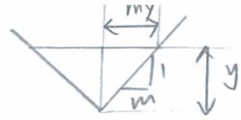
17/18 Sem I CV2015 Solution.

1. (a) There is a change of regime from laminar BL to turbulent BL on the sphere. With turbulent BL, the point of separation now moves from 80° to about 120° from the front stagnation point, causing wake size and pressure drag to reduce.

1b) "Most efficient cross section" is cross section of a given area tunnel, which flow rate is maximized under this cross section. P is minimized while R_h is maximized.

$$Q = \frac{A}{n} R_h^{2/3} S_0^{1/2}$$

(c)



$$P = 2y\sqrt{1+m^2} \quad -(1)$$

$$A = my^2$$

$$y = \sqrt{\frac{A}{m}} \quad -(2)$$

Combine (1) & (2).

$$P = 2\sqrt{A}\sqrt{\frac{1+m^2}{m}}$$

$$P^2 = 4A\,\frac{1+m^2}{m}$$

$$\frac{dP^2}{dm} = 4A \cdot \frac{m(2m)-(1+m^2)}{m^2}$$

$$= 4A \cdot \frac{m^2-1}{m^2}$$

Let $\frac{dP}{dm} = 0$

$$m^2 - 1 = 0$$

$$m = \pm 1$$

(d) $E = y + \frac{q^2}{2gy^2}$

$$q = y\sqrt{2g(E-y)}$$

$$\frac{dq}{dy} = \sqrt{2g}\left(\sqrt{E-y} - \frac{1}{2}\frac{y}{\sqrt{E-y}}\right)$$

Let $\frac{dq}{dy} = 0$

we get $y_c = \frac{2E}{3}$

$$q_{max} = y_c\sqrt{2g\left(\tfrac{3}{2}y_c - y_c\right)}$$

$$= \sqrt{g y_c^3}$$

(e)

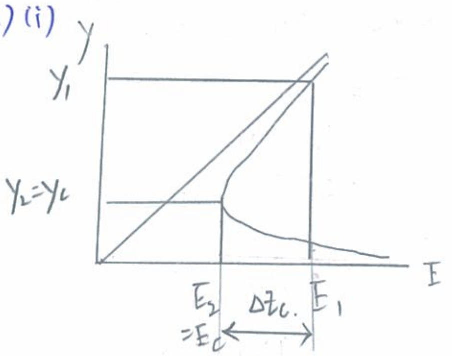
$$W - F_D - B = 0$$

$$\rho_s \cdot \frac{\pi}{6} D^3 \cdot g - \rho_f \frac{\pi}{6} \cdot D^3 \cdot g - C_D\left(\frac{\rho V_t^2}{2}\right) \cdot \frac{\pi}{4} D^2 = 0$$

Arrange,

$$V_t = \sqrt{\frac{4}{3} \cdot \frac{1}{C_D}\left(\frac{\rho_s - \rho_f}{\rho_f}\right) g D}$$

2(a)(i)

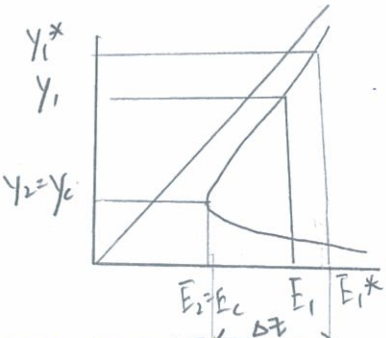


Case 1: $\Delta z = \Delta z_{critical}$

$E_2 = E_c$

$y_2 = y_c$

Choking only.



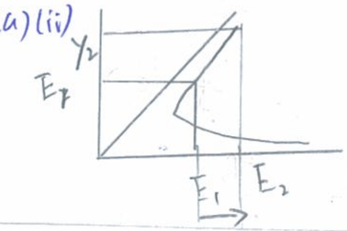
Case 2: $\Delta z > \Delta z_c$

Then $E_2 = E_c$, $y_2 = y_c$

E_1 will increase to E_1^*, y_1 increase to y_1^*

$$E_1^* = E_c + \Delta z.$$

2(a)(ii)



When section depress, E become larger and never reach E_c.

∴ Choking will not happen

2(b)(i) $Q = \frac{A}{n}\left(\frac{A}{P}\right)^{2/3} S_0^{1/2}$

$$16 = \frac{8y_n}{0.0136}\left(\frac{8y_n}{8+2y_n}\right)^{2/3}(0.001)^{1/2}$$

$y_n = 1\text{m}$.

$y_n > y_c$, Mild slope.

$$y_c = \sqrt[3]{\frac{q^2}{g}}$$

$$= \sqrt[3]{\frac{(\frac{16}{8})^2}{9.81}}$$

$$= 0.741\text{m}$$

(ii) $y_2 = y_c$

$E_2 = E_c$

$= 3/2\,(0.741)$

$= 1.112\text{m}$.

$$E_1 = y_n + \frac{V^2}{2g}$$

$$= 1 + \frac{(16)^2}{2g(1)^2(8)^2}$$

$$= 1.204\text{m}$$

$\Delta z = 1.204\text{m} - 1.112\text{m}$

$= 0.092\text{m}$.

(iii) $q_2 = \frac{16}{8-1.5} = 2.46\ \text{m}^3/\text{s}/\text{m}$

$$E_{c2} = \sqrt[3]{\frac{2.46^2}{9.81}} \times 3/2$$

$= 1.277\text{ m} > E_1 = 1.204$

∴ Ponding.

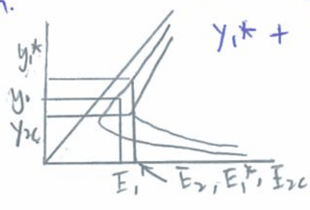
$$y_2 = y_{c2} = \sqrt[3]{\frac{2.46^2}{9.81}} = 0.851$$

$E_1^* = E_{c2}$

$$y_1^* + \frac{V^2}{2g} = 1.277$$

$$y_1^* + \frac{(\frac{16}{8})^2}{2g(y_1^*)^2} = 1.277$$

$$y_1^* = 1.11\text{m}$$



3(a) ① The headloss at a section is the same as for a uniform flow having the same velocity and hydraulics radius of the section.

② The channel slope S_0 is small

③ The roughness coefficient, n is independent of the depth of flow and is constant throughout the channel under consideration.

(b) $$H = z + y + \frac{V^2}{2g}$$

$$\frac{dH}{dx} = \frac{d}{dx}\left(z + y + \frac{V^2}{2g}\right)$$

$$\frac{dH}{dx} = -S_f, \quad \frac{dz}{dx} = -S_0, \quad \frac{d}{dx}\left(y + \frac{V^2}{2g}\right) = \frac{dE}{dx}.$$

$$-S_f = \frac{dE}{dx} - S_0$$

$$\frac{dE}{dx} = S_0 - S_f$$

$$\frac{dE}{dy} \cdot \frac{dy}{dx} = S_0 - S_f$$

$$\frac{dE}{dy} = \frac{d}{dy}\left(y + \frac{Q^2}{2gA^2}\right)$$

$$= 1 - \frac{Q^2}{2g} \cdot \left(\frac{2}{A^3} \cdot \frac{dA}{dy}\right)$$

$$= 1 - \frac{Q^2 B}{gA^3}$$

$$= 1 - Fr^2$$

$$\therefore (1 - Fr^2)\frac{dy}{dx} = S_0 - S_f$$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2} \quad \text{4.}$$

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(c) (i) Assume Mild,

$$Q = \frac{A}{n}\left(\frac{A}{P}\right)^{2/3} S^{1/2}$$

$$= \frac{4.6y}{0.015}\left(\frac{4.6y}{4.6+2y}\right)^{2/3}(0.024)^{1/2}$$

$$E = y + \frac{V^2}{2g}$$

$$10 = y + \frac{Q^2}{2g(4.6y)^2}$$

$$10 = y + \frac{0.024}{2g(0.015^2)}\left(\frac{4.6y}{4.6+2y}\right)^{4/3}$$

$$y = 2.716\text{m}$$

$$Q = 149.36 \text{ m}^3/\text{s}. \qquad q = \frac{149.36}{4.6} = 32.47 \text{ m}^3/\text{s/m}.$$

(ii) $$y_c = \sqrt[3]{\frac{q^2}{g}}$$

$$= \sqrt[3]{\frac{\left(\frac{149.36}{4.6}\right)^2}{9.81}}$$

$$= 4.754\text{m} > y_n = 2.716\text{m}.$$

$\therefore$ Contradict that this is a mild slope

(iii) No. Assume steep slope.

$$E_c = E = 10\text{m}$$

$$y_c = 6.67\text{m}.$$

$$q = \sqrt{g y_c^3}$$

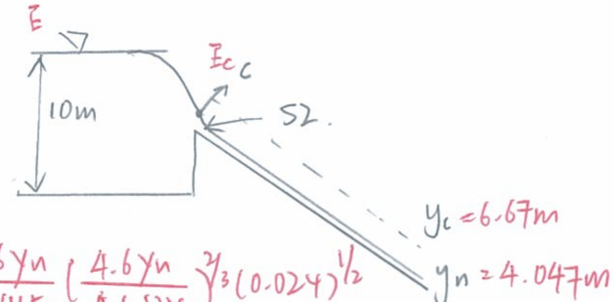
$$= \sqrt{9.81 \times 6.67^3}$$

$$= 53.95 \text{ m}^3/\text{s/m}$$

$$Q = 53.95 \times 4.6$$

$$= 248.18 \text{ m}^3/\text{s}.$$

(iv)



$$248.18 = \frac{4.6y_n}{0.015}\left(\frac{4.6y_n}{4.6+2y_n}\right)^{2/3}(0.024)^{1/2}$$

$$y_n = 4.047\text{m}$$

4(a). $\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2}$.

C1 profile

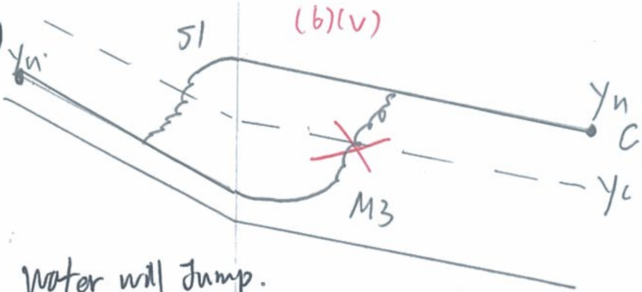
$y > y_n \rightarrow S_0 - S_f > 0$

$y > y_c \rightarrow 1 - Fr^2 > 0$

$\therefore \frac{S_0 - S_f}{1 - Fr^2} > 0$

$\therefore$ y increase when x increase, meanwhile, flow rate will decrease. Thus, C1 is decelarating profile.

(b)(i)



water will Jump.

Either S1, or M3 profile will occur.

(b)(ii) y_n (Steep)

$$Q = \frac{A}{n}\left(\frac{A}{P}\right)^{2/3} S_0^{1/2}$$

$$200 = \frac{30y_n}{0.021}\left(\frac{30y_n}{30+2y_n}\right)^{2/3}(0.01)^{1/2}$$

$y_n = 1.264$m.

y_n (Mild)

$$200 = \frac{30y_n}{0.021}\left(\frac{30y_n}{30+2y_n}\right)^{2/3}(0.0001)^{1/2}$$

$y_n = 5.523$m.

$$y_c = \sqrt[3]{\frac{q^2}{g}}$$

$$= \sqrt[3]{\frac{\left(\frac{200}{30}\right)^2}{9.81}}$$

$= 1.655$m.

y_n (mild) $> y_c$

y_n (steep) $< y_c$

$\therefore$ Steep slope is upstream of mild slope.

(b)(iii) As $y_n = 1.264$m in the beginning, in order to reach $y_n = 5.523$m in downstream, water surface must pass through $y_c = 1.655$m. From steep to mild, the only way is through a hydraulic jump. Therefore, HJ will be formed in channel.

(b)(iv) Assume jump at steep slope

$$\frac{y_{adj}}{y} = \frac{1}{2}\left(\sqrt{1+8Fr^2} - 1\right)$$

$$Fr^2 = \frac{V^2}{gy}$$

$$= \frac{200^2}{9.81(30)^2(1.264)^3}$$

$= 2.2434$

$$y_{adj} = \frac{1.264}{2}\left(\sqrt{1+8(2.2434)} - 1\right)$$

$= 2.1189$m

take note $y_c < 2.1189 < y_{n2}$

$\therefore$ Assumption is correct.

$$S_{f1} = \frac{n^2V_1^2}{y_1^{4/3}}$$

$$= \frac{0.021^2\left(\frac{200}{30\times 2.1189}\right)^2}{2.1189^{4/3}}$$

$= 1.6\times10^{-3}$

$$S_{f2} = \frac{n^2V_2^2}{y_2^{4/3}}$$

$$= \frac{0.021^2\left(\frac{200}{30\times 5.523}\right)^2}{5.523^{4/3}}$$

$= 6.58\times10^{-5}$

$$S_f = \frac{1.6\times10^{-3} + 6.58\times10^{-5}}{2}$$

$= 8.35\times10^{-4}$

$$\Delta X = \frac{E_2 - E_1}{S_0 - S_f} = \frac{5.597 - 2.623}{0.01 - 8.35\times10^{-4}} = 325\text{m}$$

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

$$= 2.1189 + \frac{200^2}{2(9.81)(2.1189)^2(30)^2}$$

$= 2.623$m

$E_2 = 5.597$m.