o.: 18 - 19 S1 Hoang Thu Minh

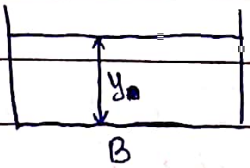
The total shear stress in a pipe can be described as

$$\tau_{total} = \tau_{lam} + \tau_{tur} = \mu\frac{du}{dy} + \eta\frac{du}{dy} \cancel{= \rho(\nu + \varepsilon)\frac{du}{dy}}$$

with η is the eddy viscosity (or coefficient of momentum transfer)
In turbulent pipe flow, there is a continuous mixing of particles, with a consequent transfer of momentum. Hence, its eddy viscosity is very large. On the other hand, in laminar pipe flow, $\tau_{tur} = 0$. Hence, turbulent pipe flow has higher wall shear stress than laminar flow.

Pipeline : boundary layer thickness = ~~1m~~ 0.5m
River : boundary layer thickness = 0.5m

- For $Re \le 1$, the curve is a straight line which follows Stoke's Law $F_D = 3\pi\mu VD$. In this region, the flow around the sphere is ~~vor~~ completely viscous.
- For $Re > 1$, laminar BL starts to separate from the surface of the sphere, beginning first at the rear stagnation point where adverse pressure gradient is strongest, i.e. woke starts to form at the rear stagnation point. As Re increases more, C_D starts to level off because now pressure drag is becoming more important and drag is proportional to V^2
- ~~When~~ $C_D \approx 0.4$ for $10^3 < Re < 10^4$ because the lam BL is separating from the front half of the sphere
- At $Re = 2 \times 10^5$, for a smooth sphere, C_D suddenly by 80%



Set the area, A, to be constant

$A = By_0$, $P = B + 2y_0$, $R_h = \frac{A}{P} = \frac{By_0}{B + 2y_0}$

No.:

$$\frac{dP}{dy} = \frac{-A}{y^2} + 2$$

Equating $\frac{dP}{dy} = 0$ to get P_{min} $\Rightarrow \frac{-A}{y^2} + 2 = 0$

$\Rightarrow 2y^2 = A = By$

$\Rightarrow y = \frac{B}{2}$

$\Rightarrow$ The ~~best~~ most efficient section is half of a square

(e) $\tau_0 = \rho l^2 \left(\frac{du}{dy}\right)^2$ (Prandtl's mixing length concept)

Near the wall, viscous shear dominates. Hence:

$$\tau_0 = \mu \frac{du}{dy} = \mu\left(\frac{u}{y}\right)$$

Also, $\tau_0 = \rho u_*^2$

$\Rightarrow \mu\left(\frac{u}{y}\right) = \rho u_*^2 \Rightarrow \frac{u}{u_*} = \frac{u_* y}{\nu}$ --- (1)

Assume the mixing length, l, near wall is proportional to the distance from the wall, y, i.e. $l = Ky$

Hence, $\tau_0 = \rho l^2 \left(\frac{du}{dy}\right)^2 = \rho K^2 y^2 \left(\frac{du}{dy}\right)^2$

Also $\tau_0 = \rho u_*^2$

$\Rightarrow \rho K^2 y^2 \left(\frac{du}{dy}\right)^2 = \rho u_*^2 \Rightarrow du = \frac{u_*}{K}\frac{dy}{y}$

$\Rightarrow \int du = \int \frac{u_*}{K}\frac{dy}{y}$

$\Rightarrow u = \frac{u_*}{K} \ln y + C$

K = von Karman's const $= 0.4$

$\therefore u = 2.5 u_* \ln y + C$ --- (2)

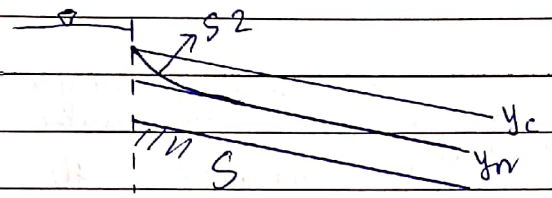
NO.:

2.

(a) – Change from horizontal, adverse or mild slope to steep slope
– Flow ~~exits~~ from mild slope exits the channel.
– Flow from reservoirs enters steep slope

(b) Water flows from reservoir into steep slope ⇒ water depth in the reservoir is at critical flow depth

$\Rightarrow y_c = 3\,m \quad \Rightarrow \quad q = \sqrt{g y_c^3} = \sqrt{9.81 \times 3^3} = 16.27\ m^2/s$



(c) $Q = 5\,m^3/s, \quad b_1 = 5\,m, \quad y_o = 2\,m$

(i) $q_1 = Q/b_1 = 1\ m^2/s \quad \Rightarrow y_{c_1} = \sqrt[3]{\frac{q_1^2}{g}} = \sqrt[3]{\frac{1^2}{9.81}} = 0.467\ m$

$E_o = y_o + \frac{q_1^2}{2gy_o^2} = 2 + \frac{1^2}{2(9.81)(2^2)} = 2.013\ m$

$E_{c_1} = \frac{3}{2} y_{c_1} = \frac{3}{2} \times 0.467 = 0.7008\ m$

For choking to occur, $\Delta Z = E_o - E_{c_1} = 1.313\ m$

∴ Minimum height of the hump $= 1.313\ m$

(ii) $q_2 = Q/b_2 = 5/4 = 1.25\ m^2/s$

$\Rightarrow y_{c_2} = \sqrt[3]{\frac{q_2^2}{g}} = \sqrt[3]{\frac{1.25^2}{9.81}} = 0.542\ m$

$\Rightarrow E_{c_2} = \frac{3}{2} y_{c_2} = \frac{3(0.542)}{2} = 0.813\ m$

Since $E_{c_1} < E_{c_2}$, choking occurs.

$\Rightarrow E_1^* = E_{c_2} + \Delta Z = 0.813 + 1.313 = 2.126\ m$

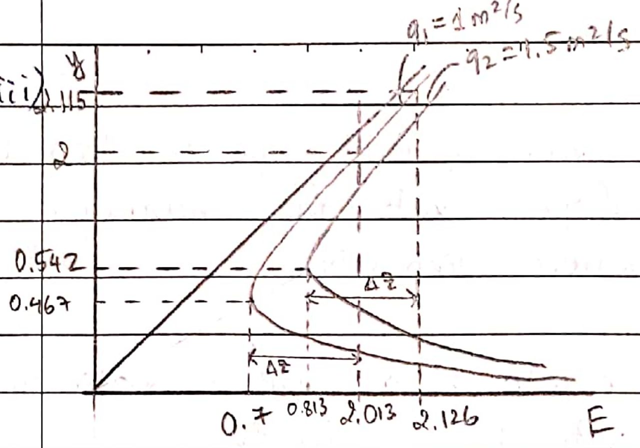
$\Rightarrow y_1^* + \frac{q_1^2}{2g{y_1^*}^2} = 2.126$

$\Rightarrow y_1^* + \frac{1^2}{2(9.81)(y_1^*)^2} = 2.126$

$\Rightarrow y_1^* = 2.115\ m$

No.:

(iii)

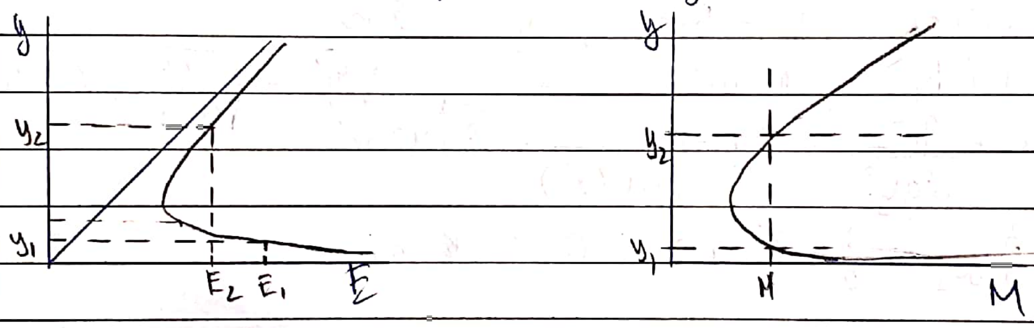


3.

(a)

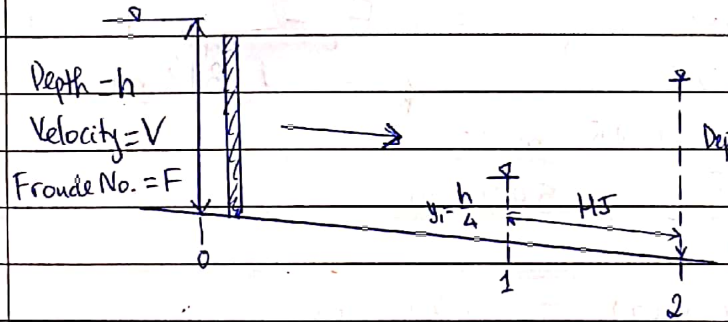
(i) Conjugate depths are two depths that cause the formation of ~~in a~~ hydraulic jump, with one subcritical flow depth abruptly changing ~~and~~ one supercritical flow depth. These two flow depths satisfy Newton's Law.

(ii)



(b)

(i)



$F^2 = \frac{V^2}{gh} \Rightarrow \frac{V^2}{2g} = \frac{F^2 h}{2}$

$E_0 = h + \frac{V^2}{2g} = h + \frac{F^2 h}{2}$

$E_1 = \frac{h}{4} + \frac{V_1^2}{2g}$

$E_0 = E_1 \Rightarrow \frac{3h}{4} + \frac{F^2 h}{2} = \frac{h}{4} + \frac{V_1^2}{2g} \Rightarrow V_1^2 = 2g\left(\frac{3h}{4} + \frac{F^2 h}{2}\right)$

$\cancel{V_1 = 2\sqrt{g\left(\frac{3h}{2} + F^2 h\right)}}$

No.:

$$\cancel{Fr_1^2 = \frac{V_1^2}{g\left(\frac{h}{4}\right)} - 2g\left(\frac{3h}{4} + \frac{F^2h}{2}\right)\frac{gh}{4}}$$

$q = Vh$

$$V_1 = \frac{q}{y_1} = \frac{Vh}{h/4} = 4V$$

$$Fr_1^2 = \frac{V_1^2}{gy_1} = \frac{(4V)^2}{g\left(\frac{h}{4}\right)} = \frac{64V^2}{gh} = 64F^2$$

SHJ equation: $y_2 = \frac{1}{2}y_1\left(\sqrt{1+8Fr_1^2} - 1\right) = \frac{1}{2}\times\frac{h}{4}\left(\sqrt{1+512F^2} - 1\right)$

$$\therefore y_2 = \frac{1}{8}h\left(\sqrt{1+512F^2} - 1\right)$$

(ii) $q = Vh = 0.7 \times 2 = 1.4\ m^2/s$

$y_1 = \frac{h}{4} = \frac{2}{4} = 0.5\ m$

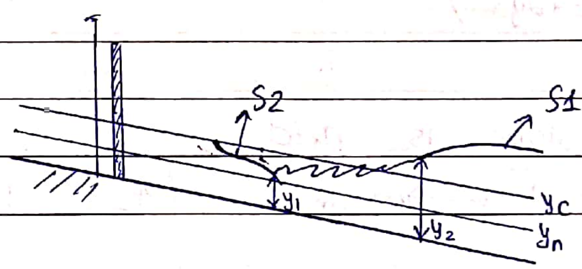
$\Rightarrow V_1 = \frac{q}{y_1} = \frac{1.4}{0.5} = 2.8\ m/s$

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{2.8}{\sqrt{9.81 \times 0.5}} = 1.264$$

$$y_{1,conj} = \frac{1}{2}y_1\left(\sqrt{1+8Fr_1^2} - 1\right) = \frac{1}{2}(0.5)\left(\sqrt{1+8\times1.264^2} - 1\right) = 0.678\,m$$

$F = \frac{V}{\sqrt{gh}} = \frac{0.7}{\sqrt{9.81\times2}} = 0.158 \Rightarrow y_2 = \frac{1}{8}h\left(\sqrt{1+512F^2} - 1\right) = 0.678\,m$

(iii) $y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{1.4^2}{9.81}} = 0.585\ m > y_n \Rightarrow$ Steep slope



No.:

4.

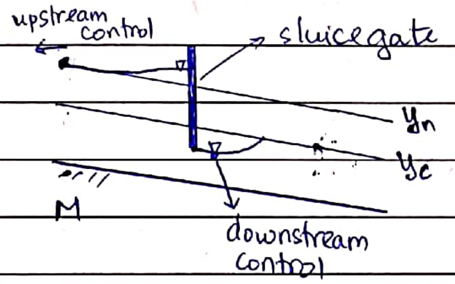
(a)

(i) Control point is any feature in the channel that dictates the relationship between the flow depth and the discharge in its neighborhood.

(ii) - To understand the extent to which the control points may interfere with or even dominate the shape of the longitudinal water surface profile

- To understand the function of the controls.

(iii)



(b) $b = 6m$, $n = 0.022$, $S_o = \tan(0.3°) = 5.236 \times 10^{-3}$

(i) $q = Q/b = 30 : 6 = 5 \ m^2/s$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{5^2}{9.81}} = 1.366 \ m$$

- Calculate normal flow depth y_n:

$$\left.\begin{array}{l} A = 6y_n \\ P = 6 + 2y_n \end{array}\right\} \Rightarrow R = \frac{A}{P} = \frac{6y_n}{6+2y_n}$$

Manning's equation: $Q = \frac{1}{n} A R^{2/3} S_o^{1/2}$

$$\Rightarrow 30 = \frac{1}{0.022} \times (6y_n)\left(\frac{6y_n}{6+2y_n}\right)^{2/3} (5.236 \times 10^{-3})^{1/2}$$

$\Rightarrow y_n = 1.514 \ m$

Since $y_n > y_c$, the slope is mild.

Since $y_1 > y_n$, ~~the flow profile is M1~~ GVF will form

(ii) $E_{A1} = y_1 + \frac{q^2}{2gy_1^2} = 2 + \frac{5^2}{2(9.81)(2^2)} = 2.319 \ m$

$V_2 = \frac{q}{y_2} = \frac{5}{3} = 1.667 \ m/s$; $V_1 = \frac{q}{y_1} = \frac{5}{2} = 2.5 \ m/s$

No.: ....

$$E_2 = y_2 + \frac{q^2}{2gy_2^2} = 3 + \frac{5^2}{2(9.81)(3^2)} = 3.142\ m$$

$$\bar{V} = \frac{1}{2}(V_1 + V_2) = \frac{1}{2}(2.5 + 1.667) = 2.0835\ m/s$$

$A_1 = 2 \times 6 = 12\ m^2$; $P_1 = 6 + 2 \times 2 = 10$; $R_1 = \frac{A_1}{P_1} = 1.2\ m$

$A_2 = 3 \times 6 = 18\ m^2$; $P_2 = 6 + 2 \times 3 = 12$; $R_2 = \frac{A_2}{P_2} = 1.5\ m$

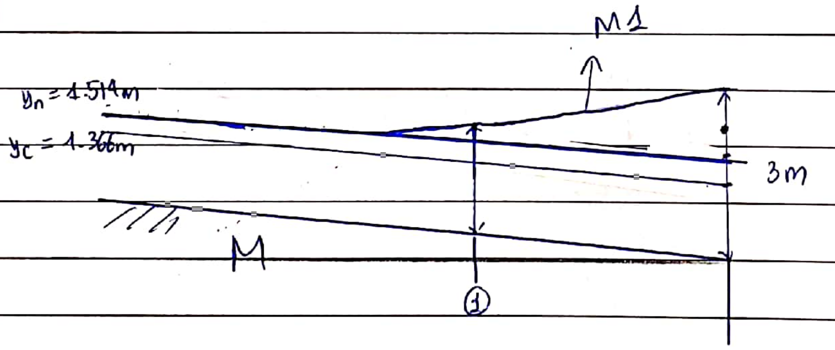
$$\bar{R} = \frac{1}{2}(R_1 + R_2) = \frac{1}{2}(1.2 + 1.5) = 1.35\ m$$

$$\bar{S_f} = \left(\frac{n\bar{V}}{\bar{R}^{2/3}}\right)^2 = \left(\frac{0.022 \times 2.0835}{1.35^{2/3}}\right)^2 = 1.4082 \times 10^{-3}$$

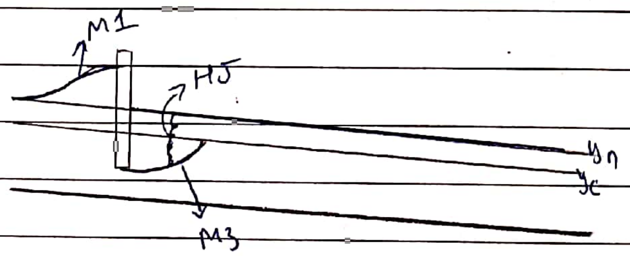
$$\frac{E_2 - E_1}{x_2 - x_1} = S_o - \bar{S_f} \Rightarrow \frac{3.142 - 2.319}{\Delta x} = 5.236 \times 10^{-3} - 1.4082 \times 10^{-3}$$

$\Rightarrow \Delta x = 215\ m$

(iii)



(iv)



$y_1 = 1.228\ m$

$V_1 = \frac{q}{y_1} = \frac{5}{1.228} = 4.072\ m/s$

$Fr_1^2 = \frac{V_1^2}{gy_1} = \frac{4.072^2}{9.81 \times 1.228} = 1.376$

Since $y_1 = 1.228\ m < y_c$, a M3 profile will form downstream of the sluice gate.

$$y_{1,conj} = \frac{1}{2}y_1\left(\sqrt{1 + 8Fr_1^2} - 1\right) = \frac{1}{2} \times 1.228\left(\sqrt{1 + 8 \times 1.376} - 1\right) = 1.514\ m = y_n$$

Hence there will be a hydraulic jump immediately downstream of the sluice gate.