Sem I Exam 2014-2015

CV2015 Hydraulics

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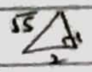
1a) Slope of water surface = bed slope = 0.001

Thickness of boundary layer = y_o (not sure)

b) 0.368 y_o from the channel bed



c) $R_h = \frac{A}{P_{wetted}} = \frac{\frac{1}{2}(y_o)(5+5+2y_o+2y_o)}{5+2y_o\sqrt{1^2+2^2}} = \frac{y_o(5+2y_o)}{5+2y_o\sqrt{5}}$

$Q = \frac{1}{n} A R_h^{2/3} S_o^{1/2} = \frac{1}{0.015} \times y_o(5+2y_o)\left[\frac{y_o(5+2y_o)}{5+2y_o\sqrt{5}}\right]^{2/3} \cdot \sqrt{0.001}$

When $y_o = 2m$,

$Q = \frac{1}{0.015} \times 2(5+2\times2)\left[\frac{2(5+2\times2)}{5+2\times2\sqrt{5}}\right]^{2/3}\sqrt{0.001}$

$= 44.988 = 45.0\ m^3/s$

d) $Q_1 = \frac{1}{n} A R_h^{2/3} S_o^{1/2} = \frac{1}{n} A R_h^{2/3}\sqrt{0.001}$

$Q_2 = \frac{1}{n} A R_h^{2/3} S_{o2}^{1/2} = \frac{1}{n} A R_h^{2/3}\sqrt{0.002}$

$\frac{Q_2}{Q_1} = \frac{\frac{1}{n} A R_h^{2/3}\sqrt{0.002}}{\frac{1}{n} A R_h^{2/3}\sqrt{0.001}} \times 100\% = 141.42\% \approx 140\%$

Flow rate is approximately 40% higher if bed slope is doubled.

e) $Q_3 = \frac{1}{n} A_3 R_{h3}^{2/3} S_o^{1/2} = \frac{1}{n}\sqrt{S} \cdot \left[y_o(5+2y_o)\right]\left[\frac{y_o(5+2y_o)}{5+2y_o\sqrt{5}}\right]^{2/3}$

$= \frac{1}{n}\sqrt{S} \cdot \left[2(5+2\times2)\right]\left[\frac{2(5+2\times2)}{5+2(2)\sqrt{5}}\right]^{2/3}$

$= \frac{1}{n}\sqrt{S}\,(21.340)$

$Q_4 = \frac{1}{n} A_4 R_{h4}^{2/3} S_o^{1/2} = \frac{1}{n}\sqrt{S}\left[4(5+2\times4)\right]\left[\frac{4(5+2\times4)}{5+2\times4\sqrt{5}}\right]^{2/3}$

$= \frac{1}{n}\sqrt{S}\,(89.866)$

$\frac{Q_4}{Q_3} = \frac{\frac{1}{n}\sqrt{S}\,(89.866)}{\frac{1}{n}\sqrt{S}\,(21.340)} = 4.211 \approx 4$ times

f) In a wide channel, $A \approx by_o$, $P \approx b$, $R_h \approx y_o$

$Q_5 = \frac{1}{n} A_5 R_{h5}^{2/3} S_o^{1/2} = \frac{1}{n}\sqrt{S}(by_o)(y_o)^{2/3}$

$Q_6 = \frac{1}{n} A_6 R_{h6}^{2/3} S_o^{1/2} = \frac{1}{n}\sqrt{S}(b2y_o)(2y_o)^{2/3}$

$\frac{Q_6}{Q_5} = \frac{\frac{1}{n}\sqrt{S}(by_o)(y_o)^{2/3}}{\frac{1}{n}\sqrt{S}(b2y_o)(2y_o)^{2/3}} = 3.175 \approx 3$ times

[HOW TO DRAW SPECIFIC ENERGY DIAGRAM. Choose a starting point, move vertically up/downwards to the next curve to find y at the same E. If there is energy loss, after reaching the new point, shift left to correspond to a decrease in E. Draw the diagrams systematically so that even if a lot of steps are involved like this question, you can still solve the questions easily and not lose track of what you are drawing]

2ai) $E = y + \frac{v^2}{2g}$

In a rectangular channel, $Q = VA = Vby$

$$\frac{Q}{b} = Vy$$

$$q = Vy$$

$$V = \frac{q}{y}$$

$$\therefore E = y + \frac{v^2}{2g} = y + \frac{q^2}{2gy^2} \quad //$$

aii) y

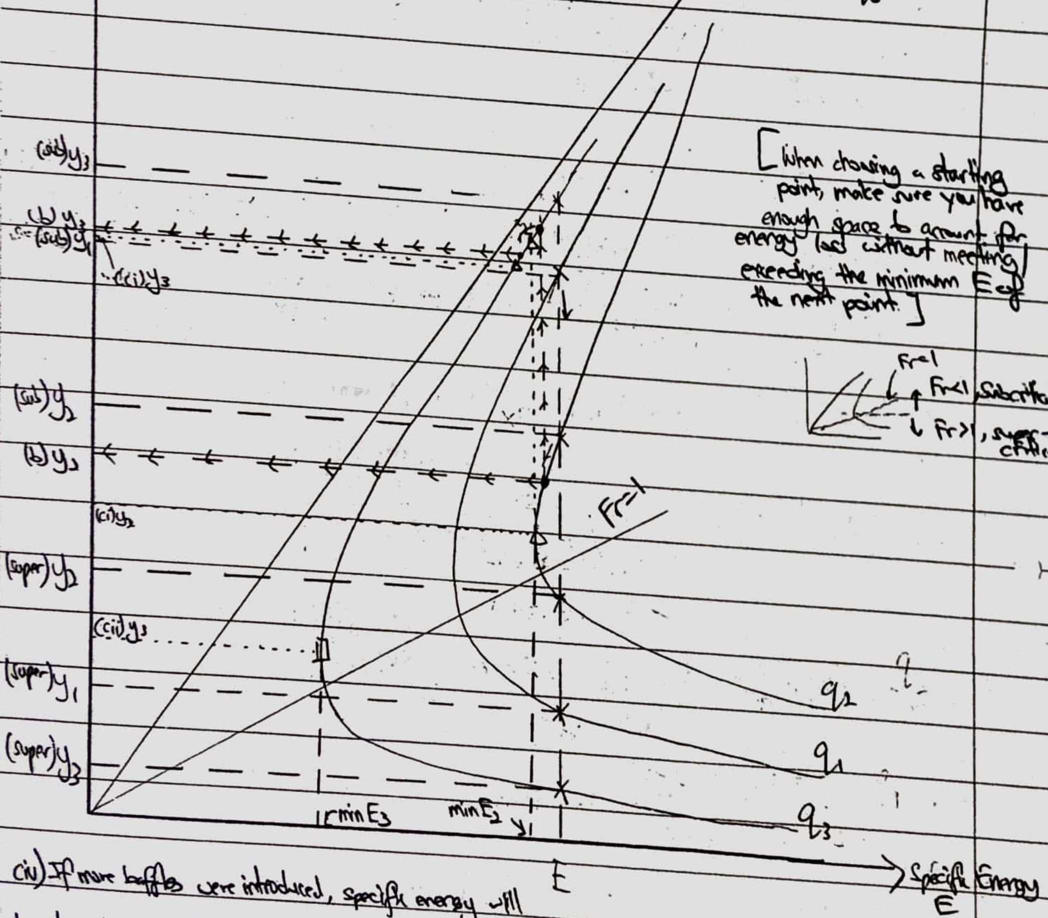
Drawn with x and normal dotted lines - - -. Specific energy stays at E. Water levels are labelled as (sub) y_1, (sub) y_2, (sub) y_3, (super) y_1, (super) y_2, (super) y_3.

b) Drawn with • and →. When channel flows from $q_1 \to q_2 \to q_3$, there is a slight loss in E. Hence, y_2 and y_3 are smaller than in (aii) where there are no energy loss.

ci) Drawn with Δ and ··· lines. Water levels are at (sub) y_1, (ci) y_2, (ci) y_3. Specific energy drops to minimum at q_3 only.

cii) Drawn with □ and ··· lines. Water levels are at (sub) y_1, (b) y_2, (cii). Specific energy drops to minimum at q_3 only.

ciii) Water levels are at (sub) y_1, (ci) y_2, (cii) y_3. Specific energy drops to minimum at both q_2 and q_3.

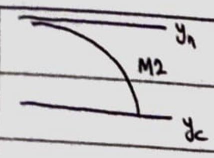


[When choosing a starting point, make sure you have enough space to account for energy loss without meeting/exceeding the minimum E of the next point.]

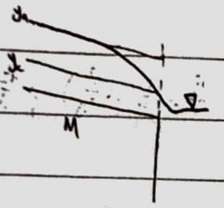
aii) $q_3 < q_1 < q_2$

civ) If more baffles were introduced, specific energy will drop to below min E_3. As a result, choking and ponding will occur. Eventually, q_1, q_2 and q_3 will increase and the curves will shift left in a way such that the new specific energy will be the minimum specific energy of q_3', the new q_3 curve.

[NOTE: During exam, you can use different coloured pens/highlighters to draw/highlight the specific energy diagram. Read the WHOLE question first and make sure you draw a LARGE diagram. Graph paper may be provided or use it if you have]



Example:



Exit from a channel with lower water level at exit

b) i) Critical depth in non-rectangular channel:

$$E = y + \frac{Q^2}{2gA^2}$$

$$\frac{d}{dy}\left(A^{-2}\right) = \frac{-2}{A^3}\left(\frac{dA}{dy}\right)$$

$$\frac{dE}{dy} = 1 + \frac{Q^2}{2g}\left(\frac{-2}{A^3}\frac{dA}{dy}\right) = 0$$

Since $dA = B\,dy$,

$$1 - \frac{Q^2 B}{gA^3} = 0$$

Thus, at critical conditions,

$$\frac{Q^2 B_c}{gA_c^3} = 1$$

$$Q^2 = \frac{gA_c^3}{B_c}$$

$$V_c^2 = \frac{gA_c}{B_c}$$

$$\left(\frac{1}{n}R_c^{\frac{2}{3}}S_c^{\frac{1}{2}}\right)^2 = \frac{gA_c}{B_c}$$

$$S_c = \frac{n^2 g A_c}{B_c R_c^{\frac{4}{3}}}$$

Wide rectangular channel: $R_c = y_c$; $A_c = B_c y_c$

$$S_c = \frac{n^2 g y_c}{y_c^{\frac{4}{3}}}$$

Hence

$$y_c^{\frac{1}{3}} = \frac{n^2 g}{S_c}$$

$$y_c = \left[\frac{n^2 g}{S_c}\right]^3$$

b) ii) $A_c = (B_c + 4y_c + B_c)(y_c)\left(\frac{1}{2}\right)$

$= 2y_c^2 + B_c y_c$

$Q^2 B_c = gA_c^3$

$2y_c^2 + 3.05y_c = \left(\frac{17^2(3.05)}{9.81}\right)^{\frac{1}{3}}$

$= 4.479$

$y_c = 0.917\text{ m}$ or -2.442 m (ignored)

$0.917^{\frac{1}{3}} = \frac{0.015^2(9.81)}{S_c}$

$S_c = 2.272 \times 10^{-3}$

3.a) i) Energy equation : $H = z + \frac{P}{\gamma} + \frac{v^2}{2g}$

Assume open channel flow with a small bed slope : $\frac{P}{\gamma} \approx y$

$$H = z + y + \frac{v^2}{2g}$$

$$\frac{dH}{dx} = \frac{d}{dx}\left(z + y + \frac{v^2}{2g}\right) \qquad \frac{dE}{dx} = \frac{d}{dx}\left(y + \frac{v^2}{2g}\right) \;;\; \frac{dz}{dx} = -S_o$$

$$-S_f = \frac{dE}{dx} - S_o$$

$$\frac{dE}{dx} = S_o - S_f$$

$$E = y + \frac{v^2}{2g}$$

$$= y + \frac{q^2}{2gy^2}$$

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3}$$

$$= 1 - \frac{v^2 y^2}{gy^3}$$

$$= 1 - \frac{v^2}{gy}$$

$$= 1 - F_r^2$$

$$\frac{dE}{dx} = \frac{dy}{dx} \cdot \frac{dE}{dy}$$

$$\frac{dy}{dx} = S_o - S_f \left(\frac{1}{1-F_r^2}\right)$$

$$= \frac{S_o - S_f}{1 - F_r^2}$$

a) ii) M2 profile :

Mild slope : $y_n > y_c$

Space 2 : y in between y_n and y_c

Thus, $y_n > y > y_c$

If $y > y_c$, $F_r < 1$; $\therefore 1 - F_r^2 > 0$

If $y < y_n$, $S_o < S_f$; $\therefore S_o - S_f < 0$

Hence, $\frac{dy}{dx} = \frac{S_o - S_f}{1 - F_r^2}$

$$= \frac{-ve}{+ve} = -ve$$

At the upstream limit, $y \to y_n$; $\therefore S_o \to S_f$

$\frac{dy}{dx} \to 0$ (y asymptotes to y_n)

At the downstream limit, $y \to y_c$; $\therefore F_r \to 1$

$1 - F_r^2 \to 0$

$\frac{dy}{dx} \to \infty$ (y cuts the y_c-line at right angle)

4.a) Assumptions: i) resistance of the channel bed is negligible and the force, $P_f = 0$.

ii) longitudinal slope of the channel bed is small and hence the component of self weight acting on the control volume can be ignored.

Lake : $E \approx y$

$$10 = 0.8 + \frac{V^2}{2(9.81)}$$

$$V = 13.435 \ ms^{-1}$$

$$q = Vy$$

$$= 13.435(0.8)$$

$$= 10.748 \ m^2/s$$

$\therefore Q = 10.748 \ m^3/s$ per unit width

Wide channel : $R_h \approx y_n$

$$V = \frac{1}{n} y_n^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$\frac{10.748}{y_n} = \frac{1}{0.025}\left(y_n^{\frac{2}{3}}\right)\left(1.5\times10^{-3}\right)^{\frac{1}{2}}$$

$$y_n^{\frac{5}{3}} = 6.9378$$

$$y_n = 3.197 \ m$$

b) $M = \frac{y^2}{2} + \frac{q^2}{gy}$

$$= \frac{3.197^2}{2} + \frac{10.748^2}{9.81(3.197)}$$

$$= 8.794 \ m^2$$

$$8.794 = \frac{y^2}{2} + \frac{10.748^2}{9.81y}$$

$$0 = 0.5y^3 - 8.794y + 11.7757$$

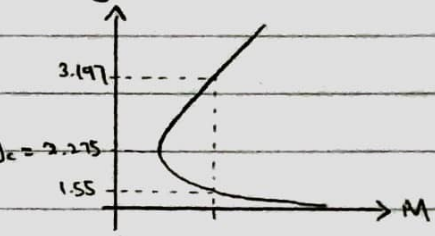
$y = 1.55 m$ or $3.197 m$

$y = 1.55 m$ is the sequent depth.

$$q^2 = gy_c^3$$

$$y_c^3 = \frac{10.748^2}{9.81}$$

$$y_c = 2.275 \ m$$



$\therefore$ the location of hydraulic jump is 1.55 m, which is between 0.8 m and 3.197 m.

c) At $y_1 = 1.55$:

$E_1 = 1.55 + \frac{10.748^2}{2(9.81)(1.55)^2}$

$= 4$ m

$V_1 = \frac{1}{n} y_1^{\frac{2}{3}} S_{f1}^{\frac{1}{2}}$

$\frac{10.748}{1.55} = \frac{1}{0.035}(1.55)^{\frac{2}{3}} S_{f1}^{\frac{1}{2}}$

$S_{f1} = 0.01675$

At $y_2 = 0.8$

$E_2 = 0.8 + \frac{10.748^2}{2(9.81)(0.8)^2}$

$= 10$m

$\frac{10.748}{0.8} = \frac{1}{0.035}(0.8)^{\frac{2}{3}} S_{f2}^{\frac{1}{2}}$

$S_{f2} = 0.1519$

$\frac{E_1 - E_2}{x_1 - x_2} = 0.0015 - \frac{S_{f1} + S_{f2}}{2}$

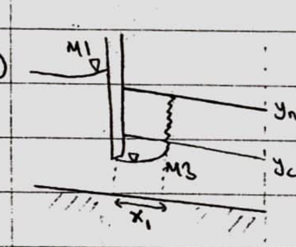
$\frac{-6}{x_1} = -0.08283$

$x_1 = 72.44$ m

$x_1 > x_2$, $y_1 > y_2 \Rightarrow \frac{dy}{dx} > 0$

$y_n > y_c \Rightarrow$ mild slope

d)



Mild slope

$y > y_c$, $F_r < 1$; $\therefore 1 - F_r^2 > 0$ $\quad \frac{dy}{dx} = \frac{-ve}{+ve} = -ve$

$y < y_c$, $F_r > 1$; $\therefore 1 - F_r^2 < 0$ $\quad \frac{dy}{dx} = \frac{-ve}{-ve} = +ve$

e) Adverse slope ($S_0 < 0$) :

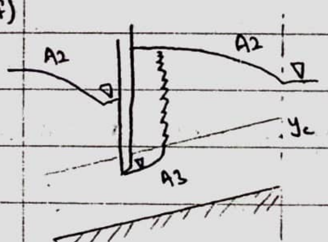
The water profile after the hydraulic jump is not constant as it don't have y_n.

The water level after hydraulic jump is higher than the water level of the reservoir.

The water level before the sluice gate is decreasing. $\frac{dy}{dx} = -ve$

The water level decreases ($\frac{dy}{dx} = -ve$) until it meets the water level of reservoir.

f)



Adverse slope