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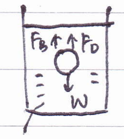
CV2015 Hydraulics. Sem 1 2015-16.

1-(a) Flow separation occurs when there is an excessive momentum loss in the lam BL moving downstream against adverse pressure gradient, where the pressure is increasing along the boundary. There will be a backflow trying to move in the direction of favourable pressure gradient ($\frac{dp}{dx} < 0$) which is characterized by the eddies. In turb flow, the velocity at a point in the flow direction fluactuates in both magnitude and direction. Hence, there is violent mixing of faster moving outer strata strata into the slower moving inner strata & vice versa resulting in an increased of mean velocity close to the boundary. This added energy allow the 'BL' stick longer against the adverse pressure gradient. Therefore, location of separation being moved further downstream to a region of higher pressure. at which angle $= 120°$ from the front stagnation point.

(b)



$+\uparrow \sum F_y = 0$: $F_B + F_D - W = 0$ $\qquad W = mg = \rho V g$

$F_D = W - F_B$ $\qquad V = \frac{4}{3}\pi r^3$

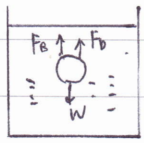
$C_D \frac{1}{2}\rho_f V_t^2 \cdot A = \rho_s V_s g - \rho_f V_s g$ $\qquad = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \frac{1}{6}\pi D^3$

$$C_D\left(\frac{\rho_f V_t^2}{2}\right)\left(\frac{\pi}{4}D^2\right) = \frac{1}{6}\pi D^3 (g)(\rho_s - \rho_f)$$

$$V_t^2 = \frac{8D}{6} g(\rho_s - \rho_f) \cdot \frac{1}{C_D} \cdot \frac{1}{\rho_f}$$

$$V_t = \sqrt{\frac{4}{3} \cdot \frac{1}{C_D} \cdot \left(\frac{\rho_s - \rho_f}{\rho_f}\right) \cdot g D}$$ proved.

(c) $\rho_s = 2560\ kg/m^3$, $\rho_g = 1260\ kg/m^3$, $\mu_g = 1.512\ Ns/m^2$, $D_{max} = ?$



$D_{max} = ?$

$Re \le 1$

$\because Re \le 1$, stoke's law applies. ($F_D = 3\pi\mu V D$)

$F_D = W - F_B$ $\qquad Re \le 1$

$3\pi\mu V D = \rho_s V_s g - \rho_g V_s g$ $\qquad \frac{\rho_g V D}{\mu_g} \le 1$

$3\pi\mu V D = \frac{\pi D^3}{6} g(\rho_s - \rho_g)$ $\qquad \frac{1260\ VD}{1.512} \le 1$

$3\pi(1.512)(1.2\times10^{-3}) = \frac{\pi(D^3)}{6}(g)(2560 - 1260)$

$\therefore D_{max} = 0.01368\ m.$ $\qquad VD \le 1.2\times10^{-3}$

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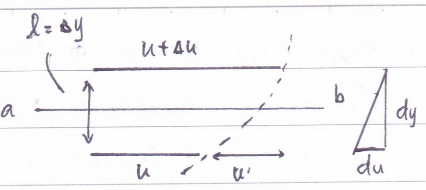
(d) u' = fluac vel in x direction

v' = " y "

z' = " z "



consider over a short time, a fluid mass move upward from below line ab with vertical vel (v') and the surrounding velocity is $(u+u')$

Momentum per unit time transported $= \rho Q(\text{vel}) = \rho(v'dA)(u)$

Force = rate of change of momentum

$\tau dA = \rho Q(\Delta \text{vel})$

$\tau dA = \rho v' dA (u+u'-u)$

$\tau = \rho v'u'$

$= -\rho \overline{v'u'}$ ← temporal average values

↑ minus sign is added becos the product of fluactuating component is always negative.

Assume mixing length, l is propotional to the distance from wall, y ($l = Ky$)

where $l = Ky$, K = von Karman Constant

$\frac{du}{dy} = \frac{|\overline{u'}|}{l}$

$\overline{u'} = l\frac{du}{dy}$

$\tau = \rho\,\overline{u'v'}$

$= \rho\, l^2 \left(\frac{du}{dy}\right)^2$

$= \rho (K^2y^2)\left(\frac{du}{dy}\right)^2$

$\left(\frac{du}{dy}\right)^2 = \frac{\tau}{\rho K^2 y^2}$

$\frac{du}{dy} = \sqrt{\frac{\tau}{\rho}}\;\frac{1}{Ky}$ (note: $\tau = \rho U_*^2$)

$du = U_* \frac{1}{Ky} dy$

$u = \int \frac{U_*}{K} \cdot \frac{1}{y}\, dy$

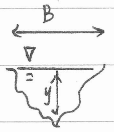
$= \frac{U_*}{0.4} \int \frac{1}{y} dy$

$= 2.5\, U_* \ln y + C$

$= \frac{1}{4} U_* \ln y + C_x$ proved.

2.

(a) $F_r = \frac{V}{\sqrt{gy}}$, $y = \frac{A}{B}$ = hydraulic mean depth.



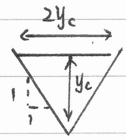
$dA = B\,dy$

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2A^2 g}$$

To find E_{min}: $\frac{dE}{dy} = 1 + \frac{Q^2}{2g}(-2)(A^{-3})\left(\frac{dA}{dy}\right) = 0$

$$1 = \frac{Q^2\left(\frac{dA}{dy}\right)}{gA^3}$$

$$\therefore \frac{Q^2 B_c}{g A_c^3} = 1$$



For most efficient cross section, it is 90° triangular channel.

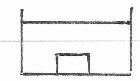
$$\frac{Q^2 B}{g A^3} = 1$$

$B_c = 2y_c$

$A = \frac{1}{2}(y_c)(2y_c) = y_c^2$

$$\frac{Q^2(2y_c)}{g(y_c^2)^3} = 1$$

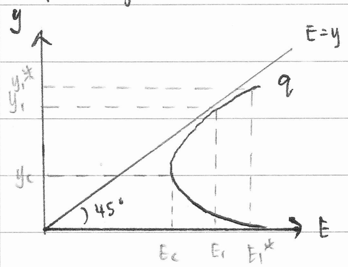
$$Q = \sqrt{\frac{g y_c^5}{2}}$$ proved.

(b) 

Hump with constant channel: $E = y + \frac{V^2}{2g} = y + \frac{q^2}{2gy^2}$ $(q = Vy)$

(i) $q_1 = q_2$, $E_1 = E_2 + \Delta z$

Sp E diagram:

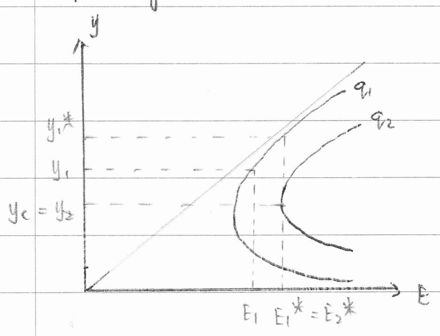


Choking & ponding occurs when $\Delta z > (\Delta z)_{crit}$.

If there is a hump which high enough so that $\Delta z > (\Delta z)_{crit}$, choking will occur at the downstream level which is at the hump and increase the real energy at downstream to be $E_2^* = E_c$.

Due to this, ponding has to happen at upstream and increases the water level from y_1 to y_1^* and therefore energy upstream to be $E_1^* = E_c + \Delta z$.

(ii) Sp E diagram:



Contraction : $E_1 = E_2$, $q_1 \neq q_2$

At a certain energy $E_1 < E_{c2}$, when the flow enters the contracted section, q_1 changes to q_2. Since $E_1 < E_{c2}$, real E_2 has to increase $E_2^* = E_c$ and also increase in critical depth $y_2 = y_{c2}$ in contraction.

Since $E_1 = E_2$, ponding will occur at upstream and E_1 therefore increases to E_1^* so that $E_1^* = E_2^*$. (Note: y_1 increases to y_1^* as shown in figure.)

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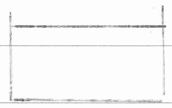
2(c)



4m

$Q = 2 m^3/s$.

(i) $(\Delta Z)_{crit} = ?$

$q = \frac{Q}{b} = \frac{2}{4} = \frac{1}{2} m^2/s$

$E_1 = E_2 + (\Delta Z)_{crit}$

$y_c = \sqrt[3]{\frac{q^2}{g}}$

$y_1 + \frac{q^2}{2g y_1^2} = y_c + \frac{q^2}{2g y_c^2} + (\Delta Z)_{crit}$

$= \sqrt[3]{\frac{(0.5)^2}{g}}$

$1 + \frac{(0.5)^2}{2g(\cancel{\ }1)^2} = 0.2943 + \frac{0.5^2}{2g(0.2943)^2} + (\Delta Z)_{crit}$

$= 0.2943 m$.

$\therefore (\Delta Z)_{crit} = 0.5713 m$.

(ii) hump and contraction :

$E_1 = E_2 + \Delta Z$, $q_1 \neq q_2$

$y_{2c} = \sqrt[3]{\frac{q_2^2}{g}}$

$q_2 = \frac{Q}{b} = \frac{2}{2} = 1 m^2/s$

$1.0127 = E_2 + 0.5713$

$= \sqrt[3]{\frac{1^2}{g}}$

$\therefore E_2 = 0.4414 m$.

$\because E_2 < E_{2c}$

$= 0.4671 m$.

$\therefore$ ponding and choking occurs.

$E_{2c} = \frac{3}{2} y_{2c}$

$= \frac{3}{2}(0.4671)$

$= 0.7 m$.

$E_1 = y_1 + \frac{q^2}{2g y_1^2}$

$= 1 + \frac{0.5^2}{2g(1)^2}$

$= 1.0127 m$.

$E_1^* = E_{2c} + (\Delta Z)_{crit}$

$y_1^* + \frac{q_1^2}{2g(y_1^*)^2} = 0.7 + 0.5713$.

$\frac{19.62(y_1^*)^3 + 0.01274}{19.62(y_1^*)^2} = 1.2713$

$19.62(y_1^*)^3 - 19.62(1.2713)(y_1^*)^2 + 0.01274 = 0$

By trial and error,

$y_1^* = 1.2709 m$ or $y_1^* = 0.023 m$.

(impossible).

So, the flow depth at u/s is 1.2709m and flow at uncontracted section is 0.4671m. #.

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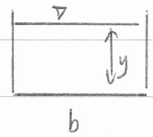
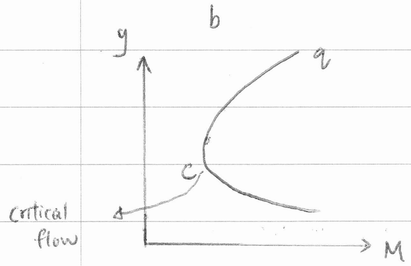
3(a)





$$M = \frac{y^2}{2} + \frac{q^2}{gy}$$

$$\frac{dM}{dy} = \frac{1}{2}(2y) + \frac{q^2}{g}(-y^{-2}) = 0$$

$$y - \frac{q^2}{gy^2} = 0$$

$$y = \frac{q^2}{gy^2}$$

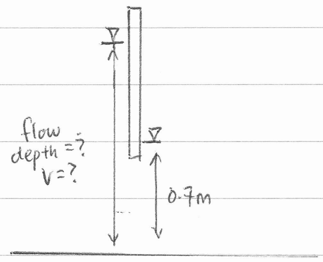
$$gy^3 = q^2$$

$$y^3 = \frac{q^2}{g}$$

$$\therefore y = \sqrt[3]{\frac{q^2}{g}}.$$

Hence, the minimum momentum function, M corresponds to the critical flow condition.

3(b) (i)



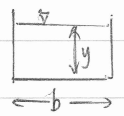
$y_1 = 0.7m$

$E_0 = 4m$

Assume there is no energy loss.

Denote E_0 be the specific energy of u/s

E_1 be the specific energy of d/s.



$Q = AV = byV$

$q = \frac{Q}{b} = Vy$

$$E_1 = y_1 + \frac{V^2}{2g} = y_1 + \frac{q^2}{2gy_1^2}$$

$$E_0 = E_1 = 4m$$

$$4 = 0.7 + \frac{q^2}{2g(0.7)^2}$$

$$\frac{q^2}{2g(0.7)^2} = 3.3$$

$$q = 5.633\ m^2/s.$$

$$E_0 = y_0 + \frac{q^2}{2gy_0^2}$$

$$4 = y_0 + \frac{5.633^2}{2gy_0^2}$$

$$\frac{1.617 + y_0^3}{y_0^2} = 4$$

$$y_0^3 - 4y_0^2 + 1.617 = 0$$

$$y_0 = 3.89\ m.$$

$$q = Vy$$

$$V = \frac{5.633}{3.89}$$

$$= 1.45\ m/s.$$

$\therefore$ flow depth and flow velocity just u/s of sluice gate is 3.89m and 1.45 m/s.

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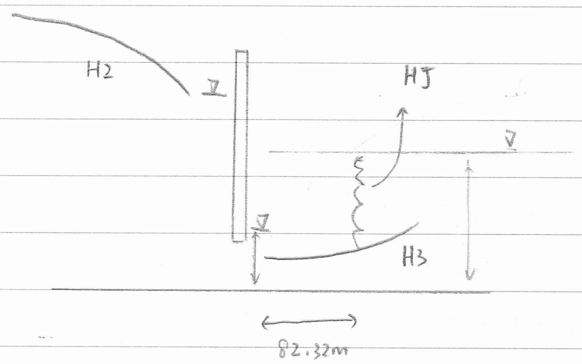
(ii) The channel is wide, Manning roughness coefficient, $n=0.03$. $y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{3.633^2}{g}} = 1.79$

No	y	v	E	ΔE	S_f	Avg (S_f)	$S_0 - S_f$	ΔX	X
1	0.9								
2	1.79								

Using excel or refer to tut 11/12 question 1(b),
procedure is more or less the same.

∴ distance d/s of sluice gate is 82.32m. (I use excel to calculate)

(iii)



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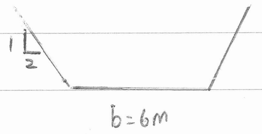
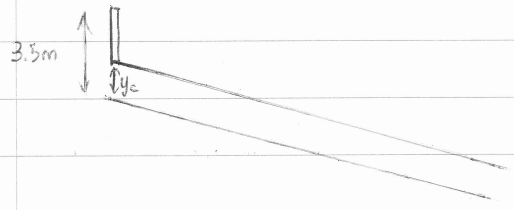
4.





(a) critical flow condition.

(b) Assume no energy loss. , Q=?

$$y_c = \sqrt[3]{\frac{q^2}{g}}$$

$$E = y + \cancel{\frac{V^2}{2g}}^{\to 0} = 3.5m$$

$$\therefore E = y_c$$

$$3.5 = \sqrt[3]{\frac{q^2}{g}}$$

$$(3.5)^3 = \frac{q^2}{g}$$

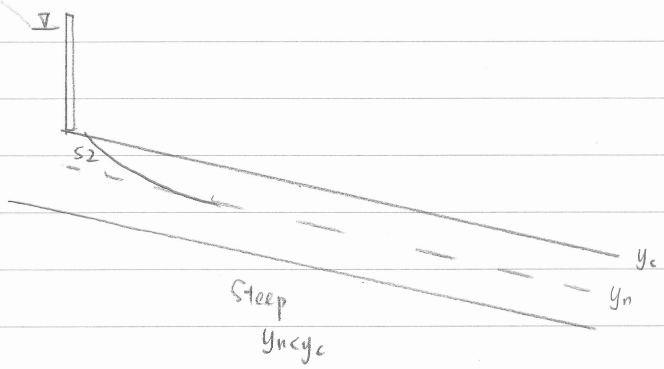
$$q = \sqrt{(3.5)^3 g}$$

$$= 20.51 \, m^2/s.$$

$$q = \frac{Q}{b}$$

$$Q = 20.51(6) = 123.06 \, m^3/s.$$

(c)



(d) If slope is mild which means $y_n > y_c$, the magnitude of the expected flow rate will smaller than the steep slope because normal depth is now become larger, which means the velocity of the flow will be small in the mild slope. According to the formula, $Q = AV$, when $V\downarrow$, $Q\downarrow$.

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(e)