \Rightarrow Fr_1^2 = \frac{V_1^2}{gy_1} = \frac{5.26^2}{9.81 \times 1.90} = 1.484$

SHJ equation: $y_2 = \frac{1}{2} y_1 \left(\sqrt{1 + 8Fr_1^2} - 1\right) = \frac{1}{2} \times 1.90\left(\sqrt{1+8\times1.484} - 1\right)$

$= 2.46 \ m$

Since the limits of the S1 profile is $y_c = 2.17m$ and $y_{n3} = 6.71m$

y_2 can be found on this profile.

Hence, the hydraulic jump will form in reach 2.

(iv) ~~Downst~~ Upstream $y_1 = 2.46m$

Downstream $y_2 = y_{n3} = 6.706m$

$A_1 = 2.46 \times 5 = 12.3 \ m^2$, $P_1 = 2(2.46) + 5 = 9.92m$, $R_1 = \frac{A_1}{P} = 1.24 \ m$

$A_2 = 6.706 \times 5 = 33.53 \ m^2$, $P_2 = 2(6.706) + 5 = 18.412m$, $R_2 = A_2 / P_2 = 1.821 \ m$

$\bar{R} = \frac{R_1 + R_2}{2} = 1.5305m$

$V_1 = 10 / 2.45 = 4.082 \ m/s$

$V_2 = 10 / 6.71 = 1.490 \ m/s$

$\Rightarrow \bar{V} = 2.786 \ m/s$

$\bar{S_f} = \left(\frac{n\bar{R}^{2/3}}{\bar{V}}\right)^2 = \left(\frac{0.02 \times 1.5305^{2/3}}{2.786}\right)^2 = 9.0895 \times 10^{-5}$

$E_1 = y_1 + \frac{q^2}{2gy_1^2} = 2.46 + \frac{10^2}{2(9.81)(2.46^2)} = 3.3022m$

$E_2 = y_2 + \frac{q^2}{2gy_2^2} = 6.706 + \frac{10^2}{2(9.81)(6.706)^2} = 6.8193 \ m$

No.:

$$\frac{E_2 - E_1}{x_2 - x_1} = S_0 - S_f \Rightarrow x_2 - x_1 = \frac{6.8193 - 3.3022}{0.01 - 9.0895 \times 10^{-5}} = 354.9 \text{ m}$$

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