EN3601 Hydraulics 2011-2012 Sem 1.

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1(a) $F_D = \frac{1}{2} C_D A \rho_f V_t^2$

$G - F_b$

$$F_D = G - F_b$$

$$\Rightarrow \frac{1}{2} C_D A \rho_f V_t^2 = \rho_s \frac{4}{3}\pi \left(\frac{D}{2}\right)^3 g - \frac{4}{3}\pi \left(\frac{D}{2}\right)^3 \rho_f g$$

$$= \frac{1}{6}\pi D^3 g(\rho_s - \rho_f)$$

$$\Rightarrow V_t = \sqrt{\frac{\pi D^3 g(\rho_s - \rho_f)}{3 C_D (\frac{1}{4}\pi D^2)\rho_f}} = \sqrt{\frac{4}{3}\frac{1}{C_D}\frac{\rho_s - \rho_f}{\rho_f} g D}.$$

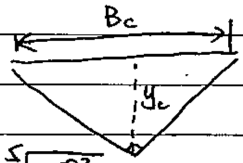
1(b). Specific energy $E = y + \frac{V^2}{2g}$, $V = \frac{Q}{A}$.

$$\Rightarrow E = y + \frac{Q^2}{2A^2 g}.$$

Critical flow $\Rightarrow E = E_{min} \Rightarrow \frac{dE}{dy} = 0$, $A = A_c$, $B = B_c$

$$\Rightarrow 1 + \frac{Q^2}{2g} \cdot -2A^{-3} \cdot \frac{dA}{dy} = 0$$ where $A = By \Rightarrow \frac{dA}{dy} = B_c$ at critical condition.

$$\Rightarrow 1 - \frac{Q^2 B_c}{g A_c^3} = 0 \Rightarrow 1 = \frac{Q^2 B_c}{g A_c^3}$$



For a 90° triangular channel, $B_c = 2y_c$, $A_c = y_c^2$

hence $1 = \frac{Q^2 \cdot 2y_c}{g \cdot y_c^6} \Rightarrow 2Q^2 = g y_c^5 \Rightarrow y_c = \sqrt[5]{\frac{2Q^2}{g}}$

$$= \sqrt[5]{\frac{2}{g}}\, Q^{2/5}$$

$$\approx 0.73\, Q^{2/5}$$

1(c).

loop.	Pipe	K.	Q_1	KQ_1^2	KQ_1	Q_2
	AC	170.	0.2_c	6.8	34	0.131_c
I_c	CB.	122.	0.1_c	1.22	12.2	0.031_c
	AB.	68.	0.1_A	-0.68.	6.8	0.169_A
				7.34	53	
II_c	ADC.	170.	0.2_c	6.8	34	0.2_c
	AC.	170.	0.2_A	-6.8	34	0.2_A
				0	68.	

$$\Delta Q_I = -\frac{\Sigma KQ^2}{2\Sigma KQ} = -\frac{7.34}{2\times 53} = -0.069$$

$$\Delta Q_{II} = \frac{-\Sigma KQ|Q|}{2\Sigma KQ} = 0$$

1(d) $b = 4m$ $Q_1 = 32 m^3/s$. $y_1 = 2m$. $\Delta x = 100m$ $Q_2 = 20 m^3/s$.
$\Delta Z = 0m$

Since no hump or depression, $E_1 = E_2$.

$q_1 = Q_1 / b = 32/4 = 8 m^2/s$.

$q_2 = Q_2 / b = 20/4 = 5 m^2/s$

$E_1 = y_1 + \frac{q_1^2}{2gy_1^2} = 2 + \frac{8^2}{2\times 9.81 \times 2^2} = 2.8155 m$

$E_2 = y_2 + \frac{q_2^2}{2gy_2^2} \Rightarrow 2.8155 = y_2 + \frac{25}{2\times 9.81 \times y_2^2}$

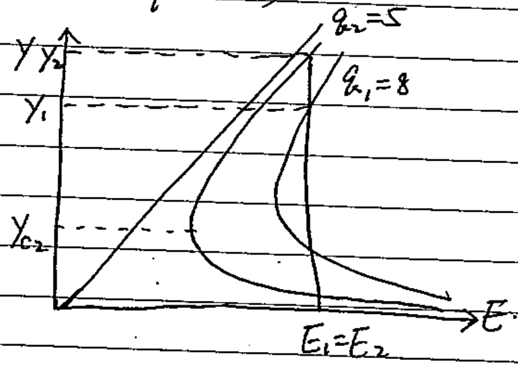
$\Rightarrow$ By trial & error (by solving a cubic equation)

$y_2 = 2.63 m$. or $0.79 m$

$y_{c1} = \sqrt[3]{\frac{q_1^2}{g}} = \sqrt[3]{\frac{8^2}{9.81}} = 1.87 m$

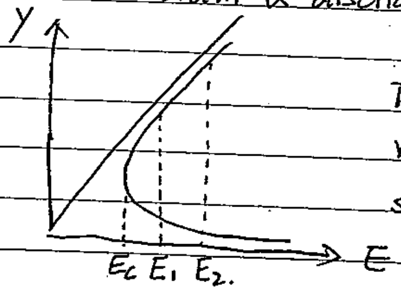
$y_{c2} = \sqrt[3]{\frac{q_2^2}{g}} = \sqrt[3]{\frac{25}{9.81}} = 1.37 m$

Hence $y_2 = 2.63m$ is possible.



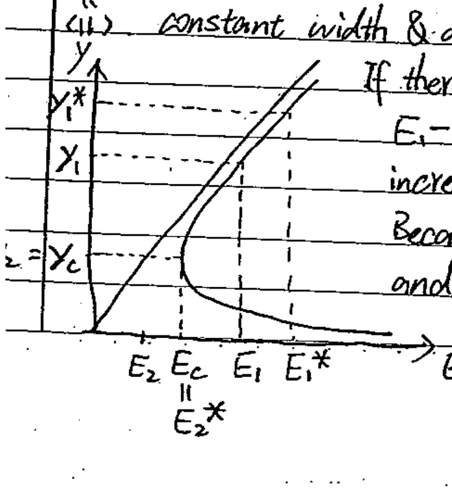
2(a)

(i) depression happens $\Rightarrow$ $E_1 + \Delta Z = E_2$.
constant width & discharge $\Rightarrow$ constant q.



The vertical energy line can only move to the right, which makes it impossible to be smaller than the critical energy E_c, then choking will never occur.

(ii) constant width & discharge $\Rightarrow$ constant q.

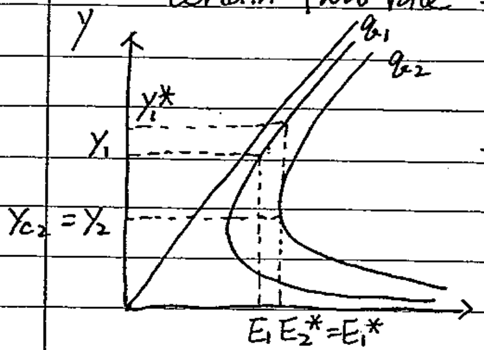


If there is a hump high enough so that ~~$E_1 - E_2 = \Delta Z \geq$~~ $E_1 - \Delta Z = E_2 < E_c$, chocking will occur at the hump and increase the real energy at downsteam to be $E_2^* = E_c$. Because of that, ponding has to happen at upstream and energy upstream also increase to $E_1^* = E_2^* + \Delta Z$

2(a) cont'

(iii) contracted section $\Rightarrow b_1 > b_2$. $\left.\right\} \Rightarrow q_1 < q_2$

certain flow rate $\Rightarrow$ constant Q



At a certain energy $E_1 < E_{c2}$, when the flow enters the contracted section, q_1 changes to q_2. Since $E_1 < E_{c2}$, real E_2 has to increase to $E_2^* = E_{c2}$ and critical depth $y_2 = y_{c2}$ in the contraction. Since $E_1 = E_2$, ponding will occur upstream and E_1 increases to $E_1^* = E_2^*$, y_1 increases to y_1^* as shown in the figure.

2(b) $E_1 + \Delta z = E_2$ $\quad q_1 < q_2$. $\quad Q = 10 m^3/s$.

$B_1 = 5m$. $\quad y_1 = 2m$. $\quad B_2 = 4m$. $\quad \Delta z = 0.3m$

(i) $q_1 = Q/B_1 = 2 m^2/s$. $\quad q_2 = Q/B_2 = 2.5 m^2/s$

$$E_1 = y_1 + \frac{q_1^2}{2gy_1^2} = 2 + \frac{2^2}{2\times 9.81\times 2^2} = 2.051 m$$

$$y_{2c} = \sqrt[3]{\frac{q_2^2}{g}} = \sqrt[3]{\frac{2.5^2}{9.81}} \approx 0.86 m$$

$$E_2 = E_1 + \Delta z = y_2 + \frac{q_2^2}{2gy_2^2} \Rightarrow 2.051 + 0.3 = y_2 + \frac{2.5^2}{2\times 9.81\, y_2^2}$$

By trial & error, $y_2 = 2.29 m$ or $0.40 m$

Drop in water surface $= y_1 + \Delta z - y_2$

$= 2 + 0.3 - 2.29 = 0.01 m$.

chocking does not occur since $y_2 = 2.29m > y_{2c} = 0.86m$

(ii) $\begin{cases} E_2 = E_1 + \Delta z' \\ y_1 + \Delta z' = y_2' \end{cases} \Rightarrow \begin{cases} 2.051 + \Delta z' = y_2' + \frac{2.5^2}{2\times 9.81\, y_2'^2} \\ 2 + \Delta z' = y_2'. \end{cases}$

$\Rightarrow 0.051 = \frac{2.5^2}{2\times 9.81\, y_2'^2} \Rightarrow y_2' = 2.5m \Rightarrow \Delta z' = 0.5m$

Hence the bed should be lowered $0.5 - 0.3 = 0.2m$ to produce no change in water surface.

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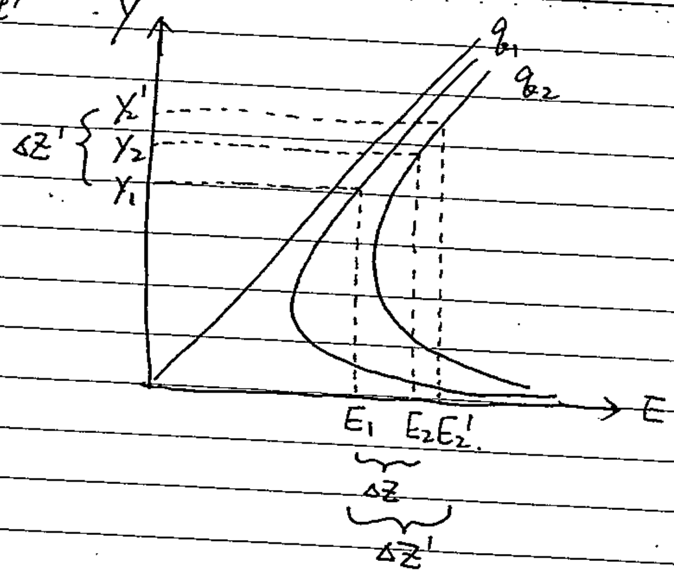
2(b) cont'

(iii)



3(a) (i) The energy or Bernoulli equation gives:

$$H = z + \frac{p}{\gamma} + \frac{V^2}{2g} = z + y + \frac{V^2}{2g} = z + E$$

Differentiate the equation wrt x

$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dE}{dx}.$$

$\frac{dH}{dx} = -S_f$ energy gradient or energy slope.

$\frac{dz}{dx} = -S_o$ slope of channel bed

Then $-S_f = -S_o + \frac{dE}{dx}$ or $\frac{dE}{dx} = -S_f + S_o$

Do partial differentiation, $\frac{dE}{dx} = \frac{dE}{dy} \cdot \frac{dy}{dx}$. *

Since $E = y + \frac{V^2}{2g} = y + \frac{q^2}{2gy^2}$,

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} = 1 - \frac{V^2y^2}{gy^3} = 1 - \frac{V^2}{gy} = 1 - Fr^2$$

Equation * becomes: $-S_f + S_o = (1 - Fr^2) \cdot \frac{dy}{dx}$

$$\Rightarrow \frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

(ii) S1 profile $\Rightarrow y > y_c > y_n$

$y > y_c \Rightarrow 1 - Fr^2 > 0.$

$y > y_n \Rightarrow S_o - S_f > 0$

$\Rightarrow \frac{dy}{dx} > 0 \Rightarrow$ depth of water surface increases

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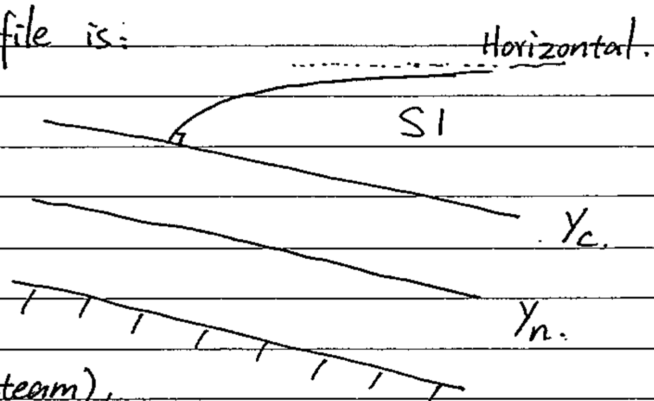
3(a) (ii) cont'.

When $y \to y_c$, $1-Fr^2 \to 0$, $\frac{dy}{dx} = \frac{S_0 - S_f}{1-Fr^2} \to \infty$

Hence ~~at~~ ~~at~~ from a certain depth y, the flow upsteam will decrease slowly ($\frac{dy}{dx} > 0$) and finally reaches y_c at a right angle ($\frac{dy}{dx} \to \infty$ when $y \to y_c$)

Therefore the S1 ~~P~~ profile is:



When $y \to \infty$ (at down steam),

$Fr = \frac{V}{\sqrt{gy}} \to 0$, $S_f = \left(\frac{nq}{y^{5/3}}\right)^2 \to 0$ (from manning's equation)

$\frac{dy}{dx} = \frac{S_0 - S_f}{1-Fr^2} \to \frac{S_0}{1} = S_0 \Rightarrow$ y asymptotes to the horizontal position

3(b). Figure Q3(a).

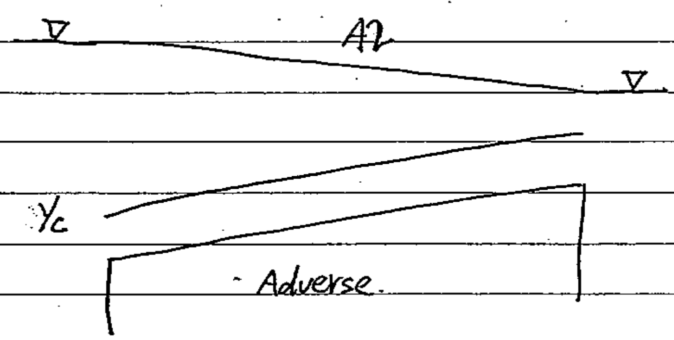
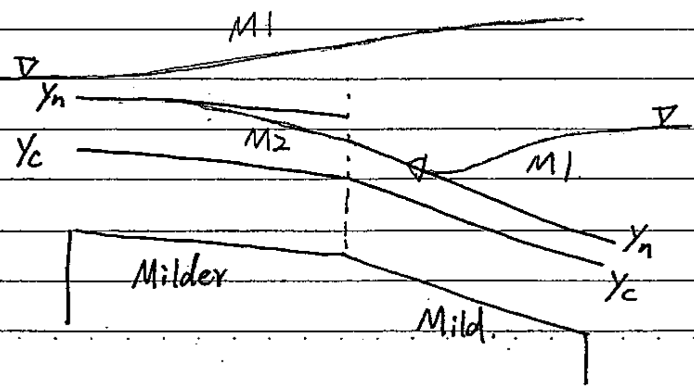


Figure Q3(b)



* No hydraulic jump is able to form, so I think it's an impossible scenario. >.<

4(a) (i) cross-section area of the channel $A = 3y^2$

From momentum eqn: $F_1 - F - F_2 = \rho Q(V_2 - V_1)$

$\Rightarrow \frac{1}{2}\rho g A_1 Y_1 - \frac{1}{2}\rho C_D A_s V_1^2 - \frac{1}{2}\rho g A_2 Y_2 = \rho Q(V_2 - V_1)$ ①

Since $Q = V_1 A_1 = V_1 3Y_1^2 = V_1 \times 3 \times 0.4^2 = 0.48V_1$
$= V_2 A_2 = V_2 3Y_2^2 = V_2 \times 3 \times 2^2 = 12V_2$ $\Big\}$ $V_1 = 25V_2$.

Eqn ① $\Rightarrow \frac{1}{2}\times 9.81 \times 3 \times 0.4^3 - \frac{1}{2}\times 0.41 \times 3 \times 0.3^2 \times (25V_2)^2$
$-\frac{1}{2}\times 9.81 \times 3 \times 2^3 = 12V_2(V_2 - 25V_2)$

$\Rightarrow 0.94176 - 34.59375V_2^2 - 117.72 = -288V_2^2$

$253.40625V_2^2 = 116.77824$

$\Rightarrow V_2 \approx 0.67885 m/s$

$Q = 12V_2 \approx 8.146 m^3/s$ #

(ii) If there's no sill. $F_1 - F_2 = \rho Q(V_2 - V_1)$

$\frac{1}{2}\rho g A_1 Y_1 - \frac{1}{2}\rho g A_2 Y_2 = \rho Q(V_2 - V_1)$

$\frac{1}{2}\times 9.81 \times 3 \times 0.4^3 - \frac{1}{2}\times 9.81 \times 3Y_2^3 = 8.146^2\left(\frac{1}{3Y_2^2} - \frac{1}{3\times 0.4^2}\right)$

By trial & error, $Y_2 \approx 2.09m$.

(iii) By putting an object in the channel, the downstream depth can be changed.

4(b) $q = 6m^2/s$ $Y_{nA} = 1m$ $Y_{nc} = 2m$.

(i) $Y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{36}{9.81}} = 1.54m$.

$Y_{nA} < Y_c < Y_{nc} \Rightarrow$ AB is steep, BC is mild slope.
From steep flow to mild slope, a hydraulic jump will occur.

(ii) $Fr_A^2 = \frac{V_A^2}{gY_A} = \frac{q^2}{gY_{nA}^3} = \frac{6^2}{9.81\times 1} = 3.67$

$Fr_C^2 = \frac{q^2}{gY_{nc}^3} = \frac{6^2}{9.81\times 2^3} = 0.46$

$Y_{conj,A} = \frac{1}{2}Y_{nA}\left[\sqrt{1+8Fr_A^2} - 1\right] = 2.265m > 2m$

$Y_{conj,c} = \frac{1}{2}Y_{nc}\left[\sqrt{1+8Fr_c^2} - 1\right] = 1.163m > 1m$

$\Big\} \Rightarrow$ hydraulic jump will form downstream