NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2012-2013

CV2012 – STRUCTURAL ANALYSIS II

April – May 2013 **Time Allowed: 2½ hours**

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **SIX (6)** pages.
2. Answer **ALL FOUR (4)** questions.
3. All questions carry equal marks.
4. An Appendix of **ONE (1)** page is attached together with this paper.
5. This is an Open Book Examination with restriction to only **ONE (1)** sheet of A4-size paper containing any reference materials.

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1. (a) The beam shown in Figure Q1(a) is subjected to a vertical unit load moving from points A to F. Draw the influence lines for:

- Vertical reaction at support A
- Shear force at point C
- Vertical reaction at support D
- Bending moment in the beam at point D
- Shear force to the left of point D

If the beam is subjected to a uniform dead load of 8 kN/m, a uniform live load of 10 kN/m, and a concentrated live load of 15 kN, determine the maximum vertical reaction at support D.

(13 marks)

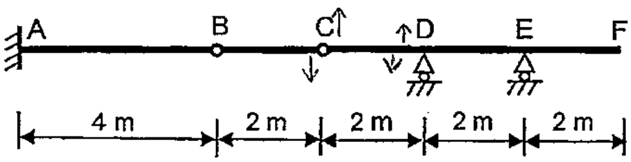


Figure Q1(a)

Note: Question No. 1 continues on page 2

(b) A floor-girder system is shown in Figure Q1(b). A vertical unit load is moving from points A to E on the floor slabs. The load is transferred to the girder GHJKL through the floor beams. Draw the influence lines for:

- Vertical reaction at support H
- Shear force in panel JK
- Bending moment in the girder at point J

Two concentrated loads of 18 kN and 24 kN travel on the floor slabs. Determine the maximum positive bending moment in the girder at point J.

(12 marks)

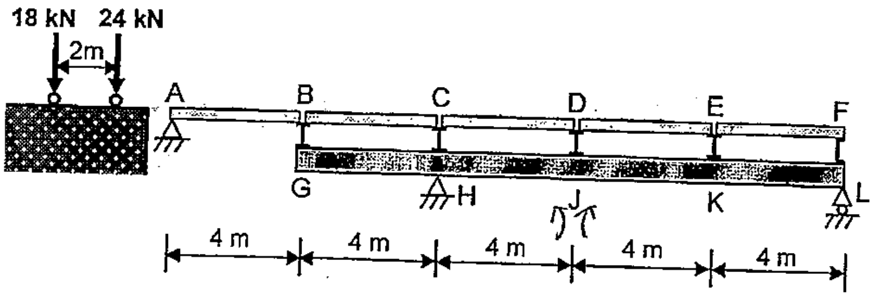


Figure Q1(b)

2. (a) The continuous beam ABCD shown in Figure Q2(a) has a flexural rigidity EI. The beam is subjected to a uniform load from points B to D.

   (i) Using the force method (flexibility method), determine the reaction at support B.

   (ii) Draw the shear force and bending moment diagrams for the beam.

(18 marks)

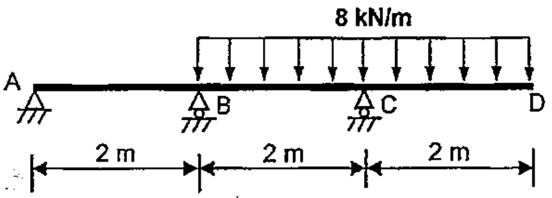


Figure Q2(a)

(b) The beam ABCD shown in Figure Q2(b) is identical to that in Figure Q2(a), except that the roller support at point B is replaced by a rigid link BG, which is connected to a simply supported beam FGH with flexural rigidity $2EI$. Determine the reaction at point B using the force method (flexibility method).

(7 marks)

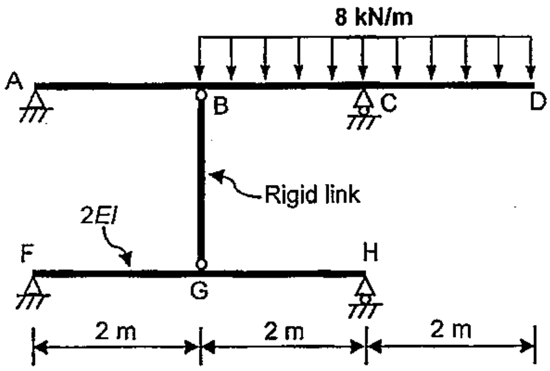


Figure Q2(b)

3. (a) A uniform beam with fixed ends at A and B carries a triangular load over half of the span as shown in Figure Q3(a). Describe a method to evaluate the fixed end moments at A and B (which can be shown to be $FEM_{AB} = -\frac{23}{960}wL^2$ and $FEM_{BA} = \frac{7}{960}wL^2$, respectively).

[Note: You only need to write the relevant integrals and/or equations without the need to evaluate/solve them.]

(5 marks)

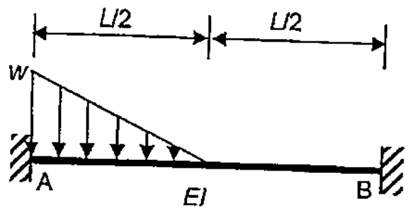


Figure Q3(a)

Note: Question No. 3 continues on page 5

(b) Figure Q3(b) shows a rigid frame with its loading arrangement. Support C is on roller and Supports A and D are fixed. The flexural rigidity for each member is shown in the figure.

(i) By making use of the fixed end moments given in part (a), show that only one degree of freedom is required to solve this problem using the slope deflection method. Identify the unknown degree of freedom.

(ii) Calculate all the member end moments using the slope deflection method.

(iii) If the flexural rigidity for member AB is increased from *EI* to 2*EI* (i.e. 2*EI* for the whole 12 m length), explain if the fixed end moments in part (a) can still be used. Rewrite the equation(s) needed to find the unknown degree of freedom.

(Note: There is no need to solve the equation.)

(iv) If only the flexural rigidity under the triangular load on member AB is increased from *EI* to 2*EI* (i.e. 2*EI* for the left 6 m and *EI* for the other 6 m), explain if the fixed end moments in part (a) can still be used. What is the minimum number of degrees of freedom required now? Identify the unknown degrees of freedom.

(Note: There is no need to write or solve any equations.)

(20 marks)

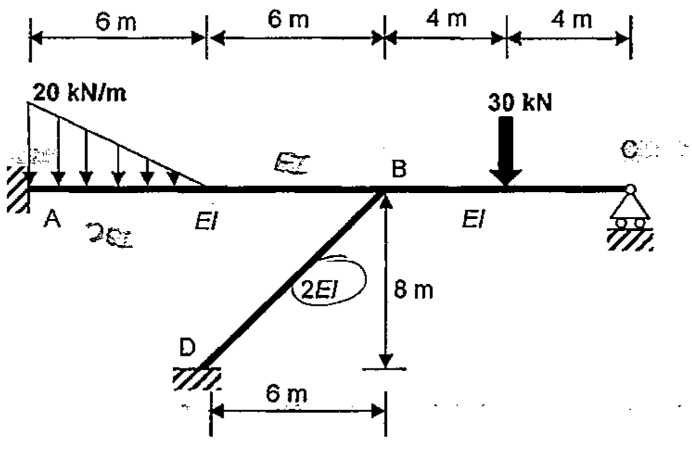


Figure Q3(b)

4. (a) Figure Q4 shows a rigid frame with its loading. The support at A is on roller and the support at E is pinned while the supports at D and F are fixed. The flexural rigidity for each member is shown in the figure.

(i) Calculate all the member end moments using the moment distribution method.

(ii) Draw the bending moment diagram for the beam ABCD.

(15 marks)

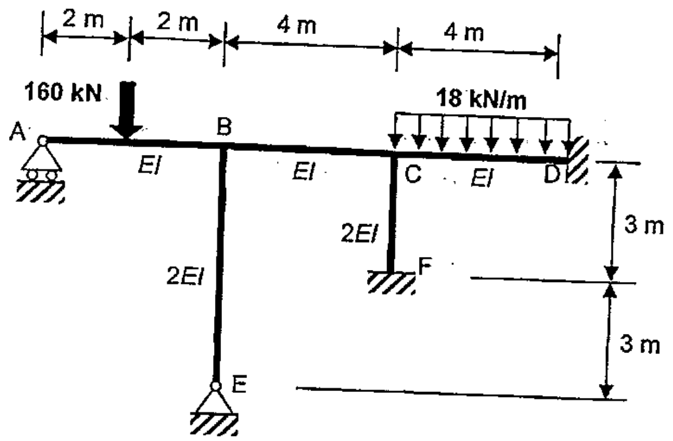


Figure Q4

(b) For each of the two cases below, re-draw the bending moment diagram for the beam ABCD if it is different from the one in part (a)(ii). Re-calculate all the member end moments using the moment distribution method, if necessary.

Case 1: Support A in Figure Q4 is modified from a roller to a hinge (i.e. similar to Support E) while supports D, E and F remain the same.

Case 2: Support E in Figure Q4 is modified from a hinge to a roller (i.e. similar to Support A) while supports A, D and F remain the same.

(10 marks)

END OF PAPER

Slope Deflection Equations:

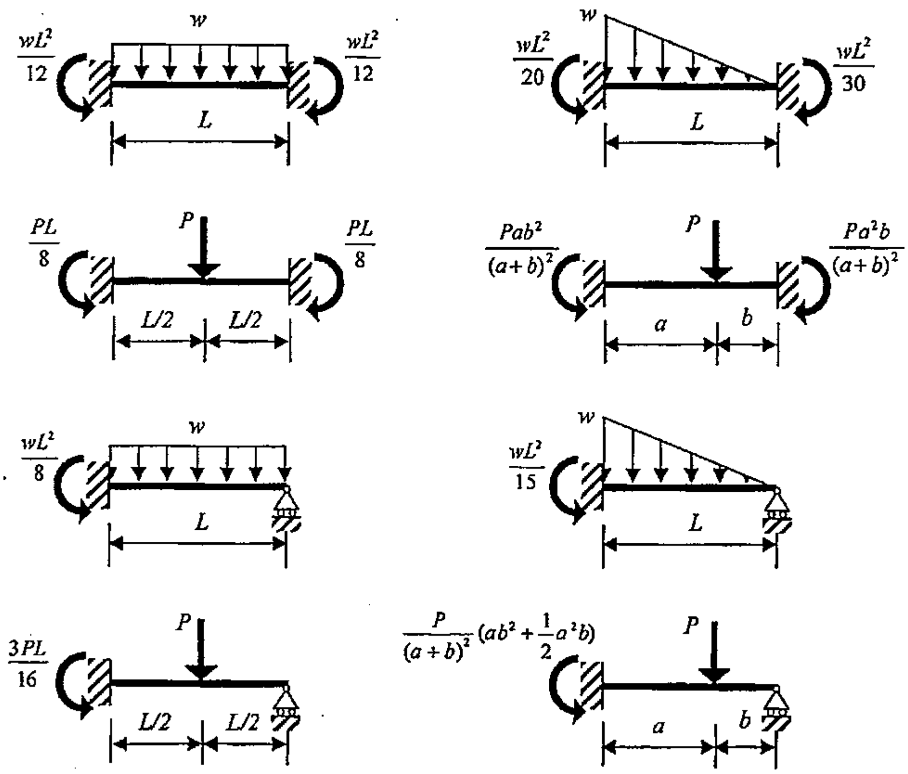
$$M_N = \frac{2EI}{L}\left[2\theta_N + \theta_F - 3\psi\right] + FEM_N \text{, or}$$

$$M_N = \frac{3EI}{L}\left[\theta_N - \psi\right] + FEM_N \quad \text{.............. for member pinned at the far end}$$

where

M_N	=	internal moment at the near end of the span
EI	=	flexural rigidity
L	=	beam length
θ_N, θ_F	=	near and far end slope or angle of the span at the supports
Δ	=	relative lateral displacement at the ends
ψ	=	Δ/L = span rotation of its cord due to a linear displacement
FEM_N	=	fixed end moment at the near end support

Fixed End Moments:



Past Year Paper Solution

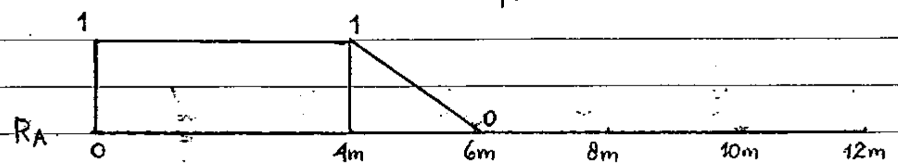
Yes, U Can!

(1) CV 2012 - Structural Analysis II

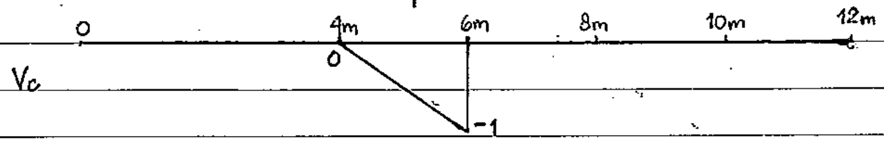
Semester 2 Examination 2012 - 2013

1. a) (i) Influence line for vertical reaction at support A :

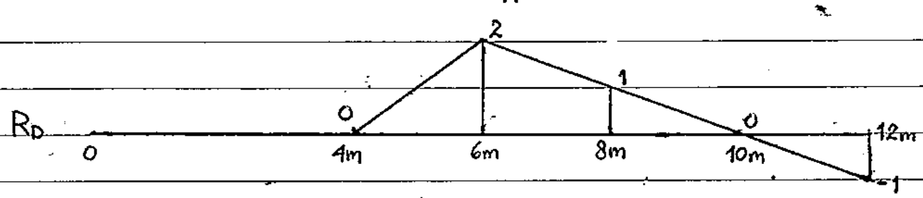
*Notes : Apply Müller - Breslau Principle



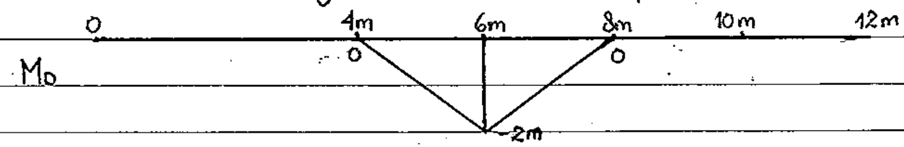
(ii) Influence line for shear force at point C :



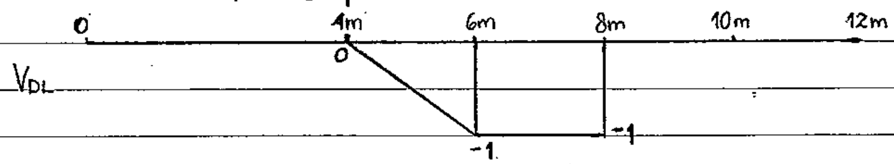
(iii) Influence line for vertical reaction at support D :



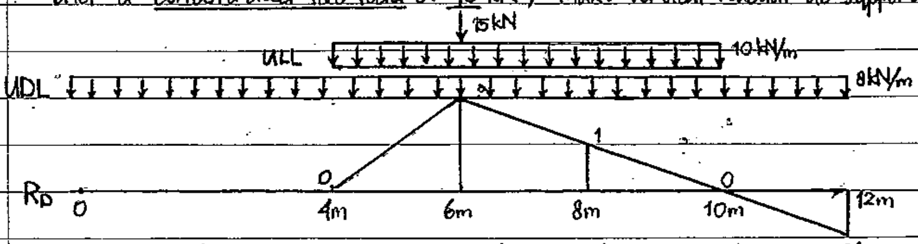
(iv) Influence line for bending moment in the beam at point D :



(v) Shear force to the left of point D :



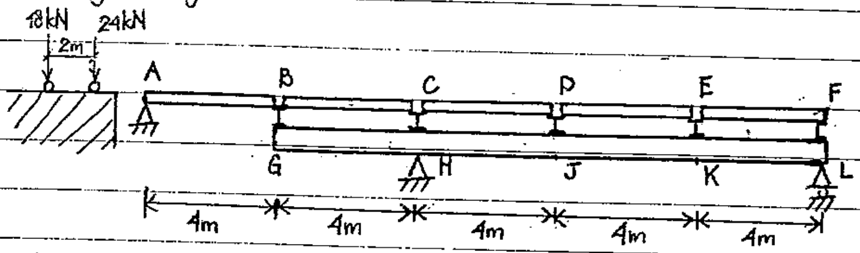
* If the beam is subjected to a uniform dead load of 8 kN/m, a uniform live load of 10 kN/m, and a concentrated live load of 15 kN, max. vertical reaction at support D :



$\therefore R_{D,max} (+) = \left(\frac{1}{2}\times 6\times 2 + \frac{1}{2}\times 2\times(-1)\right)(8) + \left(\frac{1}{2}\times 6\times 2\right)(10) + (2)(15)$

$= 130 \text{ kN}$

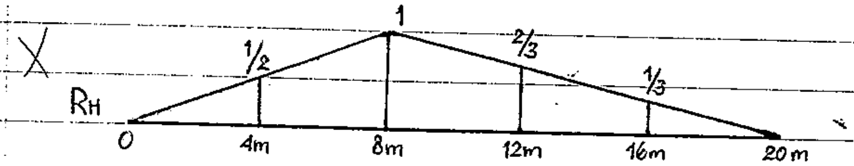
1. b) * Floor girder system :



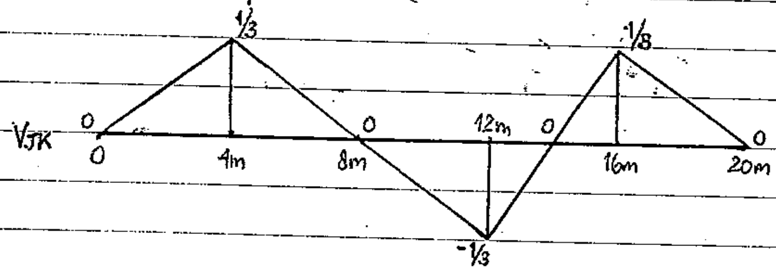
** I think there is a typo error in the question

* Note that a vertical unit load is moving from points A to F, hence the influence lines for :

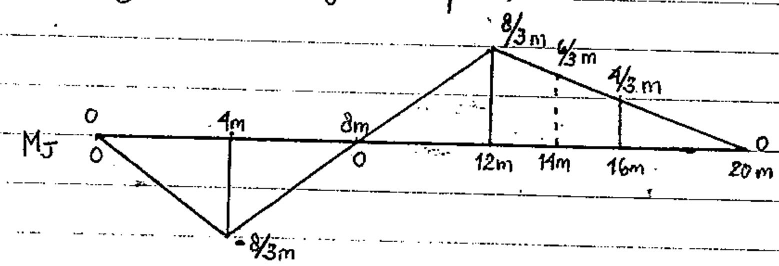
(i) Vertical reaction at support H :



(ii) Shear force in panel JK :



(iii) Bending moment in the girder at point J :

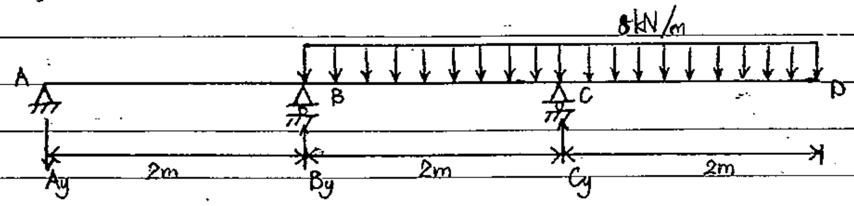


* Two concentrated loads of 18 kN & 24 kN travel on the slabs, hence :

$\therefore\ M_{J,max} = (18)(8/3) + (24)(6/3) = \underline{\underline{96\ kN.m}}$

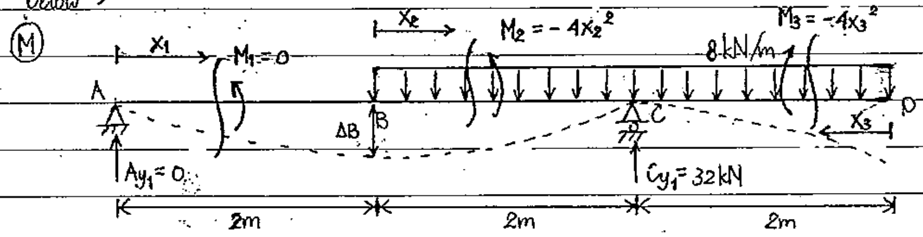
(occurs when load 18kN is on point D)

2\. a) Given : Fig. Q2(a) – Indeterminate structure to the 1st degree



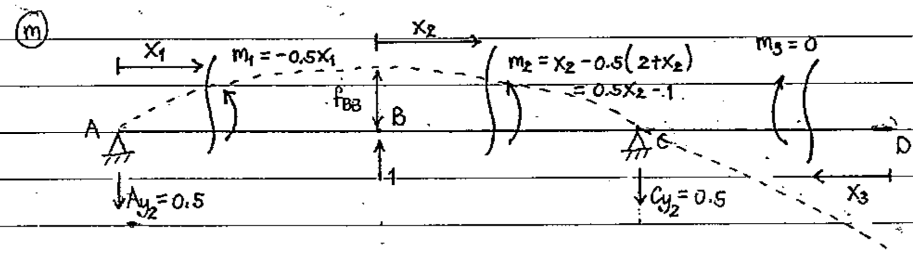
(i) To determine the reaction at support B using force method, we treat reaction at B as redundancy. By using superposition method, the continuous beam is equivalent to the combination of 2 systems below :

(M)



+

(m)



$* \; \Delta B = \int_0^L \frac{M.m}{E.I}\, dx.$

$= \frac{1}{EI} \int_0^2 (-4x_2^2)(0.5x_2 - 1)\, dx_2$

$= \frac{1}{EI} \int_0^2 (-2x_2^3 + 4x_2^2)\, dx_2$

$= \frac{1}{EI} \left(-0.5x_2^4 + \frac{4}{3}x_2^3\right)\Big|_0^2$

$= \frac{1}{EI}\left(-\frac{1}{2}(16) + \frac{4}{3}(8)\right)$

$= \frac{8}{3EI}$

$* \; f_{BB} = \int_0^L \frac{m^2}{EI}\, dx.$

$= \frac{1}{EI}\left[\int_0^2 (-0.5x_1)^2 dx_1 + \int_0^2 (0.5x_2 - 1)^2 dx_2\right]$

$= \frac{1}{EI}\left[\int_0^2 (0.25x_1^2)\, dx_1 + \int_0^2 (0.25x_2^2 - x_2 + 1)\, dx_2\right]$

$= \frac{1}{EI}\left[\left(\frac{0.25}{3}x_1^3\right)\Big|_0^2 + \left(\frac{0.25}{3}x_2^3 - \frac{1}{2}x_2^2 + x_2\right)\Big|_0^2\right]$

$= \frac{1}{EI}\left[\frac{0.25}{3}(8) + \left(\frac{0.25}{3}(8) - \frac{1}{2}(4) + 2\right)\right]$

$= \frac{4}{3EI}$

$*$ Compatibility Equation :

$\Delta B + f_{BB} . B_y = 0.$

$-\frac{8}{3EI} + \left(\frac{4}{3EI}\right) B_y = 0.$

$B_y = 2\,kN$

2. a) (ii) * Find support reactions at A & C :

* $\sum M_A = 0$.

$(8\times4)(4) - (2)(2) - C_y(4) = 0$.

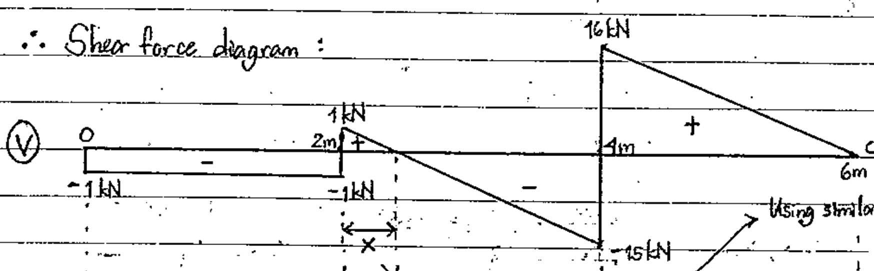
$C_y = 31\text{ kN}$.

* $\sum F_y = 0$

$2 + 31 + A_y - (8\times4) = 0$.

$A_y = -1\text{ kN}$

∴ Shear force diagram :

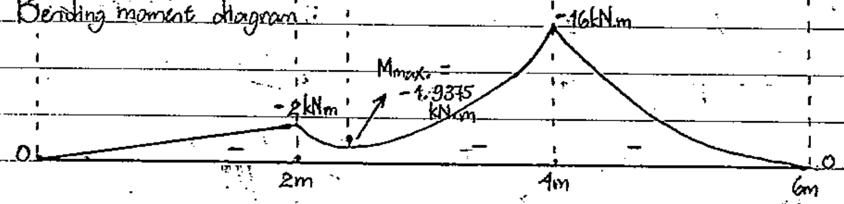


Using similar triangle relationship :

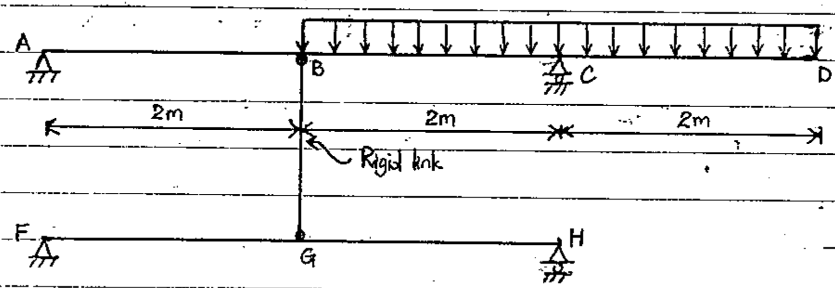
$\frac{x}{2-x} = \frac{1}{15}$

$x = \frac{1}{8} = 0.125\text{ m}$

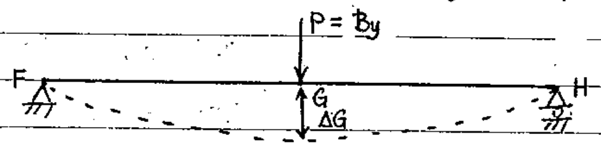
∴ Bending moment diagram :



2. b) Given : Fig. Q2 (b) - Indeterminate structure to the 1st degree.



* Since beam ABCD is identical to that in Fig. Q2 (a), we could simplify beam FGH as :



* From formula of deformation ($\delta_{max} = \frac{PL^3}{48EI}$ — at mid span)

$\Delta G = \frac{-B_y(4)^3}{48(2EI)}$

$= \ominus \frac{2B_y}{3EI}$ → downwards

* Therefore, the support at point B now may move downward as much as $\Delta G = \frac{2B_y}{3EI}$, hence using the same compatibility equation from (Q2 a) (i), we can find the support reaction at B :

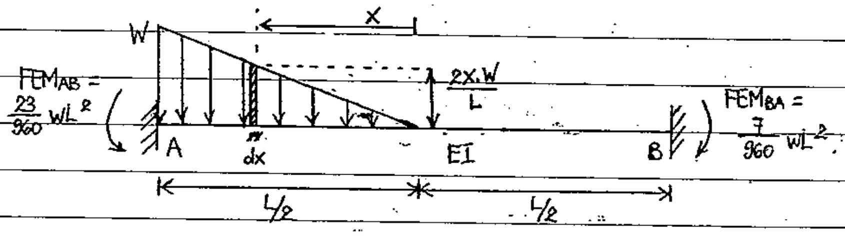
2. b) * $\Delta B + f_{BB} \cdot B_y = \Delta G$

$$-\frac{8}{3EI} + \left(\frac{4}{3EI}\right) B_y = -\frac{2B_y}{3EI}$$

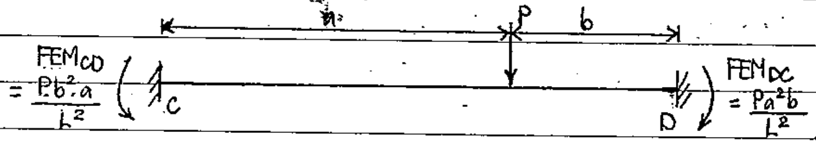
$$\left(\frac{6}{3EI}\right) B_y = \frac{8}{3EI}$$

$$B_y = \frac{4}{3}\,\text{kN}$$

3. a) Given : Fig. Q3(a)



* To find FEM_{AB} & FEM_{BA} for the structure above, we could use method of superposition of the structure below :



∴ Therefore, ① $FEM_{AB} = \int_0^{L/2} \frac{P \cdot b^2 \cdot a}{L^2}$ → where $P = \frac{2xW}{L} \cdot dx$; $b = x + L/2$; $a = L/2 - x$.

$$= \int_0^{L/2} \frac{\left(\frac{2xW}{L}\right)\left(x+\frac{L}{2}\right)^2\left(\frac{L}{2}-x\right)}{L^2} \cdot dx$$

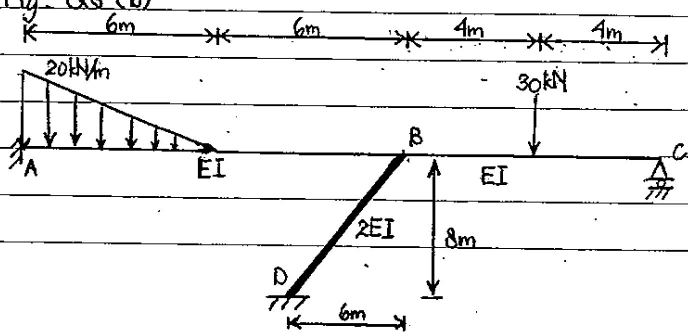
$$= -\frac{23}{960} WL^2$$

② $FEM_{BA} = \int_0^{L/2} \frac{P \cdot a^2 \cdot b}{L^2}$ → where $P = \frac{2xW}{L} dx$; $b = x + L/2$; $a = L/2 - x$.

$$= \int_0^{L/2} \frac{\left(\frac{2xW}{L}\right)\left(\frac{L}{2}-x\right)^2\left(x+\frac{L}{2}\right)}{L^2} dx$$

$$= \frac{7}{960} WL^2$$

3. b) * Given : Fig. Q3(b)



Yes, U Can!

3. b) (i) * Since A & D is fixed support, $\theta_A = \theta_D = 0$.

* No span rotation throughout the whole structure, $\psi_{AB} = \psi_{BA} = \psi_{BC} = \psi_{CB} = \psi_{BD} = \psi_{DB} = 0$.

* Hence, there are 2 degrees of freedom, which are θ_B & θ_C.

* However, the rotation θ_C can be eliminated as it is not needed for further calculation.

Proof : by using the slope deflection method formula (analyze span BC):

① $M_N = 2\frac{EI}{L}(2\theta_N + \theta_F - 3\psi) + FEM_N$.

$M_{BC} = 2\left(\frac{EI}{L}\right)(2\theta_B + \theta_C - 3(0)) + FEM_{BC}$.

$= 2\left(\frac{EI}{L}\right)(2\theta_B + \theta_C) + FEM_{BC}$

② $M_F = 2\frac{EI}{L}(2\theta_F + \theta_N - 3\psi) + FEM_F$.

$M_{CB} = 2\left(\frac{EI}{L}\right)(2\theta_C + \theta_B - 3(0)) + FEM_{CB}$.

$= 2\left(\frac{EI}{L}\right)(2\theta_C + \theta_B) + FEM_{CB} = 0.$ → Since C is pin support.

$\therefore M_{BC} = M_{BC} - \frac{1}{2}\underbrace{M_{CB}}_{=0}$

$= 2\left(\frac{EI}{L}\right)\left(\frac{3}{2}\theta_B\right) + FEM_{BC} - \frac{1}{2}\underbrace{FEM_{CB}}_{=0}$ → since C is pin support.

$= 3\left(\frac{EI}{L}\right)\theta_B + FEM_{BC}$

Therefore, there is only 1 degree of freedom reqired to solve the problem which is θ_B.

↳ Shown!

(ii) * Fixed end moments :-

* $FEM_{AB} = -\frac{23}{960}wL^2 = \frac{-23}{960}(20)(12)^2 = -69$ kN.m.

* $FEM_{BA} = \frac{7}{960}wL^2 = \frac{7}{960}(20)(12)^2 = 21$ kN.m.

* $FEM_{BC} = -\frac{3PL}{16} = -\frac{3(30)(8)}{16} = -45$ kN.m.

* $FEM_{BD} = FEM_{DB} = 0$ → since there is no load on span BD.

* End moments of all the members :

① For span AB,

* $M_{AB} = 2\left(\frac{EI}{12}\right)(2(0) + \theta_B - 3(0)) + (-69)$.

$= \left(\frac{EI}{6}\right)\theta_B - 69$ ... (1)

* $M_{BA} = 2\left(\frac{EI}{12}\right)(2\theta_B + 0 - 3(0)) + 21$.

$= \left(\frac{EI}{3}\right)\theta_B + 21$ ... (2).

Yes, U Can!

3 b) (ii) ② For span BC,

$$* M_{BC} = 3\left(\frac{EI}{8}\right)\theta_B + (-45)$$
$$= \left(\frac{3EI}{8}\right)\theta_B - 45 \ \ldots (3)$$

$$* M_{CB} = 0.$$

③ For span BD,

$$* M_{BD} = 2\left(\frac{2EI}{10}\right)(2\theta_B + 0 - 3(0)) + 0.$$
$$= \left(\frac{4EI}{5}\right)\theta_B \ \ldots (4).$$

$$* M_{DB} = 2\left(\frac{2EI}{10}\right)(2(0) + \theta_B - 3(0)) + 0.$$
$$= \left(\frac{2EI}{5}\right)\theta_B \ \ldots (5).$$

* Moment Equilibrium at joint B:

$$M_{BA} + M_{BC} + M_{BD} = 0.$$
$$\left(\frac{EI}{3}\right)\theta_B + 21 + \left(\frac{3EI}{8}\right)\theta_B - 45 + \left(\frac{4EI}{5}\right)\theta_B = 0.$$
$$\left(\frac{40+45+96}{120}\right) EI.\theta_B = 24.$$
$$\theta_B = \frac{2880}{181EI}.$$

* Substitute back θ_B to eq. (1), (2), (3), (4), (5):

$$* M_{AB} = \left(\frac{EI}{6}\right)\left(\frac{2880}{181EI}\right) - 69 = \underline{\underline{-66.35}} \text{ kN.m.}$$

$$* M_{BA} = \left(\frac{EI}{3}\right)\left(\frac{2880}{181EI}\right) + 21 = \underline{\underline{26.30}} \text{ kN.m.}$$

$$* M_{BC} = \left(\frac{3EI}{8}\right)\left(\frac{2880}{181EI}\right) - 45 = \underline{\underline{-39.03}} \text{ kN.m.}$$

$$* M_{CB} = \underline{\underline{0}}$$

$$* M_{BD} = \left(\frac{4EI}{5}\right)\left(\frac{2880}{181EI}\right) = \underline{\underline{12.73}} \text{ kN.m.}$$

$$* M_{DB} = \left(\frac{2EI}{5}\right)\left(\frac{2880}{181EI}\right) = \underline{\underline{6.36}} \text{ kN.m.}$$

(iii) If the flexural rigidity for member AB is increased from EI to 2EI, all the fixed end moments in part (a) can still be used, since fixed end moments are not dependent to the rigidity of the members.

3. b) (iii) * Equations needed to find the unknown degree of freedom :

$$* \ M_{AB} = 2\left(\frac{2EI}{12}\right)(2(0) + \theta_B - 3(0)) + (-69)$$
$$= \left(\frac{EI}{3}\right)\theta_B - 69 \ \ldots (1)$$

$$* \ M_{BA} = 2\left(\frac{2EI}{12}\right)(2\theta_B + 0 - 3(0)) + 21$$
$$= \left(\frac{2EI}{3}\right)\theta_B + 21 \ \ldots (2)$$

$$* \ M_{BC} = \left(\frac{3EI}{8}\right)\theta_B - 45 \ \ldots (3)$$

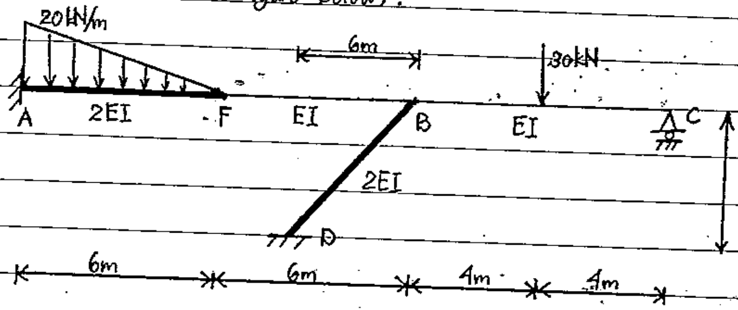
$$* \ M_{BD} = \left(\frac{4EI}{5}\right)\theta_B \ \ldots (4)$$

$$* \ M_{DB} = \left(\frac{2EI}{5}\right)\theta_B \ \ldots (5)$$

$$* \ M_{BA} + M_{BC} + M_{BD} = 0 \ \ldots (6)$$

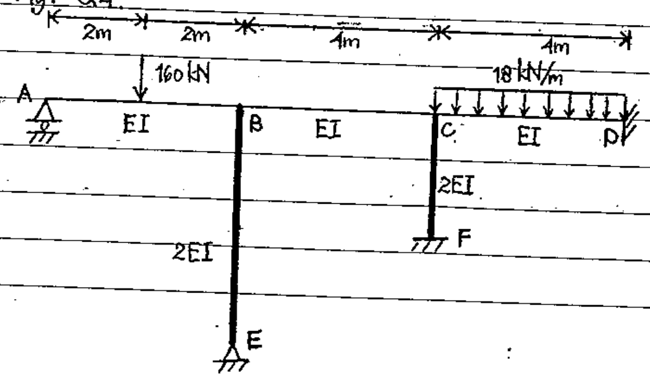
Same as part (ii).

(iv) If only the flexural rigidity under the triangular load on member AB is increased from EI to 2EI, the fixed end moments in part (a) cannot be used, since member AB will not behave uniformly. And hence, by using slope deflection method, member AB needs to be splitted into 2 parts. (i.e. AF & FB → see the figure below).



∴ Therefore, from the figure (illustration) above, the minimum number of degrees of freedom required is 2, i.e. θ_F & θ_B.

4. a) Given : Fig. Q4.



(9) Yes, U Can!

4. a) (i) * Fixed End Moments :

$$* \; FEM_{BA} = \frac{3PL}{16} = \frac{3(160)(4)}{16} = 120 \text{ kN.m}$$

$$* \; FEM_{CD} = -\frac{wL^2}{12} = -\frac{(18)(4)^2}{12} = -24 \text{ kN.m}$$

$$* \; FEM_{DC} = \frac{wL^2}{12} = \frac{(18)(4)^2}{12} = 24 \text{ kN.m}$$

* Notes :
① Fixed - fixed end : $K = \frac{4EI}{L}$.
② Fixed - pinned end : $K = \frac{3EI}{L}$
③ Symmetrical beam & loading : $K = \frac{2EI}{L}$.
④ Anti - symmetric loading : $K = \frac{6EI}{L}$.

* Stiffness Factor :

$$* \; K_{AB} = K_{BA} = \frac{3EI}{4} = \frac{3EI}{4}$$

$$* \; K_{BC} = K_{CB} = \frac{4EI}{4} = EI$$

$$* \; K_{CD} = K_{DC} = \frac{4EI}{4} = EI.$$

$$* \; K_{CF} = K_{FC} = \frac{4(2EI)}{3} = \frac{8EI}{3}.$$

$$* \; K_{BE} = K_{EB} = \frac{3(2EI)}{6} = EI.$$

* Distribution Factor (DF) :

$$* \; DF_{AB} = \frac{3/4 \, EI}{3/4 \, EI} = 1$$

$$* \; DF_{BA} = \frac{3/4 \cdot EI}{3/4 \cdot EI + EI + EI} = 0.2727$$

$$* \; DF_{BC} = \frac{EI}{3/4 \, EI + EI + EI} = 0.3636$$

$$* \; DF_{CB} = \frac{EI}{EI + 8/3 \, EI + EI} = 0.2143.$$

$$* \; DF_{CD} = \frac{EI}{EI + 8/3 \, EI + EI} = 0.2143$$

$$* \; DF_{DC} = \frac{EI}{EI + \infty} = 0.$$

$$* \; DF_{CF} = \frac{8/3 \, EI}{EI + 8/3 \, EI + EI} = 0.5714.$$

$$* \; DF_{FC} = \frac{8/3 \, EI}{8/3 \, EI + \infty} = 0.$$

$$* \; DF_{BE} = \frac{EI}{3/4 \, EI + EI + EI} = 0.3636$$

$$* \; DF_{EB} = \frac{EI}{EI} = 1.$$

* Moment Distribution Method :

Joint	A	B		E	B	C		D	C	F
Member	AB	BA	BE	EB	BC	CB	CD	DC	CF	FC
DF	1	0.2727	0.3636	1	0.3636	0.2143	0.2143	0	0.5714	0
FEM		120					-24	24		
Dist.		-32.7	-43.6		-43.6	5.1	5.1		13.7	
CO					2.6	-21.8		2.6		6.9
Dist.		-0.7	-0.9		-0.9	4.7	4.7		12.5	
CO					2.4	-0.5		2.4		6.25
Dist.		-0.65	-0.87		-0.87	0.11	0.11		0.29	
$\sum M$	0	85.95	-45.37	0	-40.37	-12.39	-14.09	29	26.49	13.15

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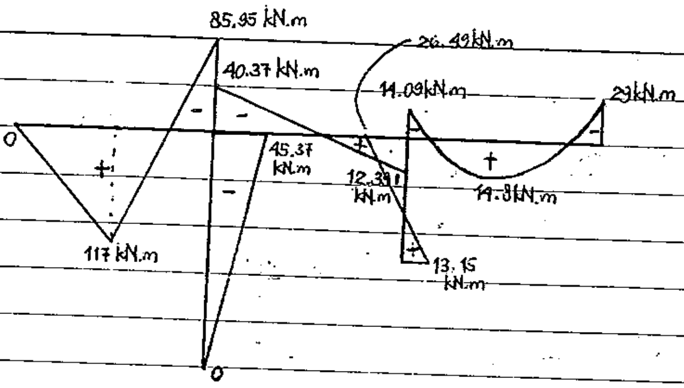
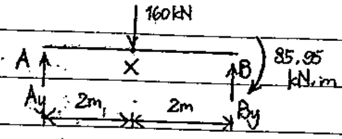
* Bending Moment Diagram :

Yes, U Can!

4. a) (ii)



* Find support reaction at A :



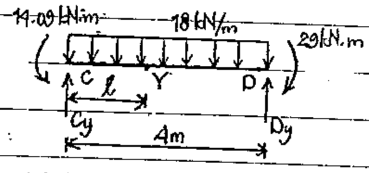
$\Sigma M_B = 0.$

$A_y(4) + 85.95 - (160)(2) = 0.$

$A_y = 58.5\ kN.$

* $M_X = (58.5)(2) = 117\ kN.m$

* Analyze span CD :



→ where Y is the point where $M_Y = M_{max}$.

* $\Sigma M_D = 0.$

$C_y(4) - 14.09 + 29 - \frac{1}{2}(18)(4)^2 = 0.$

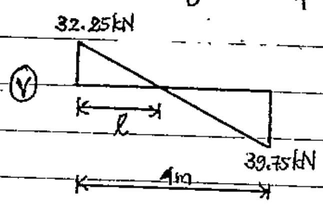
$C_y = 32.25\ kN.$

* $\Sigma F_y = 0.$

$D_y + 32.25 - (18)(4) = 0.$

$D_y = 39.75\ kN.$

* Shear force diagram of span CD :



* $\frac{\ell}{32.25} = \frac{4-\ell}{39.75}.$

$\ell = 1.79m.$

* Hence, $M_Y = M_{max} = (32.25)(1.79) - 14.09 - \frac{1}{2}(18)(1.79)^2 = 14.8\ kN.m.$

4. b) Case 1 : the bending moment diagram remains the same as part (a).

Case 2 : the bending moment diagram remains the same as part (a).