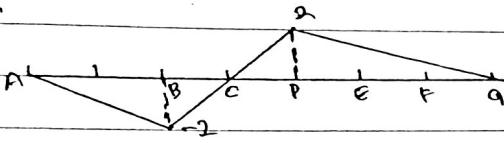
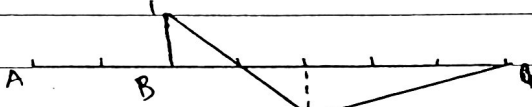


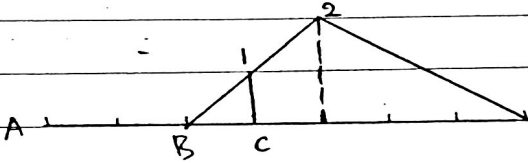
1 a. MA



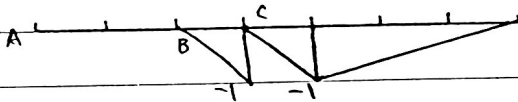
VB



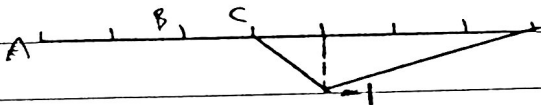
RE



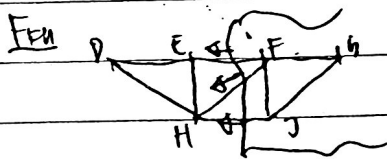
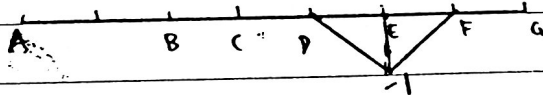
VC



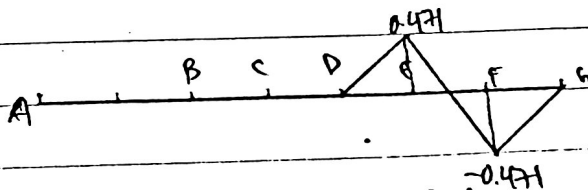
MC



FEH

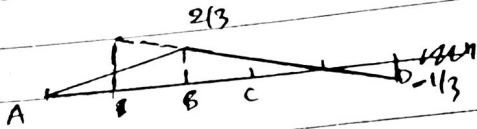


x	FEH	$\frac{1}{3} = FEH \text{ SM AT}$
0	0	$FEH = 0.471$
1	0.471	
2	-0.471	
3	0	

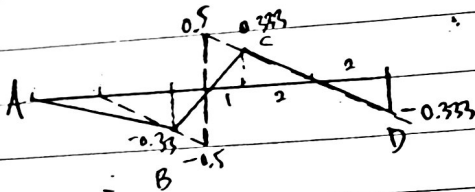


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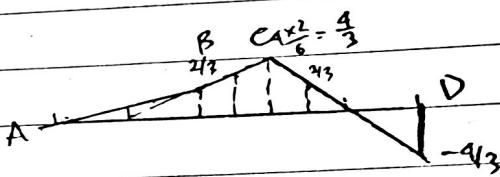
1b. RE



FG



Mg

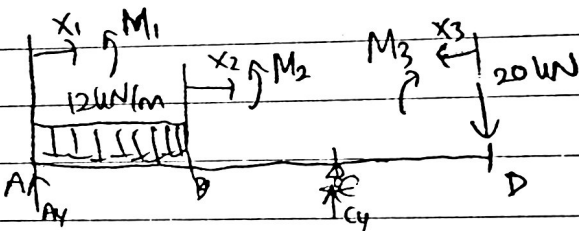


max bending moment:

① 6kN at C  $\rightarrow 1 \times 9 + \frac{1}{2} \times 6^2 = 17 \text{ kNm} \rightarrow \text{Max}$

② 9kN at C  $\rightarrow \frac{1}{2} \times 9^2 + \frac{2}{3} \times 6^2 = 16 \text{ kNm}$

2. a.



$\sum M_C = 0$

$A_y \times 6 + 20 \times 3 = 36 \times 4.5$

$6A_y = 162 - 60$

$A_y = 102/6 = 17$

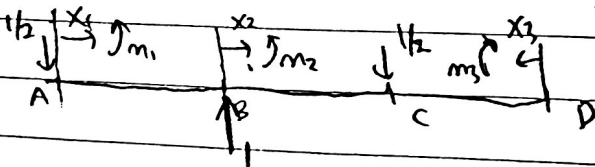
$M_1 = -\frac{12 \times x_1^2}{2} + 17x_1 = -6x_1^2 + 17x_1$

$C_y = 12 \times 3 + 20 - 17 = 39$

$M_2 = -36(1.5 + x_2) + 17(3 + x_2)$

$= -54 - 36x_2 + 51 + 17x_2 = -3 - 19x_2$

$M_3 = -20x_3$



$m_1 = -1/2 x_1$

$m_2 = -1/2(3 + x_2) + 1x_2 = -3/2 + 1/2 x_2$

$m_3 = 0$

$$\Delta C = \int \frac{mM}{EI} dx = \frac{1}{EI} \int_0^3 (-6x_1^2 + 17x_1)(-1/2 x_1) dx_1 + \int_0^3 (-3 - 19x_2)(-3/2 + 1/2 x_2) dx_2$$

$$\frac{1}{EI} \int_0^3 3x_1^3 - \frac{17}{2} x_1^2 dx_1 + \int_0^3 \left( \frac{9}{2} - 3/2 x_2 + \frac{57}{2} x_2 - \frac{19}{2} x_2^2 \right) dx_2$$

$$\frac{1}{EI} \left( \frac{3}{4} x_1^4 - \frac{17}{6} x_1^3 \Big|_0^3 + \frac{9}{2} x_2 + \frac{27}{2} x_2^2 - \frac{19}{6} x_2^3 \Big|_0^3 \right)$$

$$\frac{1}{EI} \left( \frac{3}{4} 3^4 - \frac{17}{6} 3^3 + \frac{9}{2} \times 3 + \frac{27}{2} \times 3^2 - \frac{19}{6} \times 3^3 \right)$$

$$\frac{1}{1000} (60.75 - 76.5 + 13.5 + 121.5 - 85.5)$$

$$= \frac{33.75}{1000}$$

$$f_{cc} = \int \frac{m^2}{EI} dx = \left( \int_0^3 (-1/2 x_1)^2 dx_1 + \int_0^3 (-3/2 + 1/2 x_2)^2 dx_2 \right) \frac{1}{EI}$$

$$= \left( \frac{1}{4} x_1^3 \Big|_0^3 + \frac{9}{4} x_2 - 3/4 x_2^2 + \frac{1}{4} x_2^3 \Big|_0^3 \right) \frac{1}{EI}$$

$$= (2.25 + 6.75 - 6.75 + 2.25) \frac{1}{EI}$$

$$= \frac{4.5}{1000}$$

$$\Delta C + C_y f_{cc} = \frac{-C_y \times 3}{1000}$$

$$\frac{33.75}{1000} + C_y \times \frac{4.5}{1000} = \frac{-C_y \times 3}{1000}$$

$$\frac{33.75}{1000} = \frac{-7.5 C_y}{1000}$$

$$C_y = 4.5 \quad (\downarrow)$$

$$b) \Delta C + C_y f_{cc} = \frac{-3 C_y}{1000} - 0.015$$

$$0.04875 = \frac{-7.5 C_y}{1000}$$

$$C_y = 6.5 \quad (\downarrow)$$

Your gift, your choice, your fund.

3. a. FEM<sub>AB</sub>:  $\int_0^{L/2} w \frac{x}{L} \frac{x(L-x)^2}{L^2} dx$

$$\frac{2w}{L^3} \int_0^{L/2} x^2 (L^2 - 2xL + x^2) dx$$

$$\frac{2w}{L^3} \left( \frac{1}{3} x^3 L^2 - \frac{1}{2} x^4 L + \frac{1}{5} x^5 \right) \Big|_0^{L/2}$$

$$\frac{2w}{L^3} \left( \frac{1}{3} \frac{L^3}{8} L^2 - \frac{1}{2} \frac{L^4}{16} L + \frac{1}{5} \frac{L^5}{32} \right)$$

$$= \frac{2w}{L^3} \times \frac{20 - 15 + 3}{480} L^5 = \frac{1}{30} wL^2$$

FEM<sub>BA</sub>:  $\int_0^{L/2} w \frac{2x}{L} \frac{x^2(L-x)}{L^2} dx$

$$\frac{2w}{L^3} \int_0^{L/2} x^3 (L-x) dx$$

$$\frac{2w}{L^3} \left( \frac{1}{4} x^4 L - \frac{1}{5} x^5 \right) \Big|_0^{L/2}$$

$$\frac{2w}{L^3} \times \frac{5-2}{320} L^5$$

$$= \frac{3}{160} wL^2$$

b- DOF:  $\theta_c$

$$FEM_{AC} = \frac{-wL^2}{30}$$

$$FEM_{CA} = \frac{wL^2}{20}$$

$$M_{AC} = 2E \left( \frac{2I}{6} \right) [\theta_c] - \frac{wL^2}{30} = \frac{2EI}{3} \theta_c - 24$$

$$M_{CA} = 2E \left( \frac{2I}{6} \right) (2\theta_c) + \frac{wL^2}{20} = \frac{4}{3} EI \theta_c + 36$$

$$M_{CB} = 2E \left( \frac{I}{6} \right) (2\theta_c) = \frac{2EI}{3} \theta_c$$

$$M_{BC} = 2E \left( \frac{I}{6} \right) \theta_c = \frac{EI}{3} \theta_c$$

$$M_{CA} + M_{CB} = 0$$

$$\frac{4}{3} EI \theta_c + 36 + \frac{2EI}{3} \theta_c = 0$$

$$\theta_c = -\frac{18}{EI}$$

$$M_{AC} = \frac{2EI}{3} \left( -\frac{18}{EI} \right) - 24 = -36$$

$$M_{CA} = \frac{4}{3} EI \left( -\frac{18}{EI} \right) + 36 = 12$$

$$M_{CB} = \frac{2EI}{3} \left( -\frac{18}{EI} \right) = -12$$

$$M_{BC} = \frac{EI}{3} \theta_c \left( -\frac{18}{EI} \right) = -6$$

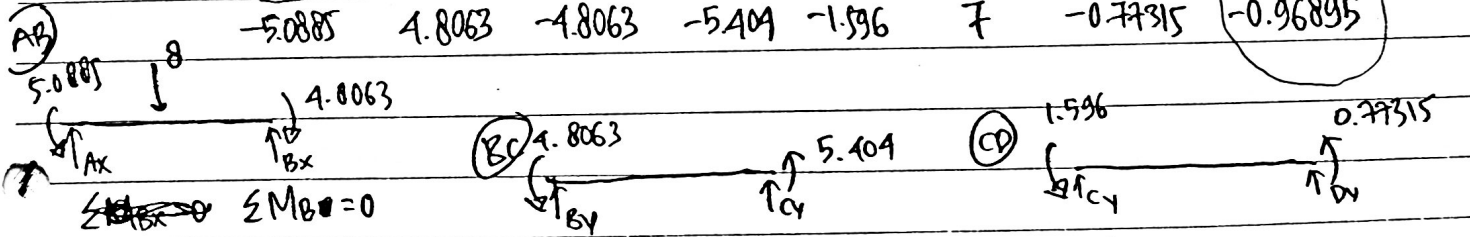
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4. a.  $K_{AB} = \frac{4EI}{5}$   $DF_{AB} = 0$   $DF_{CB} = \frac{4EI/6}{4EI/6 + 4EI/4 + 4EI/5} = 0.27$   
 $K_{BC} = \frac{4EI}{6}$   $DF_{BA} = \frac{4EI/5}{4EI/5 + 4EI/6} = 0.545$   $DF_{CD} = \frac{4EI/5}{4EI/6 + 4EI/4 + 4EI/5} = 0.324$   
 $K_{CE} = \frac{4EI}{4}$   $DF_{BC} = 1 - 0.545 = 0.455$   $DF_{CE} = \frac{4EI/4}{4EI/6 + 4EI/4 + 4EI/5} = 0.406$   
 $K_{CD} = \frac{4EI}{5}$   
 $FEM_{AB} = \frac{-8 \times 5}{8} = -5$   $FEM_{BC} = \frac{6EI \Delta}{L^2} = \frac{6EI \times \frac{24}{6}}{6^2} = -4$   
 $FEM_{BA} = 5$

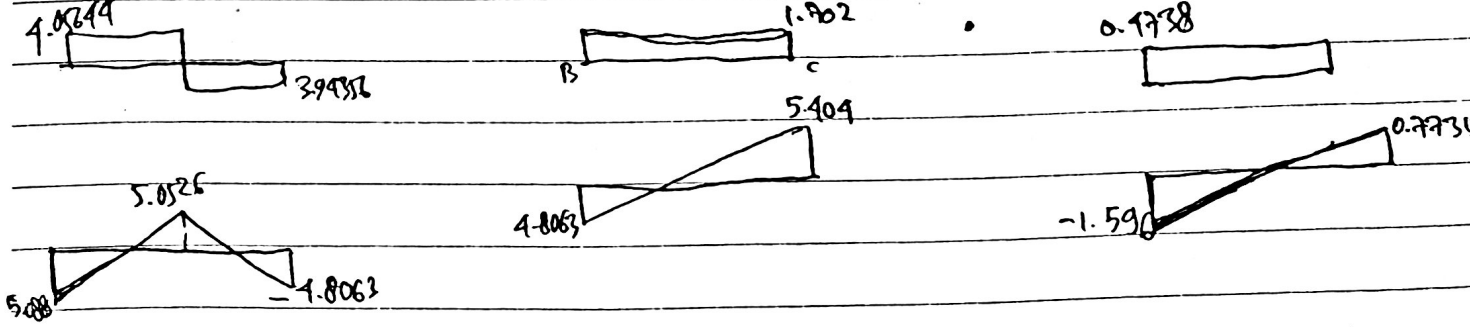
$FEM_{CB} = -4$

$FEM_{CE} = \frac{6EI \times \frac{24}{6}}{4^2} = 9$   $FEM_{EC} = 9$

Joint	A	B		C		D	E	
	AB	BA	BC	CB	CD	CE	DC	EC
DF	0	0.545	0.455	0.27	0.324	0.405	0	0
	-5	5	-4	-4		9		9
		-0.545	-0.455	-1.35	-1.62	-2.03		
		-0.2725	-0.675	-0.2275		-0.81		-1.015
		0.368	0.307	0.0614	0.0737	0.0921		
		0.184	0.0307	0.1535		0.03685		0.04605
		-0.0167	-0.01397	-0.0414	-0.0497	-0.062		
	-5.0885	4.8063	-4.8063	-5.404	-1.596	7	-0.77315	-0.96895



$5A_x + 4.8063 = 8 \times 2.5 + 5.0885$   $\sum M_B = 0$   $\sum M_D = 0$   
 $A_x = 4.05644$   $6B_y = 5.404 + 4.8063$   $5C_y = 0.77315 + 1.596$   
 $B_x = 8 - 4.05644 = 3.94356$   $B_y = 1.702$   $C_y = -1.702$   $C_y = 0.4738$   $D_y = -0.4738$

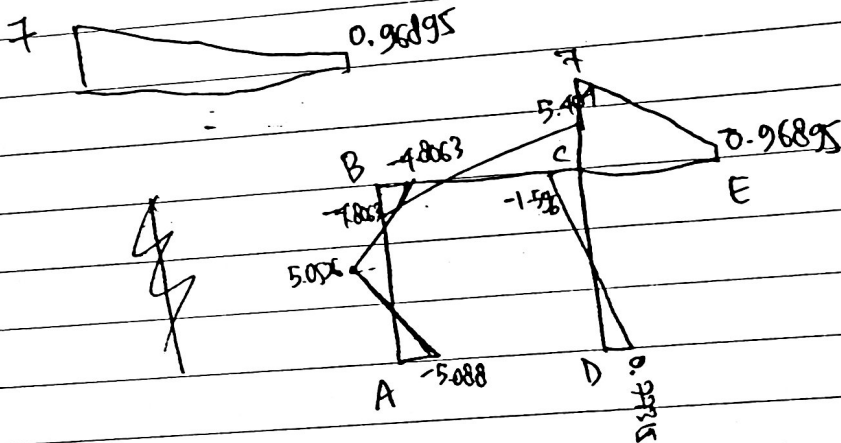




$\sum M_E = 0$

$4y + 7 = 0.96895$

$C_y = -1.50776 \quad E_y = 1.50776$



$K_{AB} = 3EJ/5$

$DF_{AB} = 0$

$DF_{CD} = \frac{8}{23}$

$K_{BC} = 3EJ/6$

$DF_{BC} = 1$  (change)

$DF_{CE} = \frac{10}{23}$

$K_{CE} = 4EJ/4$

$DF_{CE} = 1$

$FEM_{AB} = \frac{3PL}{16} = \frac{3 \times 8 \times 5}{16} = -7.5$

$K_{CD} = 4EJ/5$

$DF_{CB} = \frac{3EJ/6}{4EJ/4 + 3EJ/6 + 4EJ/5} = \frac{5}{23}$

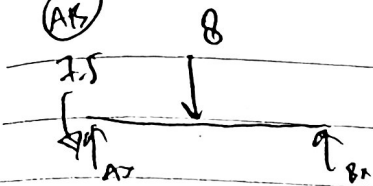
$FEM_{CB} = \frac{3EJ \Delta}{L^2} \times \frac{24}{EI} = -2$

$FEM_{CE} = \frac{6EJ}{L^2} \times \frac{24}{EI} = +9$

$FEM_{EC} = 9$

Joint	A	B	C	D	E			
	AB	BA	BC	CB	CD	CE	DC	EE
	0	1	1	$\frac{5}{23}$	$\frac{8}{23}$	$\frac{10}{23}$	0	0
FEM	-7.5	0	0	-2	0	9	0	9
Dist				-1.5217	-2.4348	-3.0435		
CO							-1.2174	1.52175
	-7.5	0	0	-3.93	-2.43	5.96	-1.21	7.48

(A/B)

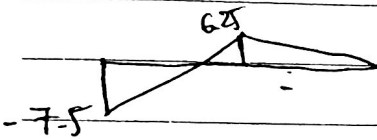
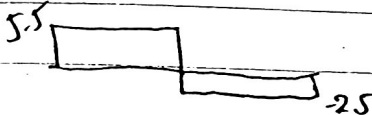


$$\sum M_B = 0$$

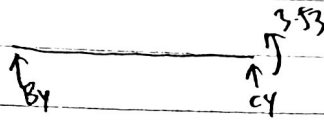
$$A_x \cdot 5 = 8 \times 2.5 + 7.5$$

$$A_x = 5.5$$

$$B_x = 2.5$$



(B/C)



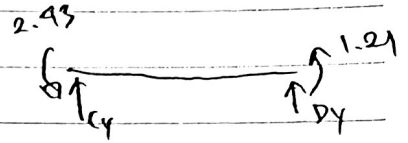
$$\sum M_C = 0$$

$$B_y \times 6 = 3.53$$

$$B_y = 0.5883 \quad C_y = -0.5883$$



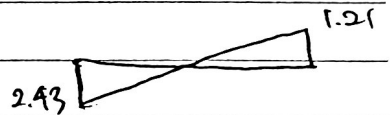
(C/D)



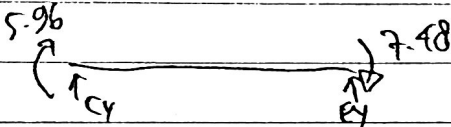
$$\sum M_D = 0$$

$$5C_y = 1.21 + 2.43$$

$$C_y = 0.728 \quad D_y = 0.721$$



(C/E)

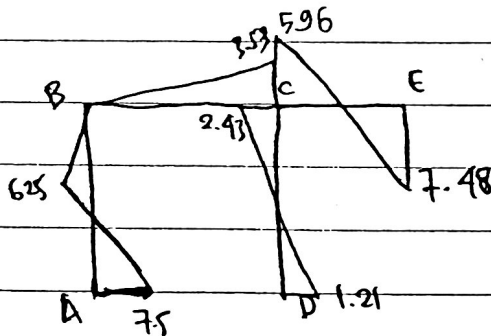
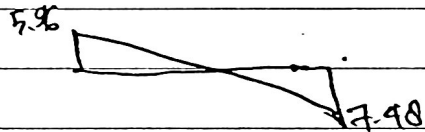
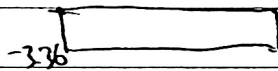


$$\sum M_E = 0$$

$$C_y \times 9 + 7.48 + 5.96 = 0$$

$$C_y = -3.36$$

$$E_y = 3.36$$



ALL THE BEST!



Josh Anderson