1)a) In turbulent flow, the velocity at a point in the flow field fluctuates in both magnitude and direction. The fluctuations result from a multitude of small eddies, created by the viscous shear between adjacent particles. For laminar flow, $\tau = \mu \frac{du}{dy}$; while for turbulent flow, $\tau = \mu \frac{du}{dy} + \eta \frac{du}{dy}$. Comparing this two equation, wall shear stress of laminar flow depends on the vicosity of the fluid and the velocity, while in a turbulent flow the wall shear stress is including the eddy viscosity which is not a constant for a given fluid, but it depends on the turbulence of the flow. In general, the total stress in turbulent flow is the sum of the laminar shear stress plus the turbulent shear stress. Therefore turbulent flow generally has a higher wall shear stress compared to laminar flow in pipe.

b) The sudden drop in C_D when $Re \geq 2\times10^5$ is caused by the change of regime from laminar boundary layer to turbulent boundary layer on the sphere. As known, Drag coefficient & depends greatly on the size of turbulent wake. As in a turbulent boundary layer, there is violent mixing of faster-moving outer strata into the slower-moving inner strata and vice versa, resulting in an increased of mean velocity close to boundary. This added energy allows the boundary layer to 'stick' longer against the adverse pressure gradient, resulting in location of point of separation moves from 80° to 120° from the stagnation point, causing wake size and pressure drag to reduce. As the wake size is smaller than laminar boundary layer case. Therefore, the is a sudden drop in C_D when $Re \geq 2\times10^5$.

c) - Shear stress at wall, $\tau_0 = \mu\left(\frac{du}{dy}\right)$

As velocity profile near the wall is a straight line.

$\therefore \tau_0 = \mu\left(\frac{u}{y}\right)$

$\frac{\tau_0}{\rho} = \frac{\nu(u)}{y}$ (As $u_*^2 = \frac{\tau_0}{\rho}$)

$\therefore u_*^2 = \frac{\nu \cdot u}{y}$

$\Rightarrow \frac{u_* \cdot y}{\nu} = \frac{u}{u_*}$

- As known $\tau \approx \tau_0 = \rho l^2 \left(\frac{du}{dy}\right)^2$, $l = Ky$ where $K = 0.4$

$\therefore \tau_0 = \rho (K^2y^2)\left(\frac{du}{dy}\right)^2$

$\frac{\tau_0}{\rho} = K^2y^2 \frac{du}{dy}$

$\int du = \int \frac{1}{K}\sqrt{\frac{\tau_0}{\rho}}\frac{dy}{y}$ (where $u_* = \sqrt{\frac{\tau_0}{\rho}}$)

$\Rightarrow u = 2.5\, u_* \ln y + C$

Yes, U can!

1) d) i) $y_0 = 0.23m$

$V = 1.2 ms^{-1}$

$q = Vy_0$

$= 0.23 \times 1.2$

$= 0.276 \ m^3s^{-1}$

$q^2 = gy_c^3$

$0.276^2 = 9.81 \ y_c^3$

$y_c = 0.198m$

$\because y_0 > y_c$

$\therefore$ it is a subcritical flow

$\therefore$ it is a mild slope.

ii) $y_2 = 0.1m$

E remains constant.

$E = y + \frac{V^2}{2g}$

$= 0.23 + \frac{1.2^2}{2 \times 9.81}$

$= 0.3034m$

$E = y_2 + \frac{V_2^2}{2g}$

$0.3034 = 0.1 + \frac{V_2^2}{2g}$

$V_2 = 1.997 ms^{-1}$

$q_2 = Vy = 0.1997 \ m^3s^{-1}$

$E = y + \frac{q^2}{2gy^2}$

$0.3034 = y + \frac{0.1997^2}{2 \times 9.81 y^2}$

$y = 0.2769$ / $y = -0.0734$ / $y = 0.1$

$\therefore y_1 = 0.2769m$

Assume i) the open channel is horizontal

ii) the channel bed is frictionless

iii) it is a steady flow

iv) no energy loss when the water passes the sluice gate.

ntusu

Yes, U Can!

1) d) iii) $q^2 = gy_c^3$

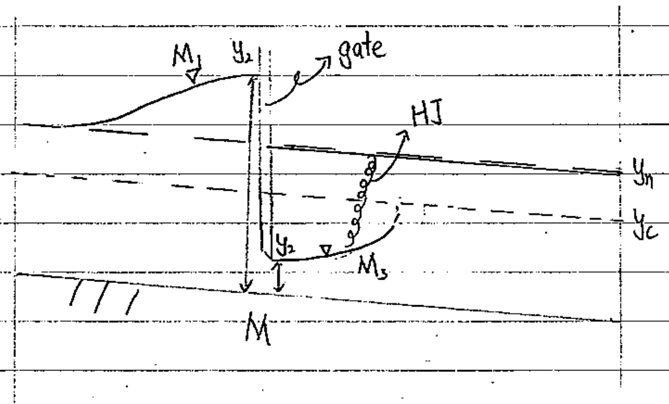
$0.1997^2 = 9.81 \times y_c^3$

$y_c = 0.16m$

As $y_1 > y_c$ $\therefore$ The flow regime upstream is a subcritical flow

And $y_2 < y_c$ $\therefore$ The flow regime down is a supercritical flow

iv)



As $y_n > y_c$

$\therefore$ it is a Mild slope.

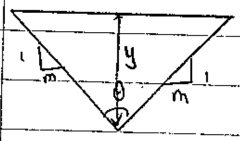
2)a) i) The curve has two asymptotes along $y=0$ and $E=y$.

ii) The curve has an upper and lower limb which represent sub-critical and super-critical flow.

iii) There is a minimum point where, E is minimum on the curve for a given q and the flow is in critical flow.

iv) For $E > E_{min}$, there are 2 possible value which satisfy the function. They are called alternate depths - one higher ($y > y_c$ in subcritical flow regime), and the other lower ($y < y_c$ in supercritical flow regime)

b)



$A = my^2$

$y = \sqrt{\frac{A}{m}}$

$P_{wet} = 2y\sqrt{1+m^2}$

$= 2\sqrt{\frac{A}{m} + Am}$

$\frac{dP_{wet}}{dm} = 2 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{A}{m} + Am}} \left(\frac{-A}{m^2} + A\right) = 0$

$\frac{-A}{m^2} + A = 0$

$m = \pm 1$

$\therefore m = 1$

when $m = 1$.

$A = y^2$

$P_{wet} = 2\sqrt{2}\,y$

$R_h = \frac{A}{P_{wet}}$

$= \frac{y^2}{2\sqrt{2}y}$

$= 0.354y$

c) $S_0 = 0.001$ - $A = 8m^2$

$n = 0.013$ $P_{wet} = 8m^2$

$b = 4.0m$

$y_0 = 2.0m$

$Q = \frac{1}{n} \times A \times R_h^{2/3} \times S_0^{1/2}$

$= \frac{1}{0.013} \times 8 \times 1^{2/3} \times 0.001^{1/2}$

$= 19.46 m^3 s^{-1}$

$q = \frac{Q}{b}$

$= 4.865 m^2 s^{-1}$

$y_c = \sqrt[3]{\frac{q^2}{g}}$

$= 1.3412m$

$E_c = \frac{3}{2} y_c$

$= 2.0118m$

i) $y_0 = y_1 = 2.0m$

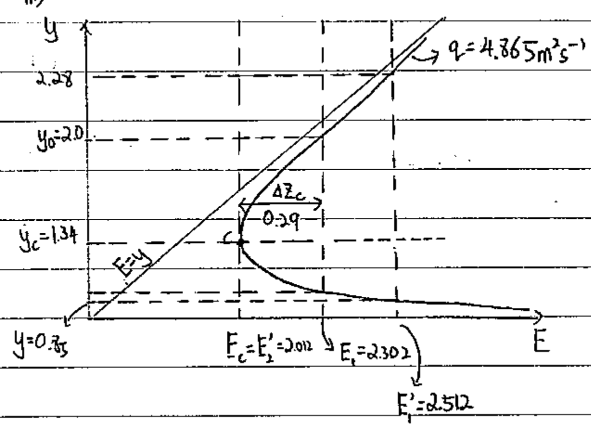
$E_1 = y_1 + \frac{q^2}{2gy_1^2}$

$= 2.302m$

$\Delta Z_c = E_1 - E_c$

$= 0.29m$

iii)



ii) As $\Delta Z > \Delta Z_c$

$\therefore y_2' = y_c = 1.3412m$

$E_2' = y_2' + \frac{q^2}{2gy_2'^2}$

$= 2.012m$

$E_1' = E_2' + \Delta Z$

$E_1' = 2.512$

$E_1' = y_1' + \frac{q^2}{2gy_1'^2}$

$2.512 = y_1' + \frac{q^2}{2gy_1'^2}$

$y_1' = 2.28$ / $y = 0.62$ / $y = 0.85$

As ~~y~~ a ponding and choking occur, upstream is a subcritical flow.

$\therefore y_1' = 2.28m$

It is not appropriate to install such a hump as ponding has occured and it might flood the surrounding.

3)a) $\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}$

As Q is constant, and the channel is prismatic and wide

$\therefore R_h = y$

$$Q = \frac{1}{n} A R_h^{2/3} S_0^{1/2} = \frac{1}{n} A \cdot R_h^{2/3} S_f^{1/2}$$

$$b y_n \, y_n^{2/3} S_0^{1/2} = b y \, y^{2/3} S_f^{1/2}$$

$$S_f = \left(\frac{y_n}{y}\right)^{10/3} S_0$$

$$F_r = \frac{V}{\sqrt{gy}}$$

$$F_r^2 = \frac{V^2}{gy}$$

$$= \frac{q^2}{gy^3}$$

$$= \frac{g y_c^3}{g y^3}$$

$$= \left(\frac{y_c}{y}\right)^3$$

$$\frac{dy}{dx} = \frac{S_0 - \left(\frac{y_n}{y}\right)^{10/3} S_0}{1 - \left(\frac{y_c}{y}\right)^3}$$

$$= S_0 \frac{1 - \left(\frac{y_n}{y}\right)^{10/3}}{1 - \left(\frac{y_c}{y}\right)^3}$$

3) b) i) $q = 80 m^2 s^{-1}$ For wide rectangular river,

$S_0 = 6 \times 10^{-4}$ $A = by$

$n = 0.015$ $P_{wet} = b$

$y_c = \sqrt[3]{\frac{q^2}{g}}$ $R_h = y$

$= 8.67 m$

$$Q = \frac{1}{n} \times A \times R_h^{2/3} \times S_0^{1/2}$$

$$Q = \frac{1}{0.015} \times b \times y_n \times y_n^{2/3} \times \sqrt{6 \times 10^{-4}}$$

$q = 1.633 \, y_n^{5/3}$

$y_n = 10.329$

As $y_c < y = 9.5 < y_n$, and $y_n > y_c$

$\therefore$ it is a M2 profile

ii) $y_1 = 9.5m$ $E_1 = y_1 + \frac{V_1^2}{2g} = 13.114 m$

$A = 9.5b$

$R_h = 9.5m$

$V_1 = \frac{q}{y} = \frac{80}{9.5} = 8.421$

$$S_{f_1} = \left(\frac{Q \times n}{R_h^{2/3} A} \right)^2 = \left(\frac{q \times n}{R_h^{2/3} \times y} \right)^2 = 0.000793$$

Similarly,

$y_2 = 9.0m$ $E_2 = 13.027$

$A = 9.0b$

$R_h = 9.0m$

$V_2 = 8.889$

$S_{f_2} = 0.00095$

$$\bar{S_f} = \frac{0.000793 + 0.00095}{2} = 0.000871$$

$$\frac{E_2 - E_1}{x_2 - x_1} = S_0 - \bar{S_f}$$

$\frac{-0.087}{x_2}$ [illegible] $= 0.0006 - 0.000871$

[illegible]

$x_2 = 32$[illegible]m

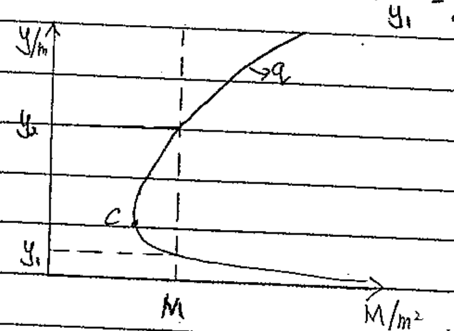
$\therefore$ Section 2 is at 320.57m downstream of Section 1. As it is a M2 profile, $\frac{dy}{dx} < 0$, depth is decreasing.

$\therefore$ To get a smaller flow depth, one must move downstream.

4)a) Hydraulic jump occurs when supercritical flow has its velocity reduced to subcritical. It is a type of water surface profile which has discontinuity in the surface, characterized by a steep upward slope of the profile, broken throughout with violent turbulence.

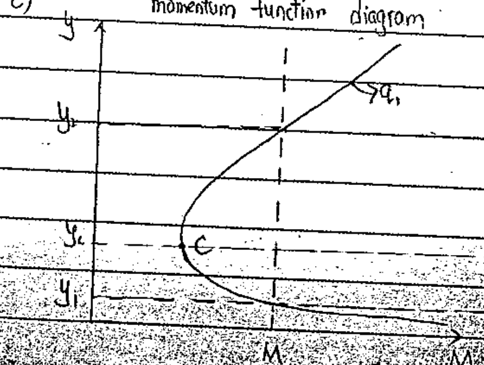
Characteristics: i) occurs where there is a conflict between upstream and downstream controls
ii) it will only 'jump' when the upstream depth meets its conjugate depth.
iii) Huge energy loss

b) Conjugate depths, y_1 and y_2 represent possible combinations of depth that could occur before and after the jump, and normally denote supercritical and subcritical flows, respectively. In a simple hydraulic jump which assume there is no momentum loss, resistance of the channel bed is negligible, the self weight acting on the control volume is ignored and the channel has a rectangular cross section, the relationship of the conjugate depths is $\frac{y_2}{y_1} = \frac{1}{2}\sqrt{1+8F_{r_1}^2} - 1$

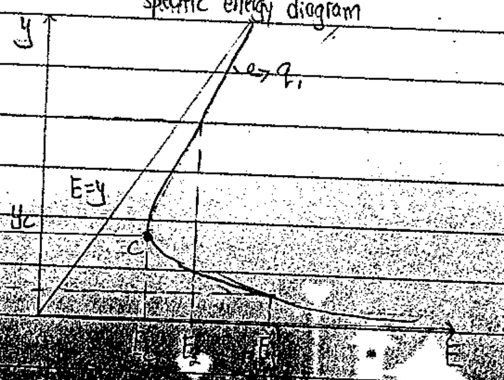


↳ y_1 and y_2 are conjugate depths

c) momentum function diagram



specific energy diagram



$M_1 = M_2$ at momentum function diagram but E_1 [illegible] specific energy diagram due to the energy loss at hydraulic jump

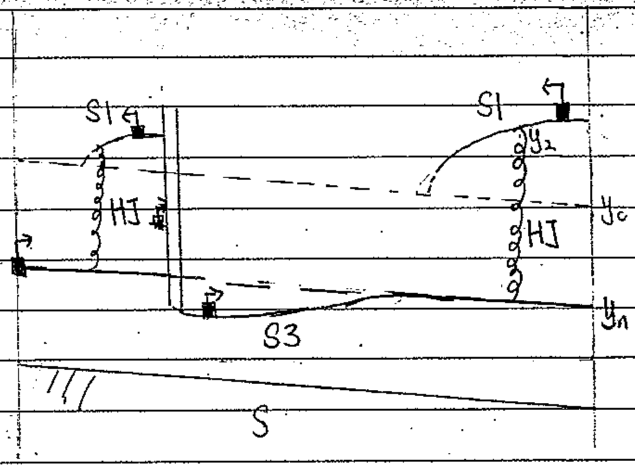
d) i) $b = 4m$

$Q = 20 m^3 s^{-1}$

$y_n = 0.5m$

$q = 5 m^2 s^{-1}$, $y_c = 1.366m$

$y_1 = y_n = 0.5m$



$$Fr_1 = \frac{V_1^2}{g y_1}$$

$$= \frac{q^2}{g y_1^3}$$

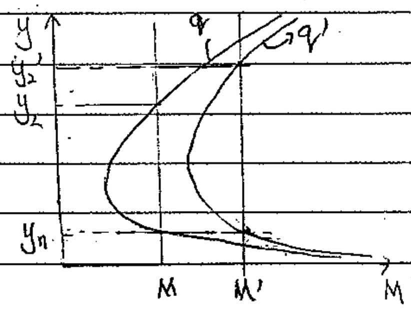
$$= 20.39$$

$$\frac{y_2}{y_1} = \frac{1}{2}(\sqrt{1 + 8 \times Fr_1^2} - 1)$$

$y_2 = 2.95m$

ii) As the downstream flow depth is higher than the flow depth y_2, the location of the will be brought forward, nearer to the sluice gate.

iii) -Increase the unit discharge of the flow by lifting up the sluice gate



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