No.: 16-17   Hoang Thu Minh

1

(a) - If $e < \delta_v$, the flow is in hydraulically smooth flow regime → viscous shear has dominant effect over the roughness shear

- If $e > 14\,\delta_v$, the flow is in fully rough flow regime ← viscous sublayer is lower than roughness

- If $\delta_v < e < 14\,\delta_v$, the flow is in transitional rough flow regime ← viscosity is not imp

(b) - For $Re \leq 1$, the flow is very viscous. The flow follows Stokes' Law: $F_D = 3\pi\mu V D$ or $C_D = 24/Re$

- For $Re > 1$, laminar BL starts to separate from the surface of the sphere. As Re increases more, C_D starts to level off because now pressure drag is becoming more important and drag is proportional to V^2. ~~F~~ When Re is from 10^3 to 10^4, $C_D \approx 0.4$

- At $Re = 2\times 10^5$, C_D suddenly reduced by 50%

(c) Manning's eq: $V = \frac{1}{n} R_h^{2/3} S_o^{1/2} \Rightarrow$ ~~$n = \frac{R_h^{2/3} S_o^{1/2}}{V}$~~ ~~(1)~~

$\tau_o = \frac{\rho f V^2}{8} \Rightarrow V = \sqrt{\frac{8\tau_o}{\rho f}}$ $\tau_o = \frac{\rho f V^2}{8}$

$\Rightarrow n = \frac{R^{2/3} S_o^{1/2}}{\sqrt{\frac{8\tau_o}{\rho f}}} = \sqrt{\frac{\rho f}{8\tau_o}}\, R_h^{2/3} S_o^{1/2}$ ~~(1)~~

$S_o \approx S = \frac{h_f}{L} = \frac{fV^2}{2gD} = \frac{fV^2}{2g(4R_h)} = \frac{fV^2}{8gR_h}$ (2)

Subst (2) into (1): $n = \frac{1}{V} \times R_h^{2/3} \sqrt{\frac{fV^2}{8gR_h}}$

$\rightarrow n = R^{1/6} \sqrt{\frac{f}{g\times 8}}$

(d)

(i) Flow from reservoir into steep slope → critical condition at entrance. Since no energy is lost, $E_o = E_c \Rightarrow E_c = 3m \Rightarrow y_c = \frac{2}{3}E_c = 2\,m$

$\Rightarrow q = \sqrt{g y_c^3} = \sqrt{9.81\times 2^3} = 8.86\ m^2/s$

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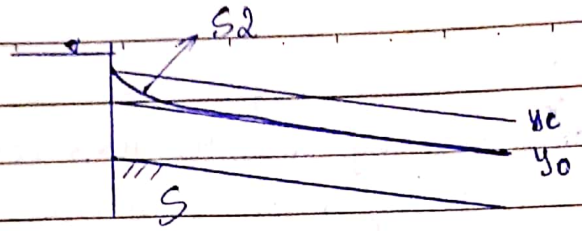
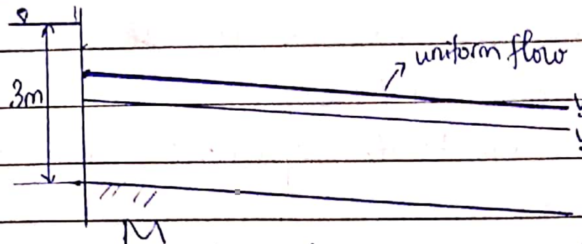


(ii)



Entrance flow depth $= y_o = 2.3m$

No energy loss $\Rightarrow E_o = E_1$

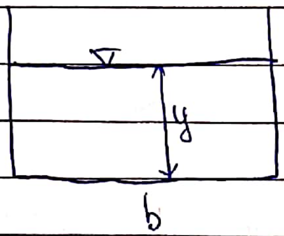
$\Rightarrow 3 = y_o + \frac{q^2}{2gy_o^2} = 2.3 + \frac{q^2}{2(9.81)(2.3^2)}$

$\Rightarrow q = 8.52\ m^2/s$

2.

(a) – Boundary conditions: the curve has 2 asymptotes which are E-axis and line $y = E$.
– Between these two limits, there is a minimum E_c or E_{min}. Any $E < E_c$ is physically impossible.
– E_c is associated with a critical flow depth y_c
– For any E larger than E_c, there are two alternative depths, one larger than y_c (subcritical flow regime) and one smaller than y_c (supercritical flow regime)

(b)



Keep the cross-sectional area, A, as constant

$A = by$

$P = b + 2y$

$R_h = \frac{A}{P} = \frac{by}{b+2y}$

$Q = \frac{1}{n} A R_h^{2/3} S_o^{1/2} = \frac{1}{n} A \left(\frac{A}{P}\right)^{2/3} S_o^{1/2} = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S_o^{1/2}$

To maximize Q, we have to minimize P.

Since $A = by \Rightarrow b = \frac{A}{y}$

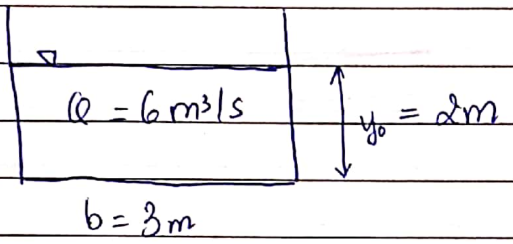
Subsitute into P : $P = \frac{A}{y} + 2y$

Differentiate P with respect to y: $\frac{dP}{dy} = \frac{-A}{y^2} + 2$

Equate $\frac{dP}{dy}$ with 0 to ~~pt~~ find P_{min} : $\frac{-A}{y^2} + 2 = 0$

$\Rightarrow \cancel{y = \sqrt{\frac{2}{A}}}$ $\left.\begin{array}{l} A = 2y^2 \\ \text{Also, } A = by \end{array}\right\} \Rightarrow b = 2y$

(c)



(i) $q = Q/b = 2\ m^2/s$

$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{2^2}{9.81}} = 0.742m$

Since $y_c < y_0$, the bed slope is Mild

(ii) $E_c = \frac{3}{2} y_c = \cancel{7}\ \frac{3}{2} \times 0.742 = 1.113\ m$

(iii) Assume flow depth at section 1 $y_1 = y_0 = 2m$

$E_1 = y_1 + \frac{q^2}{2gy_1^2} = 2 + \frac{2^2}{2 \times 9.81 \times 2^2} = 2.051\ m$

At section 2 , $E_2 = E_1 - \Delta Z = 2.051 - 1.5 = 0.551\ m$

Since $E_2 < E_c$, choking occurs.

$\Rightarrow E_2 = E_c = 1.113\ m$ and $y_2 = y_c = 0.742m$

$E_1^* = E_2 + \Delta Z = 1.113 + 1.5 = \cancel{4}\ 2.613m$

$\Rightarrow y_1^* + \frac{2^2}{2(9.81)(y_1^*)^2} = 2.613m$

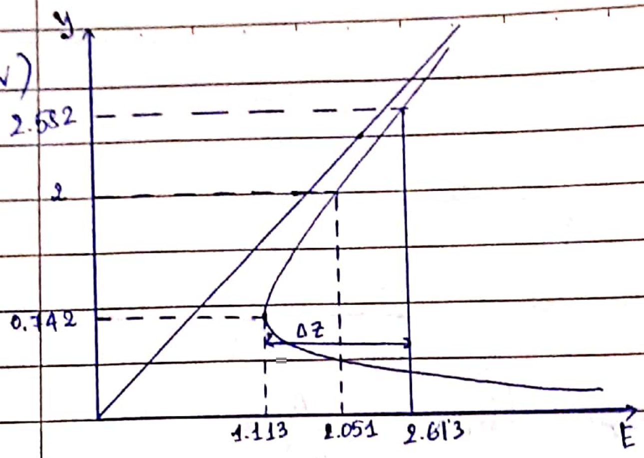
$\Rightarrow y_1^* = 2.582\ m$

$\therefore$ Flow depth at section 1 = 2.582 m

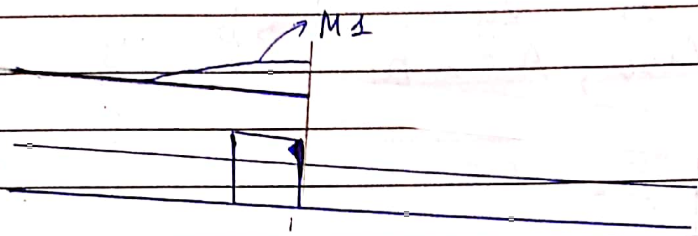
Flow depth at section 2 = ~~4~~ 0.742 m

No.:

(iv)



(v)



3.

(a) – Direct Step method assigns values to y to calculate x

– Standard step method assigns values to x to calculate y.

– We must use standard step method when the channel is not prismatic

(b) $Q = 50\,m^3/s$, $n = 0.025$, $S_0 = 3.75\times10^{-2}$, $b = 3m$

(i) $q = \frac{Q}{b} = \frac{50}{3} = 16.67\ m^2/s$

$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{16.67^2}{9.81}} = 3.048\ m$, for both reaches

– On the horizontal slope, normal flow depth does not exist

– For reach 1:

$A = by_n = 3y_n$

$P = b + 2y_n = 3 + 2y_n$

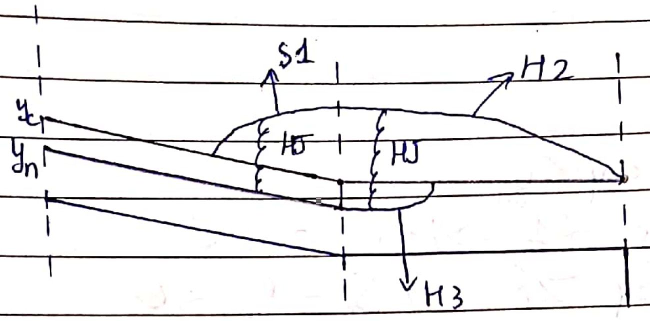
$R = \frac{A}{P} = \frac{3y_n}{3+2y_n}$

$Q = \frac{1}{n} A R^{2/3} S_0^{1/2} \Rightarrow 50 = \frac{1}{0.025} \times (3y_n)\left(\frac{3y_n}{3+2y_n}\right)^{2/3} (3.75\times10^{-2})^{1/2}$

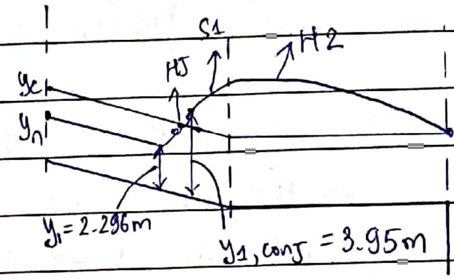
$\Rightarrow y_n = 2.296\ m$

No.:

(ii) $y_n < y_c \Rightarrow$ reach 1 has steep slope



(iii) Assume the hydraulic jump will form on the upstream slope.



$y_1 = y_n = 2.296\ m$

$V_1 = \frac{q}{y_1} = \frac{16.67}{2.296} = 7.260\ m/s$

$Fr_1^2 = \frac{V_1^2}{gy_1} = \frac{7.260^2}{9.81 \times 2.296} = 2.340$

By SHJ equation: $y_{1,conj} = \frac{1}{2} y_1 \left(\sqrt{1+8Fr_1^2} - 1\right)$

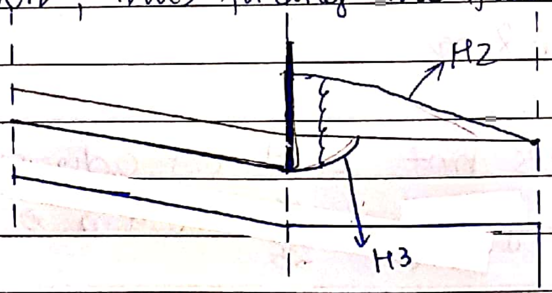
$= \frac{1}{2} \times 2.296 \left(\sqrt{1+2.34 \times 8} - 1\right)$

$= 3.95\ m$

Since the limits of S1 profile are $y_c = 3.048$ and $y_p = 5.5m$, $y_{1,conj}$ can be found on the S1 profile

$\therefore$ HJ will form on upstream slope

(iv) Put a sluice gate at the intercection of the 2 slopes, with the opening $= y_n = 2.296m$. The sluice gate will prevent the S1 profile formation, thus forcing the flow to jump on the downstream slope, where $y_{1,conj}$ can be found on H2 profile



No.:

4.

(a)

(i) $\dfrac{dy}{dx} = \dfrac{S_o - S_f}{1 - Fr^2}$

For M2 profile: $y_c < y < y_n$

- Since $y > y_c$, $Fr < 1 \Rightarrow 1 - Fr^2 > 0$
- Since $y < y_n$, $S_f > S_o \Rightarrow S_o - S_f < 0$

Hence $\dfrac{dy}{dx} = \dfrac{-ve}{+ve} = -ve$

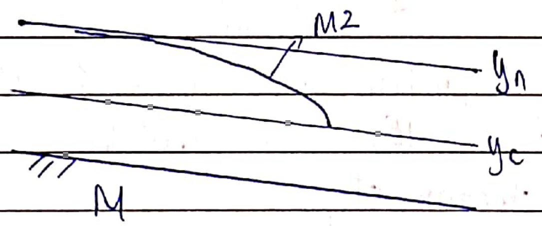
$\therefore$ M2 profile is decreasing in the downstream direction

(ii) - When $y \to y_c$, $Fr \to 1 \Rightarrow \dfrac{dy}{dx} \to \infty$

$\Rightarrow$ the flow cuts y_c-line at right angle

- When $y \to y_n$, $S_f \to 0 \Rightarrow \dfrac{dy}{dx} \to 0$

$\Rightarrow$ the flow asymptotes to y_n line



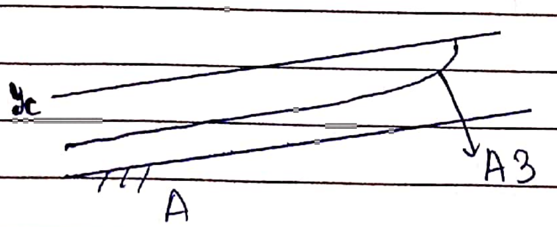
(b)

(i) $q = \dfrac{Q}{b} = \dfrac{400}{40} = 10\ m^2/s$

$y_c = \sqrt[3]{\dfrac{q^2}{g}} = \sqrt[3]{\dfrac{10^2}{9.81}} = 2.168\ m$

normal flow depth does not exist on adverse slope

Flow depth at entrance: $y = \dfrac{q}{V} = \dfrac{10}{25} = 0.4\ m < y_c \Rightarrow$ A3 profile



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(ii) ~~$y_1 = y_c = 2.168$ m~~

~~$V_1 = \frac{q}{y_1} = 4.613$ m~~ $\rightarrow$ ~~$Fr_1^2 = \frac{V_1^2}{gy_1} = 1$~~

For the hydraulic jump to form on channel 1, the A2 profile must be formed on channel 1 ~~$\rightarrow$ the flow depth at the interface of 2 channels must be larger than y_c~~

~~Assume the channel 2 to be long~~

~~Hence~~ channel 2 has steep slope so that the ~~flow depth~~ in channel 1 is A2 profile.

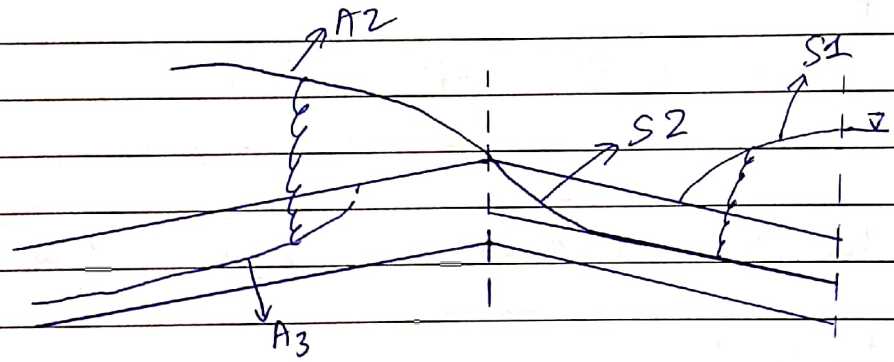
$\Rightarrow y_n < 2.168$ m.

$$\underbrace{Q}_{400} = \frac{1}{n} A R_h^{2/3} S_o^{1/2} = \frac{1}{0.025}(40 y_n)\left(\frac{40y_n}{40+2y_n}\right)^{2/3} S_o^{1/2}$$

If $y_n = 2.168\text{m} \Rightarrow S_o = 5.436 \times 10^{-3}$

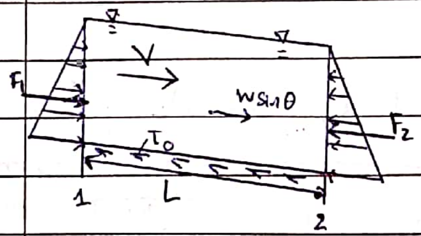
$\therefore S_o > 5.436 \times 10^{-3}$

(iii)



No.:

1(c) Given: $\tau_o = \frac{\rho f V^2}{8}$, $V = \frac{1}{n} R_h^{2/3} S_o^{1/2}$



Take control volume between section 1 & section 2 in an open channel flow with bed slope $S_o = \tan\theta$

For small θ : $\sin\theta \approx \tan\theta \Rightarrow S_o \approx S$

Momentum equation gives: $F_{net} = \dot{M}_2 - \dot{M}_1$

$\Rightarrow F_1 - F_2 + W\sin\theta -$ opposing shear force $= \rho Q(V_2 - V_1)$

Since $V_1 = V_2 = V$ and $F_1 = F_2$ (hydrostatically distributed)

$\Rightarrow$ opposing shear force $= W\sin\theta$

$\Rightarrow \tau_o PL = \rho g A L \sin\theta$ (1)

Substitute $\sin\theta \approx \tan\theta = h_f/L = S \approx S_o$ into (1):

$\tau_o PL = \rho g A L S_o$

$\Rightarrow \tau_o = \rho g \frac{A}{P} S_o = \rho g R_h S_o$

Equating with $\tau_o = \frac{\rho f V^2}{8}$: $\frac{\rho f V^2}{8} = \rho g R_h S_o$

$\Rightarrow V = \sqrt{\frac{8 g R_h S_o}{f}}$

Equating with Manning's equation $\Rightarrow \sqrt{\frac{8 g R_h S_o}{f}} = \frac{1}{n} R_h^{2/3} S_o^{1/2}$

$\Rightarrow n = \sqrt{\frac{f}{8g}} R_h^{1/6}$