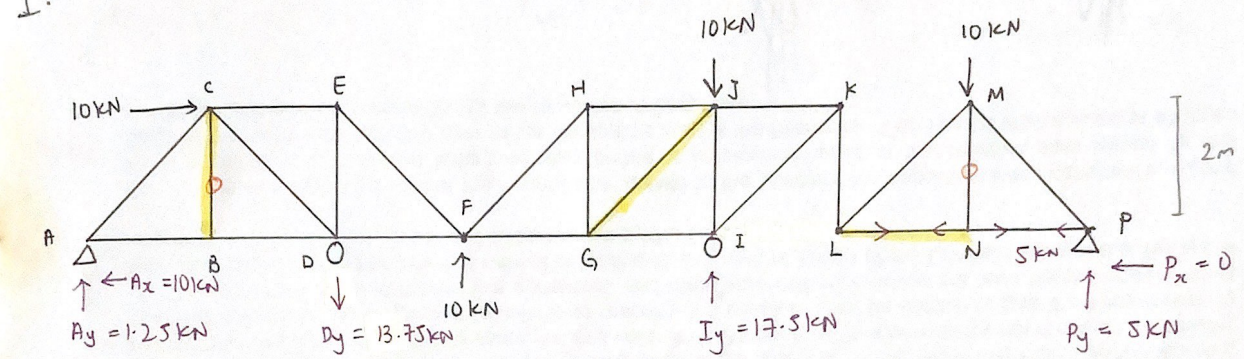
2017-2018 sem 2

1.



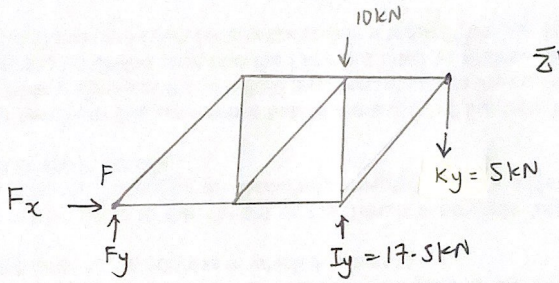
(a) External = Stable

DOF = 2j - b - r

= 2(15) - 24 - 6

= 0

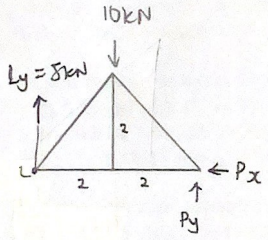
∴ Stable & statically determinate.



$\sum M_F = 0$, $I_y(4) = 10(4) + 5(6)$

I_y = 17.5kN

(b)



$\sum M_P = 0$

$L_y(4) = 10(2)$

L_y = 5kN

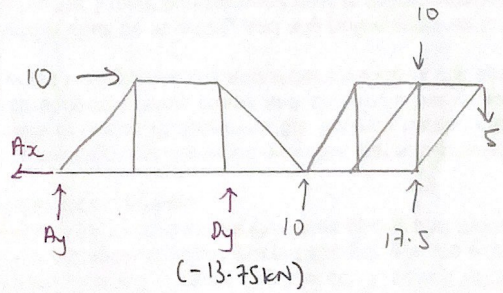
$\sum F_x = 0$

$P_x = 0$

$\sum M_L = 0$

$P_y(4) = 10(2)$

P_y = 5kN



$\sum F_x = 0$

A_x = 10 kN #

$\sum M_H = 0$

$D_y(4) + 10(6) + 17.5(10) = 10(2) + 10(10) + 5(12)$

D_y = -13.75 kN

$\sum M_D = 0$

$A_y(4) + 10(2) - 10(2) - 17.5(6) + 10(6) + 5(8) = 0$

A_y = 1.25 kN #

1c)

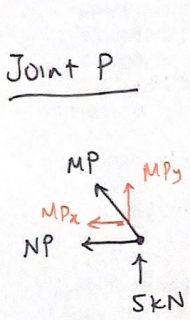
By inspection: $F_{CB} = 0\,kN$ #

$F_{LN} = 5\,kN\,(T)$

$F_{GJ} = 3.55\,kN\,(T)$ #

$|LN| = |NP|$

Joint P



$MP_y + 5 = 0$

$MP\left(\frac{2}{\sqrt{8}}\right) = -5$

$MP = -5 \times \frac{\sqrt{8}}{2}$

$MP = -\frac{5\sqrt{8}}{2}\,kN$

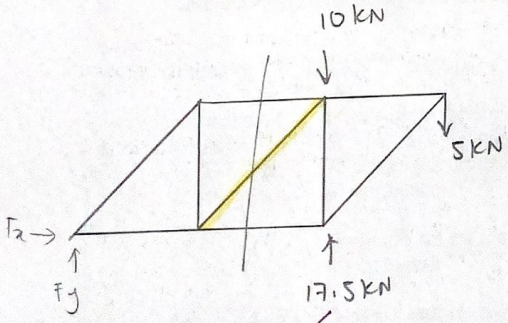
$\therefore MP_x = -NP$

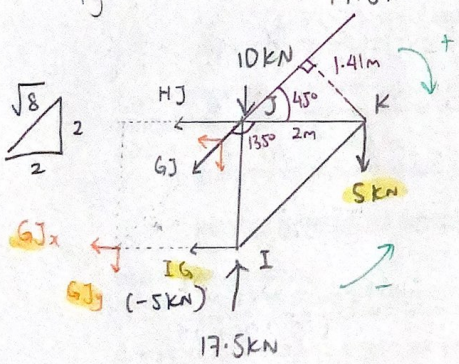
$NP = -MP_x$

$= -\left(-\frac{5\sqrt{8}}{2}\right)\left(\frac{2}{\sqrt{8}}\right)$

$= +5\,kN$

$= 5\,kN$





$\sum M_J = 0$

$IG(2) + 5(2) = 0$

$IG = -5\,kN$

$\sum M_K = 0$

$-10(2) - GJ(1.41) + (-5)(2) + 17.5(2) = 0$

$GJ = 3.55\,\text{on}\,kN\,(T)$ #

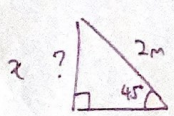
$GJ_x = GJ\left(\frac{2}{\sqrt{8}}\right)$

$GJ_y = GJ\left(\frac{2}{\sqrt{8}}\right)$

$\sum M_J = 0$

$5(2) + IG(2) + GJ_x(2) - GJ_y(2) = 0$

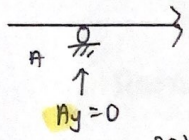
$10 + (-5)(2) + GJ\left(\frac{2}{\sqrt{8}}\right)(2) - GJ\left(\frac{2}{\sqrt{8}}\right)(2) = 0$

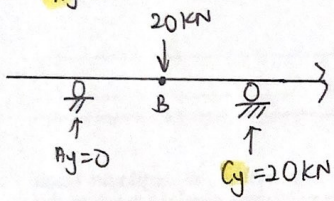


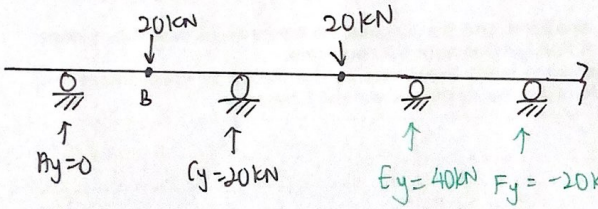
$\sin 45° = \frac{x}{2}$

$x = 1.41m$

2a)







$\sum M_E = 0$

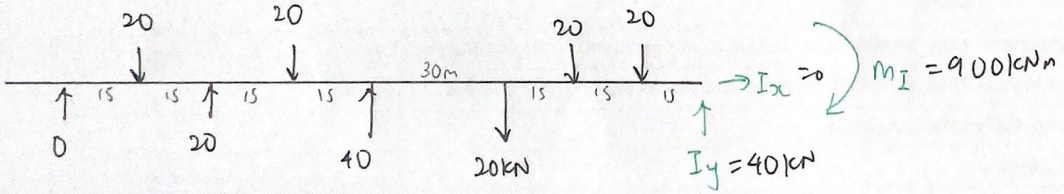
$F_y(30) + 20(15) + 20(15 \times 3) = 20(30)$

$F_y = -20\ kN$

$\sum M_P = 0$

$E_y(30) + 20(60) = 20(45) + 20(75)$

$E_y = 40\ kN$



$\sum M_I = 0$

$M_I = -40(75) - 20(105) + 20(120) + 20(90) + 20(45) + 20(30) + 20(15)$

$= 900\ kNm$

$\sum F_y = 0$

$I_y = 20(5) - 20 - 40$

$= 40\ kN$

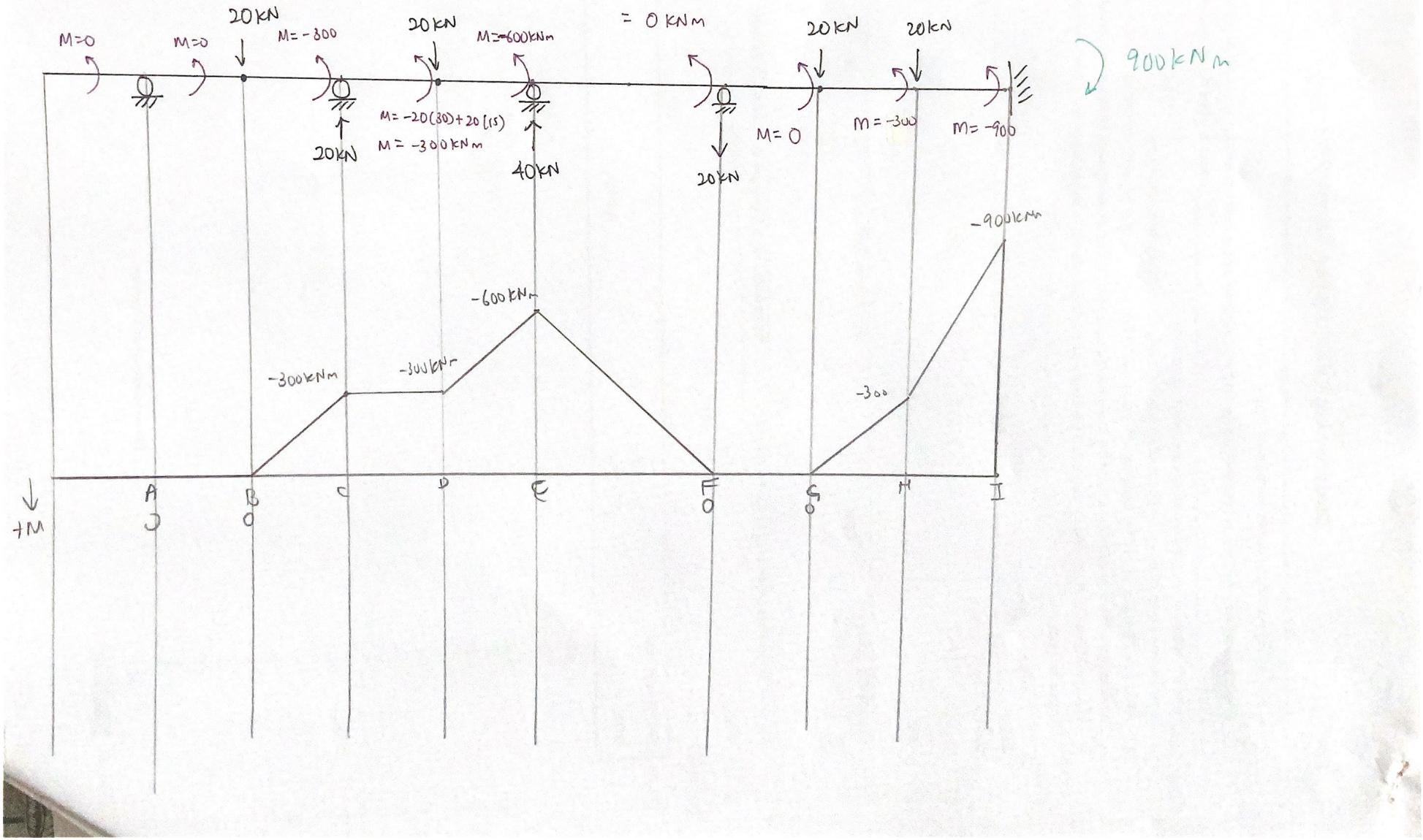
29)

$M = -20(45) - 20(15) + 20(30)$

$= -600\,kNm$

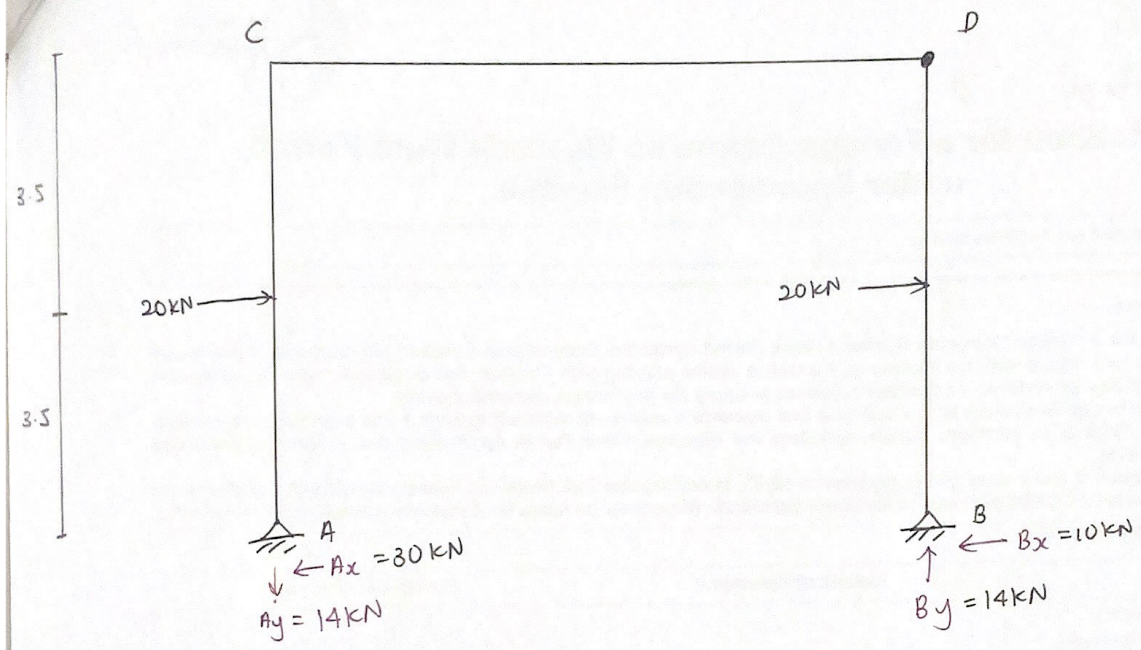
$M = -20(75) - 20(45) + 20(60) + 40(30)$

$= 0\,kNm$



2a

2b)



$\sum M_B = 0$

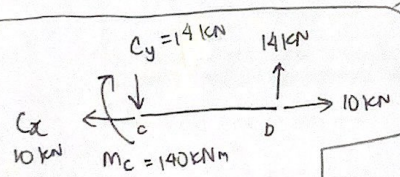
$By(10) = 20(3.5) + 20(3.5)$

$By = 14kN$

$\sum M_B = 0$

$Ay(10) = 20(3.5) + 20(3.5)$

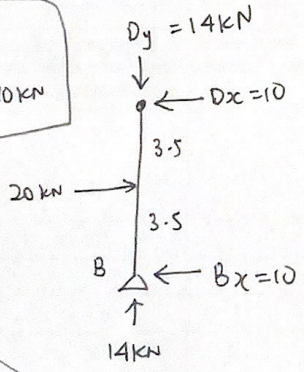
$Ay = 14kN$



$C_y = 14kN$

$C_x = 10kN$

$M_C = 14(10) = 140\ kNm$



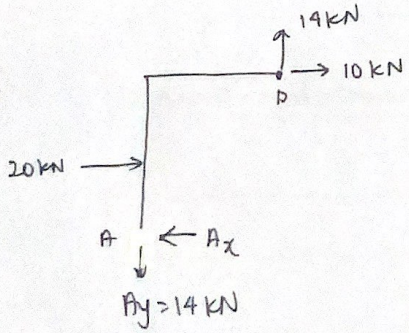
$\sum M_D = 0$

$Bx(7) = 20(3.5)$

$Bx = 10kN$

$\sum M_B = 0$

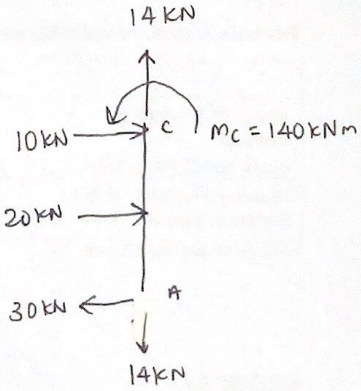
Dx



$\sum M_D = 0$

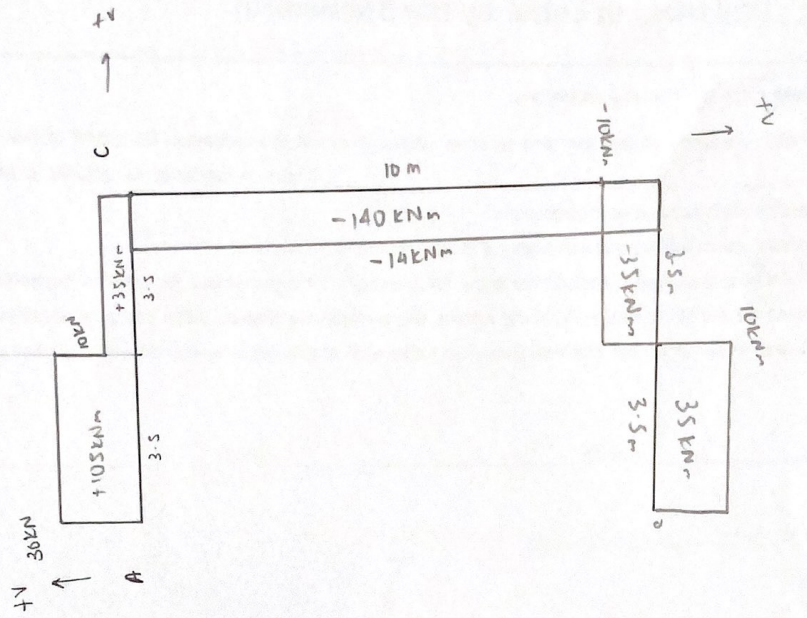
$A_x(7) = 20(3.5) + 14(10)$

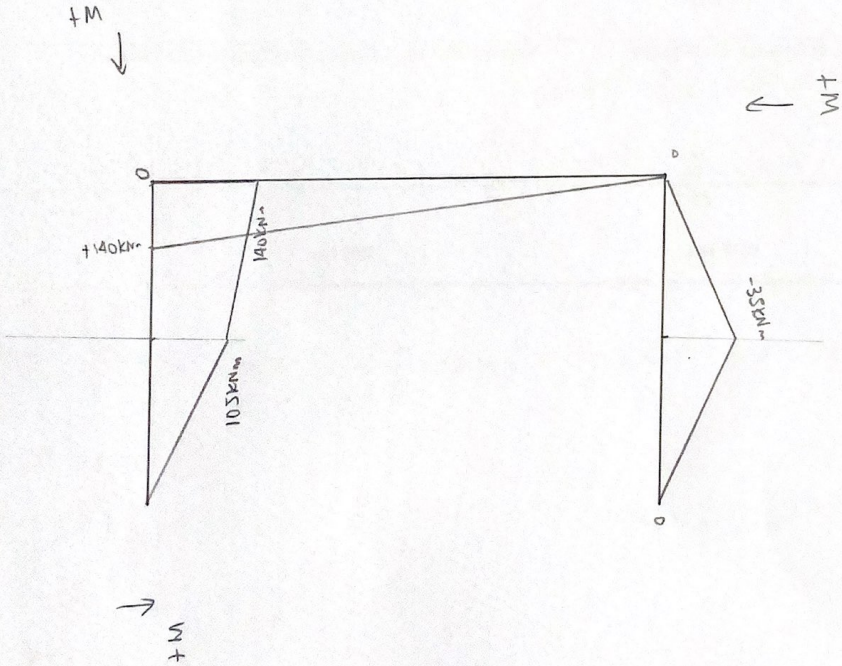
$A_x = 30kN$



SFD

2b)

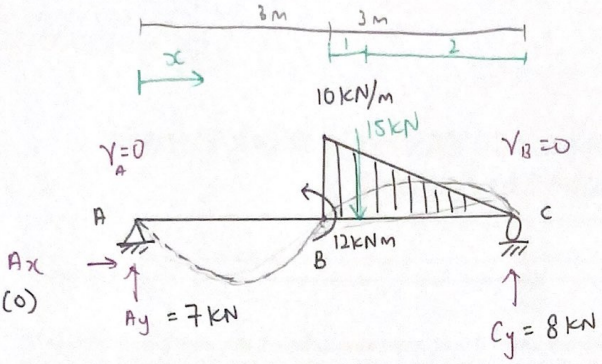

+V
C
10 m
-10KN
-140 KNm
-14KNm
-35KNm
3.5m
10KNm
35 KNm
3.5m
D
+V
+105KNm
3.5
10KN
+35KNm
3.5
36KN
+V
A


+M
+M
O
+140KN
140KNm
105KNm
-35KNm
O
O
+M

3a

$E = 200\text{GPa} = (200 \times 10^{+6})\ \text{kN/m}^2$

$I = 18 \times 10^6 \times (10^{-3})^4\ \text{m}^4 = (18 \times 10^{-6})\ \text{m}^4$



(i) $\sum M_A = 0$

$C_y(6) + 12 = 15(4)$

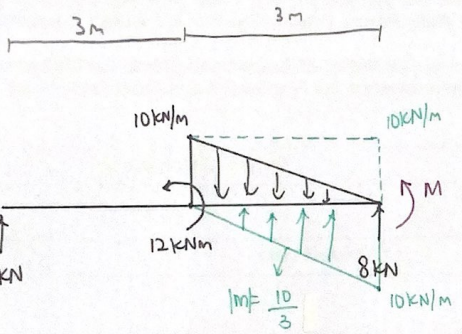
$C_y = 8\text{kN}$ #

$\sum M_C = 0$

$A_y(6) - 15(2) = 12$

$A_y = 7\text{kN}$ #

$A_x = 0\text{ kN}$ ←



(ii) $Y = ?$

$M = 7\langle x-0\rangle^1 - 12\langle x-3\rangle^0 - \frac{10}{2}\langle x-3\rangle^2 + \frac{10}{3(6)}\langle x-3\rangle^3$

$M = 7x - 12\langle x-3\rangle^0 - 5\langle x-3\rangle^2 + \frac{5}{9}\langle x-3\rangle^3$

$M = EIY''(x)$

slope: $EIY'(x) = \int M\,dx = \int 7x - 12\langle x-3\rangle^0 - 5\langle x-3\rangle^2 + \frac{5}{9}\langle x-3\rangle^3\,dx$

$EIY'(x) = \frac{7x^2}{2} - 12\langle x-3\rangle^1 - \frac{5}{3}\langle x-3\rangle^3 + \frac{5}{36}\langle x-3\rangle^4 + A$

Def.: $EIY(x) = \int \frac{7}{2}x^2 - 12\langle x-3\rangle^1 - \frac{5}{3}\langle x-3\rangle^3 + \frac{5}{36}\langle x-3\rangle^4 + A\,dx$

$EIY(x) = \frac{7}{6}x^3 - 6\langle x-3\rangle^2 - \frac{5}{12}\langle x-3\rangle^4 + \frac{1}{36}\langle x-3\rangle^5 + Ax + B$

BC1: $Y_A = Y(x=0) = 0$

$0 = 0 - 6\langle 0-3\rangle^0{}^2 - 0 + 0 + 0 + B$

$B = 0$

BC2: $Y_B = Y(x=6) = 0$

$0 = \frac{7}{6}(6)^3 - 6\langle 6-3\rangle^2 - \frac{5}{12}\langle 6-3\rangle^4 + \frac{1}{36}\langle 6-3\rangle^5 + A(6)$

$0 = 171 + A(6)$

$A = -28.5$

$\therefore Y(x) = \frac{1}{EI}\left[\frac{7}{6}x^3 - 6\langle x-3\rangle^2 - \frac{5}{12}\langle x-3\rangle^4 + \frac{1}{36}\langle x-3\rangle^5 - 28.5x\right]$ #

$\delta_B = Y(x=3)$

$= \frac{1}{EI}\left[\frac{7}{6}(3)^3 - 6\langle 3-3\rangle^0{}^2 - \frac{5}{12}\langle 3-3\rangle^4 + \frac{1}{36}\langle 3-3\rangle^5 - 28.5(3)\right]$

$= \frac{-54}{200\times10^{+6} \times 18\times10^{-6}}$

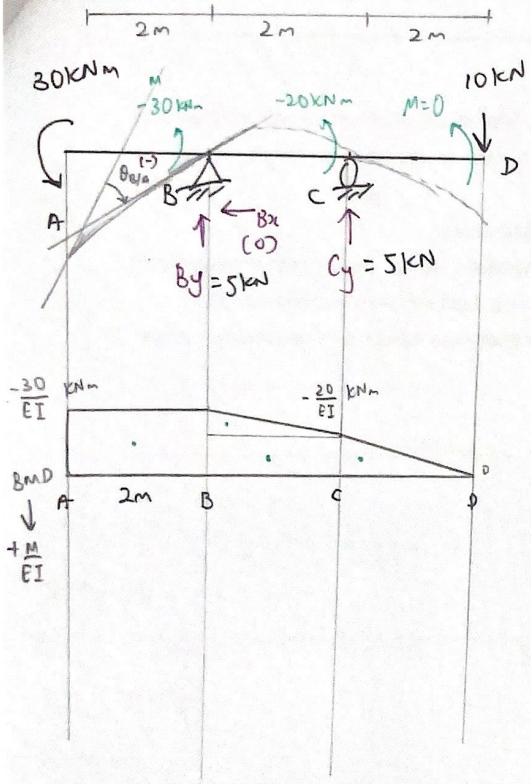
$= -0.015\text{ m}$

$= 0.015\text{ m}$ (downwards) #

3b

(3b) $E = 200\,GPa = (200\times10^{6})\ kN/m^2$ $\theta_B = ?$

$I = (18\times10^{6}\times10^{-12}) = (18\times10^{-6})\ m^4$



$\sum M_B = 0$

$Cy(2) + 30 = 10(4)$

$Cy = 5KN$

$\sum M_C = 0$

$By(2) + 10(2) = 30$

$By = 5KN$

$\sum F_x = 0$

$Bx = 0$

Thm 1: Slope at B ($\theta_{B/A}$)

$\theta_{B/A} = A_{B/A}$

$= -\frac{30}{EI} \times 2$

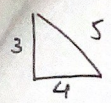
$= -\frac{30\times2}{200\times18}$

$= -\frac{1}{60}\ rad$

$= \frac{1}{60}\ rad$ (clockwise)

#

a)

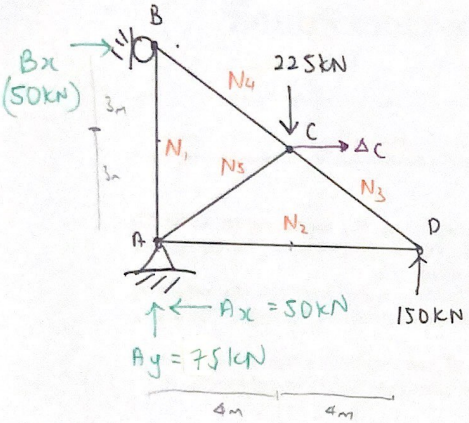


$A = (2.5 \times 10^{-3})\ m^2$

$E = (200 \times 10^6)\ kN/m^2$

$\delta_{C_H} = ?$

Real load (N_i)



$\Sigma M_A = 0$

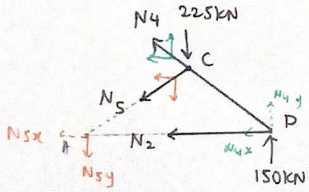
$Bx(6) + 225(4) - 150(8) = 0$

$Bx = 50kN$

$\Sigma M_B = 0$

$Ax(6) + 225(4) - 150(8) = 0$

$Ax = 50kN$



$\Sigma M_A = 0$

$N_{4y}(8) + 150(8) = 225(4)$

$N_{4y} = -37.5$

$N_4\left(\frac{3}{5}\right) = -37.5$

$N_4 = -62.5kN$

$\Sigma M_D = 0$

$N_{5y}(8) + 225(4) = 0$

$N_{5y} = -112.5$

$N_5\left(\frac{3}{5}\right) = -112.5$

$N_5 = -187.5kN$

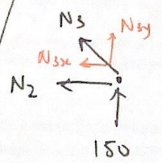
$\Sigma M_C = 0$

$N_2(3) = 150(4)$

$N_2 = 200kN$

check

Point D



$N_3 = -250kN$

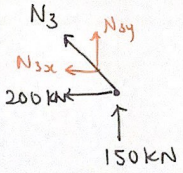
$\therefore N_2 = -N_{3x}$

$= -N_3\left(\frac{4}{5}\right)$

$= -(-250)\left(\frac{4}{5}\right)$

$= 200kN$ ✓

Joint D



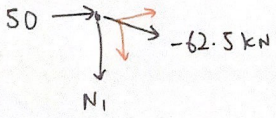
$N_{3y} + 150 = 0$

$N_{3y} = -150$

$N_3\left(\frac{3}{5}\right) = -150$

$N_3 = -250kN$

Joint B

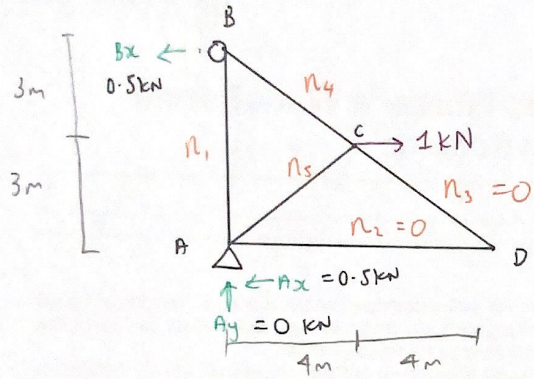


$N_1 + (-62.5)\left(\frac{3}{5}\right) = 0$

$N_1 = 37.5kN$

4a

(4a) Virtual load (n_i)



$\sum m_A = 0$

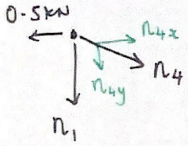
$Bx(6) = 1(3)$

$Bx = 0.5\,kN$

$\sum m_B = 0$

$Ax = 0.5\,kN$

Joint B



$n_{4x} = 0.5$

$n_4\left(\frac{4}{5}\right) = 0.5$

$n_4 = 0.625\,kN$

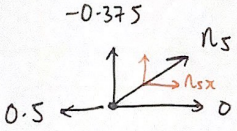
$n_{4y} + n_1 = 0$

$n_1 = -n_{4y}$

$= -0.625\left(\frac{3}{5}\right)$

$n_1 = -0.375\,kN$

Joint A



$n_{5x} = 0.5$

$n_5\left(\frac{4}{5}\right) = 0.5$

$n_5 = 0.625\,kN$

$\therefore \cancel{\Delta = \int}\ \ 1 \cdot \Delta_c = \sum_i n_i \left(\frac{N_i L_i}{A_i E_i}\right) = \frac{1}{AE} \sum n_i N_i L_i$

	n_i	N_i	L_i	$n_i N_i L_i$
1	-0.375	37.5	6	-84.375
2	0	200	-	
3	0	-280	-	
4	0.625	-62.5	5	-195.3125
5	0.625	-187.5	5	-585.9375
				-865.625

$\therefore \Delta_c = \frac{-865.625}{2.5 \times 10^{-3} \times 200 \times 10^6}$

$= -0.00173125$

$\simeq 0.00173\,m$ (left) #

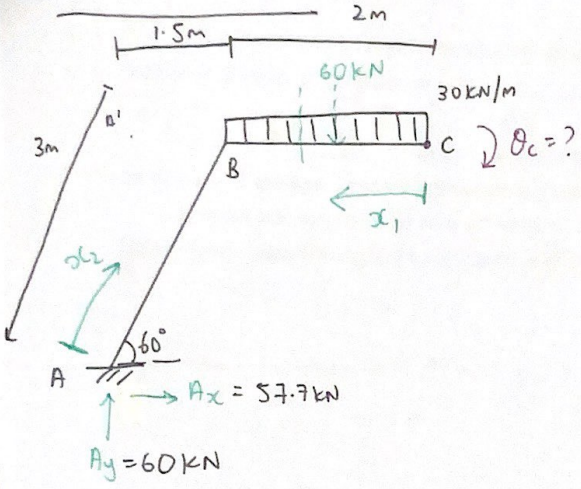
4b

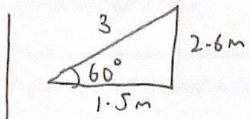
(4b) $E = 200\ GPa = (200\times10^{6})\ kN/m^2$

$I = 145\times10^{6}\times(10^{-3})^4\ m^4 = (145\times10^{-6})\ m^4$

$\theta_c = ?$

Real work (M)





$\sin60° = \frac{y}{3}$

$y = 2.6m$

$\cos60° = \frac{x}{3}$

$x = 1.5m$

$\Sigma F_y = 0$

$A_y = 60kN$

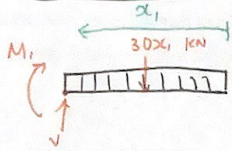
$\Sigma M_{A'} = 0$

$A_x(2.6) + 60(2.5) = 0$

$A_x = -57.7\ kN$

$(\leftarrow)$

$0 \leq x_1 \leq 2$

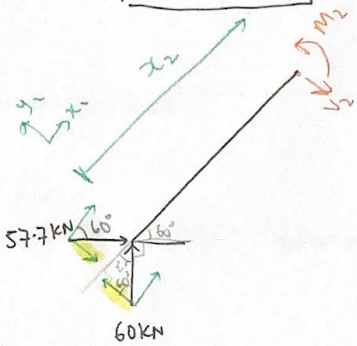


$\Sigma M_1 = 0$

$M_1 = -30x_1\left(\frac{x_1}{2}\right)$

$M_1 = (-15x_1^2)$

$0 \leq x_2 \leq 3$

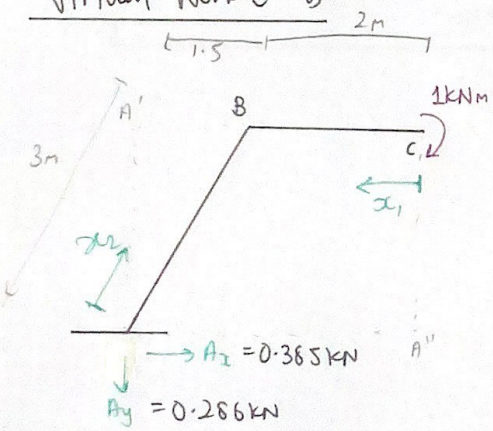


$\Sigma M_2 = 0$

$M_2 = (-57.7\sin60° + 60\cos60°)x_2$

$M_2 = (-20x_2)$

Virtual work (M_B)



$\Sigma M_{A'} = 0$

$A_x(2.6) = 1$

$A_x = 0.385kN$

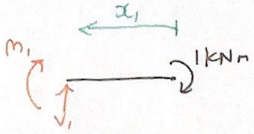
$\Sigma M_{A''} = 0$

$A_y(3.5) = 1$

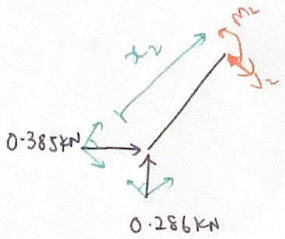
$A_y = 0.286\ kN$

$0 \leq x_1 \leq 2$

$(M_1 = -1)$



$0 \leq x_2 \leq 3$



$M_2 = -0.385\sin60°(x_2) + 0.286\cos60°(x_2)$

$M_2 = (-0.190\ x_2)$

4b

1. $\theta_c = \int_0^L \frac{M_0 M}{EI}\,dx = \frac{1}{EI}\int_0^L M_0 M$

$$\theta_c = \frac{1}{EI}\int_0^L M_0 M\,dx$$

$$\theta_c = \frac{1}{EI}\left[\int_0^2 -1(-15x_1^2)\,dx + \int_0^3 -0.19x_2(-20x_2)\,dx\right]$$

$$= \frac{1}{EI}\left[40 + \frac{171}{5}\right]$$

$$= \frac{74.2}{200 \times 145}$$

$$= 2.56 \text{ rad} \quad (\text{CW})$$

#.