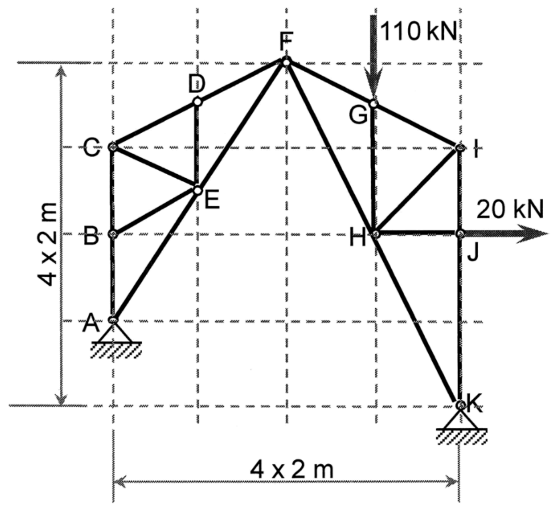
CV2011 - Structural Analysis I - PYP AY2017-2018 S1

Done by: Tham Win Fen

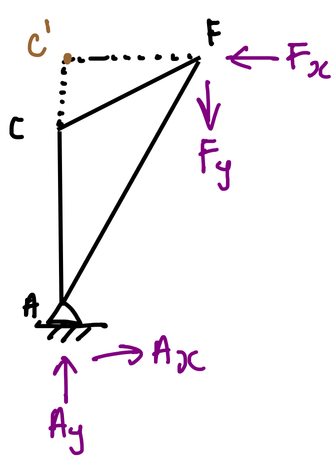
1\. (a) $j = 2$, $b = 18$, $r = 4$

$\overline{DOF} = 2(2) - 18 - 4 = 0 \Rightarrow$ Statically Determinate

(b)

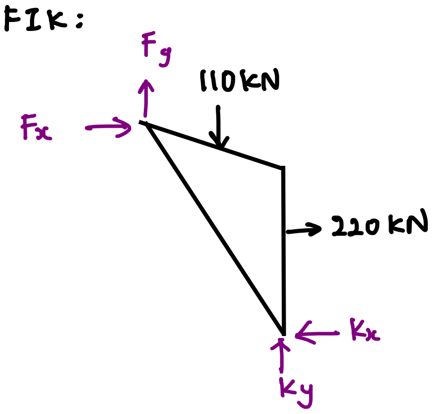


△ACF:



$\Sigma M_A = 0 \Rightarrow F_y = \frac{3}{2} F_x$ —— (1)

△FIK:



$\Sigma M_K = 0 \Rightarrow -8F_x - 4F_y + 140 = 0$

$F_y + 2F_x - 35 = 0$ —— (2)

Sub (1) in (2)

$\frac{3}{2}F_x + 2F_x - 35 = 0$

$\therefore F_x = 20\text{kN}$

$\therefore F_y = \frac{3}{2}(20) = 30\text{kN}$

Look at △ACF,

$\Sigma M_{C'} = 0$

$A_x(6) = 30(4)$

$\therefore A_x = \underline{20\text{kN}}$ #

$\Sigma F_y = 0$

$\underline{A_y = 30\text{kN}}$ #

Look at △FIK:

$\Sigma F_x = 0$

$K_x = 20 + 20$

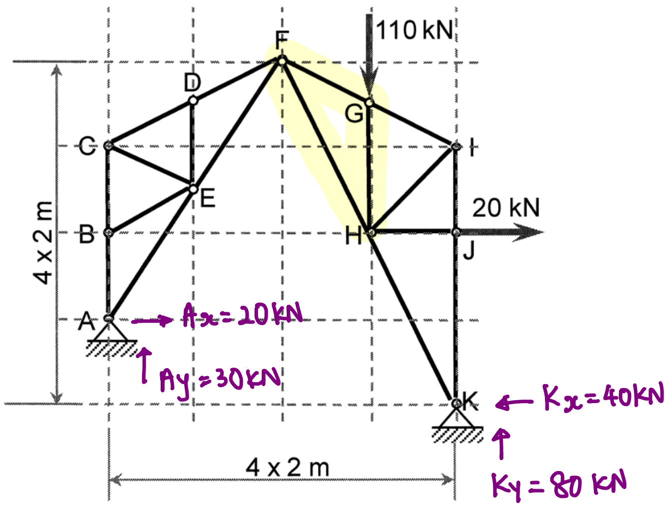
$\underline{K_x = 40\text{kN}}$ #

$\Sigma F_y = 0$

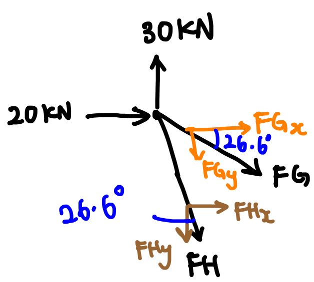
$K_y = 110 - 30$

$\underline{K_y = 80\text{kN}}$ #

1(c)



Joint F



$FG_x = FG\cos 26.6^\circ$

$FG_y = FG\sin 26.6^\circ$

$FH_x = FH\sin 26.6^\circ$

$FH_y = FH\cos 26.6^\circ$

$\therefore \Sigma F_y = 0, \quad FH_y + FG_y = 30$

$FH\cos 26.6^\circ + FG\sin 26.6^\circ = 30$

$0.894 FH + 0.448 FG = 30 \quad \text{—— (1)}$

$\Sigma F_x = 0, \quad FG_x + FH_x = -20$

$0.894 FG + 0.448 FH = -20$

$FG = -22.4 - 0.5 FH \quad \text{—— (2)}$

Sub (1) into (2)

$0.448(-22.4 - 0.5FH) + 0.894FH = 30$

$1.5 FH = 89.4$

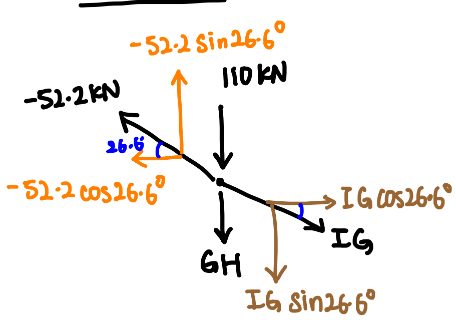
$FH = 59.6 \approx \underline{60 \text{ kN (T)}}$ #

$\therefore FG = -22.4 - 0.5(59.6)$

$= -52.2$ kN

$= \underline{52.2 \text{ kN (C)}}$ #

Joint G



$\Sigma F_x = 0$

$-52.2\cos 26.6^\circ = IG\cos 26.6^\circ$

$IG = -52.2$ kN

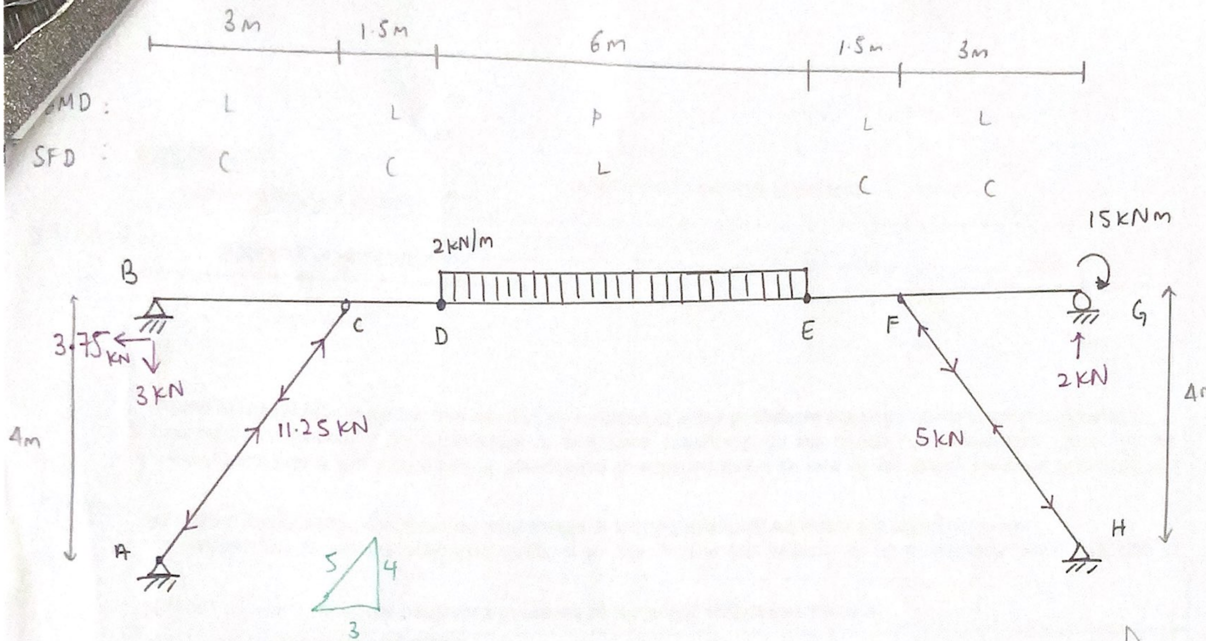
$\Sigma F_y = 0$

$-52.2\sin 26.6^\circ = 110 + GH + (-52.2)\sin 26.6^\circ$

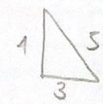
$GH = -110$ kN

$\underline{GH = 110 \text{ kN (C)}}$ #

2a)



(b) Normal, shear, BMD



(a) $\overline{DOF} = 3n - 3R - 2P - L/r$

$n = 3,\ R = 0,\ P = 3,\ L/r = 3$

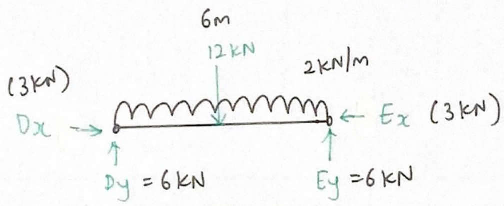
$\overline{DOF} = 3(3) - 2(0) - 2(3) - 3$

$= 0$

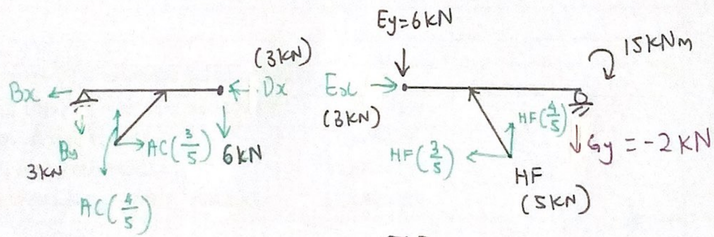
$\therefore$ Stable & statically Deter.

↳ simply supported at BD & EG.

DE supported by BC & FG



~~ΣM_D = 0~~



$\Sigma M_c = 0$

$B_y(3) = 6(1.5)$

$B_y = 3kN$

$\Sigma F_y = 0$

$3 + 6 = AC(\frac{4}{5})$

$AC = 11.25kN$

$\Sigma F_x = 0$

$B_x + D_x = AC(\frac{3}{5})$

$B_x = -3 + 11.25(\frac{3}{5})$

$B_x = 3.75kN$

$\Sigma F_x = 0$

$HF(\frac{3}{5}) = E_x$

$\therefore E_x = \frac{3}{5}(5) = 3kN$

~~$\Sigma F_y = 0$~~

$\Sigma M_F = 0$

$6(1.5) = G_y(3) + 15$

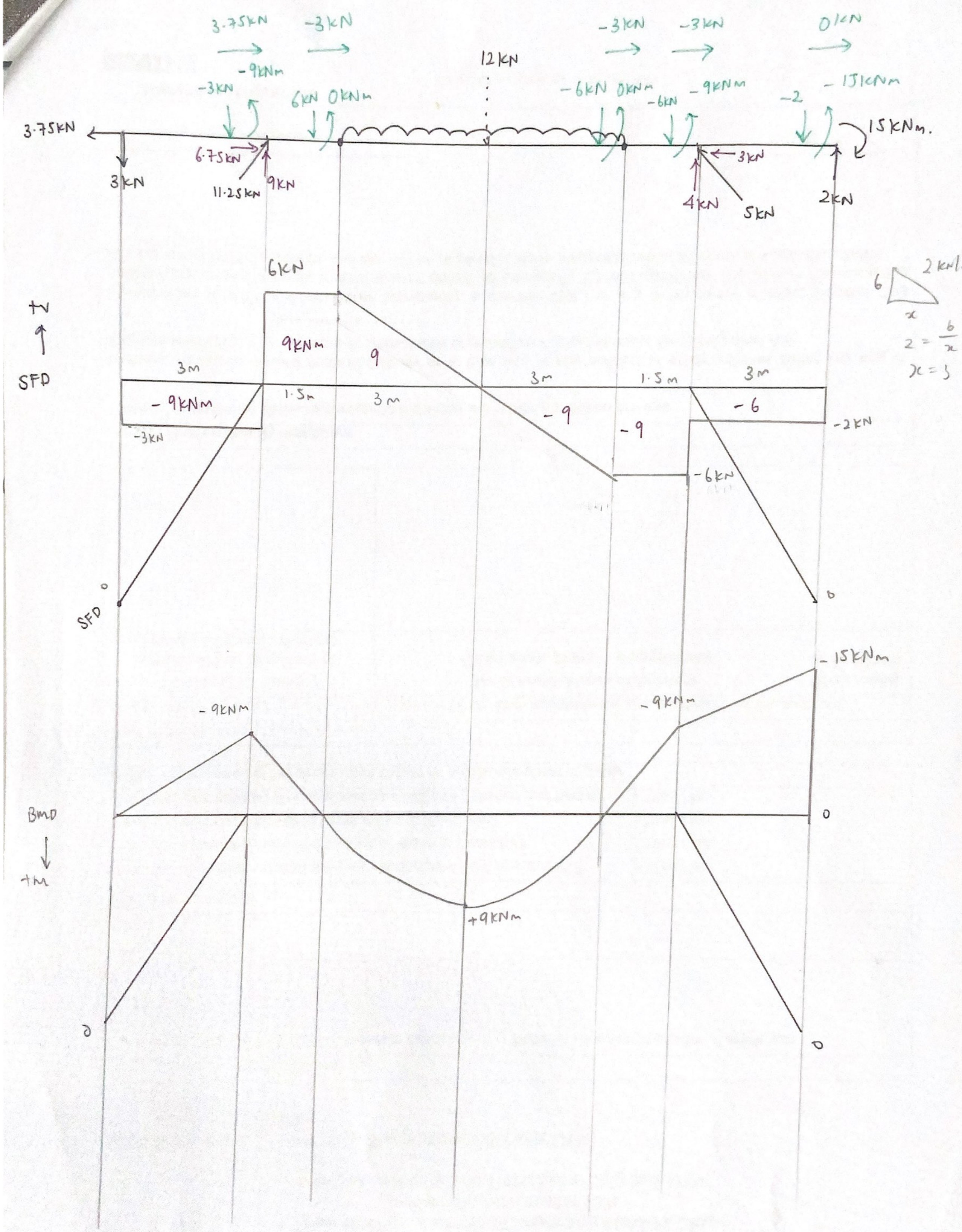
$G_y = -2$

$\Sigma F_y = 0$

$HF(\frac{4}{5}) = 6 - 2$

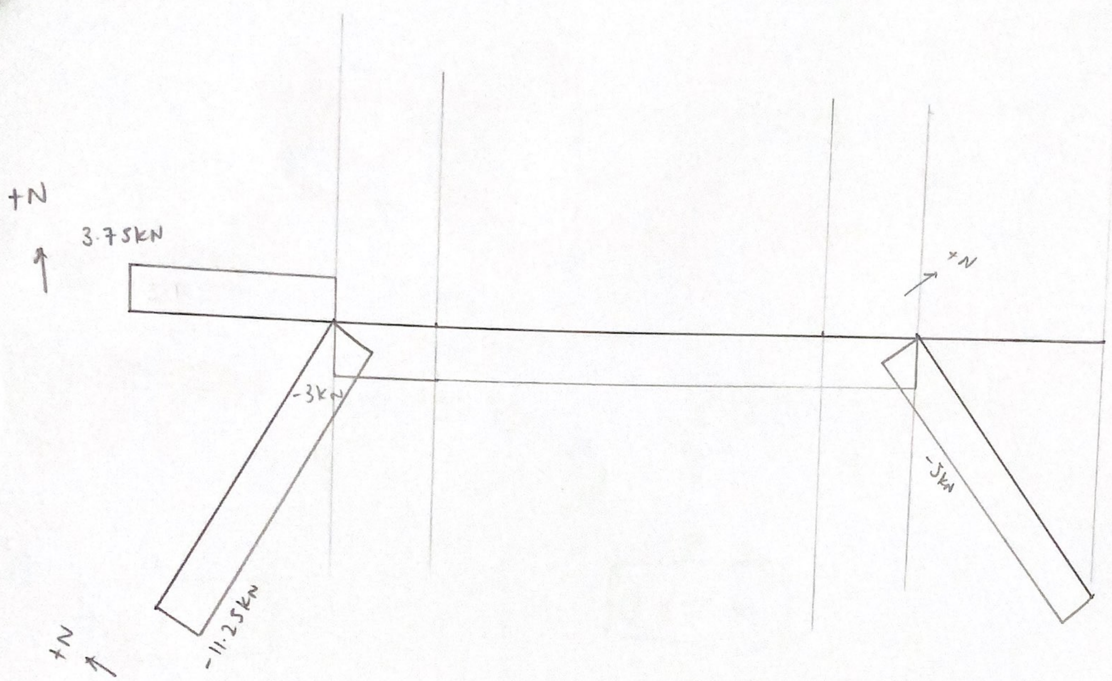
$HF = 5kN$

2b)


3.75KN
-3KN
12KN
-3KN
-3KN
0KN
-9KNm
-3KN
6KN 0KNm
-6KN 0KNm
-6KN
-9KNm
-2
-15KNm
15KNm.
3.75KN
6.75KN
3KN
11.25KN
9KN
4KN
3KN
5KN
2KN
6KN
+V
SFD
3m
9KNm
9
1.5m
3m
3m
1.5m
3m
-9KNm
-3KN
-9
-9
-6
-2KN
-6KN
0
SFD
0
6
2KN/m
x
$2 = \frac{6}{x}$
$x = 3$
-15KNm
-9KNm
-9KNm
BMD
+M
0
+9KNm
0
0

2b)

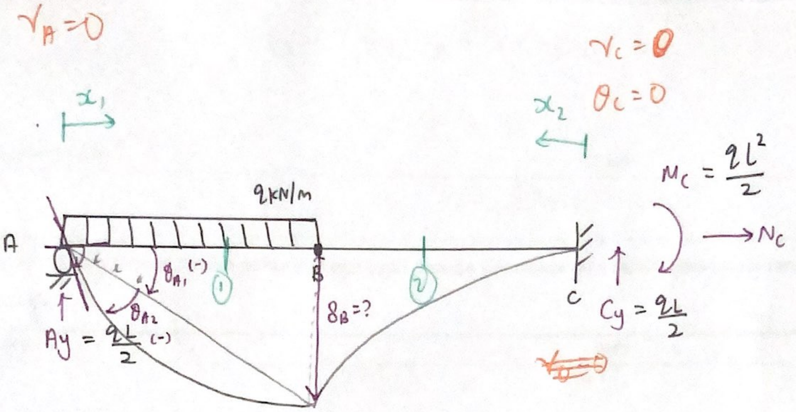
Normal



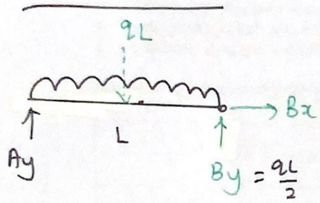
3a)

3(a)

Hinge $\theta_{BL} \neq \theta_{BR}$

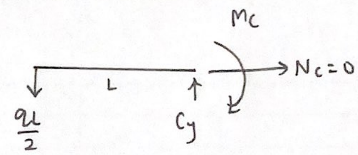


Support reactions



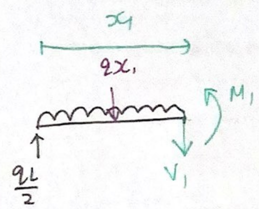
$$\Sigma M_B = 0$$
$$A_y L = qL\left(\frac{L}{2}\right)$$
$$A_y = \frac{qL}{2}$$

$M_C = \frac{qL^2}{2}$



(i) AB

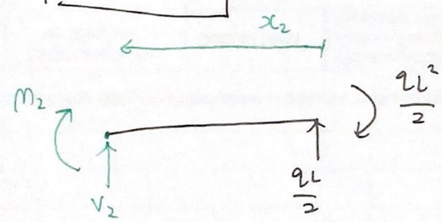
$$\boxed{0 \le x_1 \le L}$$



$$M_1 = -qx_1\left(\frac{x_1}{2}\right) + \frac{qL}{2}(x_1)$$
$$M_1 = \left[-\frac{qx_1^2}{2} + \frac{qL}{2}x_1\right] \#$$

BC

$$\boxed{0 \le x_2 \le L}$$



$$M_2 = \left[-\frac{qL^2}{2} + \frac{qL}{2}x_2\right] \#$$

(ii) $EIv''(x_1) = M_1$

$$\theta: \; EIv'(x_1) = \int -\frac{qx_1^2}{2} + \frac{qL}{2}x_1 \, dx_1$$
$$EIv'(x_1) = -\frac{qx_1^3}{6} + \frac{qLx_1^2}{4} + A$$
$$EIv(x_1) = \int -\frac{qx_1^3}{6} + \frac{qLx_1^2}{4} + A \, dx_1$$
$$EIv(x_1) = -\frac{qx_1^4}{24} + \frac{qLx_1^3}{12} + Ax_1 + B$$

$$EIv''(x_2) = M_2$$
$$EIv'(x_2) = \int -\frac{qL^2}{2} + \frac{qL}{2}x_2 \, dx_2$$
$$= -\frac{qL^2x_2}{2} + \frac{qLx_2^2}{4} + C$$
$$EIv(x_2) = -\frac{qL^2x_2^2}{4} + \frac{qLx_2^3}{12} + Cx_2 + D$$

3a)

CC1:

1: $v_A = v(x_1=0) = 0$

$0 = B$

3a)

BC2: $v_C = v(x_2=0) = 0$

$0 = D$

BC3: $\theta_C = v_C'(x_2=0) = 0$

$0 = C$

$\therefore \quad EI\,v'(x_2) = \left[-\frac{qL^2}{2}x_2 + \frac{qL}{4}x_2^2\right]$

$EI\,v(x_2) = \left[-\frac{qL^2}{4}x_2^2 + \frac{qLx_2^3}{12}\right]$

CC1: $v(x_1=L) = v(x_2=L)$

$$\frac{-qL^4}{24} + \cancel{\frac{qL^4}{12}} + AL = -\frac{qL^4}{4} + \cancel{\frac{qL^4}{12}}$$

$$A = -\frac{qL^3}{4} + \frac{qL^3}{24}$$

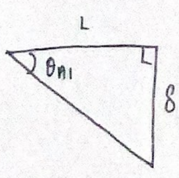
$$A = -\frac{5}{24}qL^3$$

$\therefore \quad EI\,v'(x_1) = \left[-\frac{qx_1^3}{6} + \frac{qLx_1^2}{4} - \frac{5}{24}qL^3\right]$

$EI\,v(x_1) = \left[-\frac{qx_1^4}{24} + \frac{qLx_1^3}{12} - \frac{5qL^3}{24}x_1\right]$

$\therefore \quad \delta_B = v(x_2=L) = \frac{1}{EI}\left[-\frac{qL^4}{4} + \frac{qL^4}{12}\right] = \frac{-1}{6}\left(\frac{qL^4}{EI}\right)\text{m} = \frac{1}{6}\left(\frac{qL^4}{EI}\right)$ downwards #

$\theta_A = \theta_{A1} + \theta_{A2}$



$\tan\theta_{A1} = \theta_{A1} = \frac{\delta_B}{L}$

$\theta_{A1} = -\frac{1}{6L}\left(\frac{qL^4}{EI}\right)$

$\theta_{A1} = -\frac{1}{6}\left(\frac{qL^3}{EI}\right)$

$\theta_{A2} = v'(x_1=0)$

$= \frac{1}{EI}\left[0 + 0 - \frac{5}{24}qL^3\right]$

$= -\frac{5}{24}\left(\frac{qL^3}{EI}\right)$

$\therefore \theta_A$

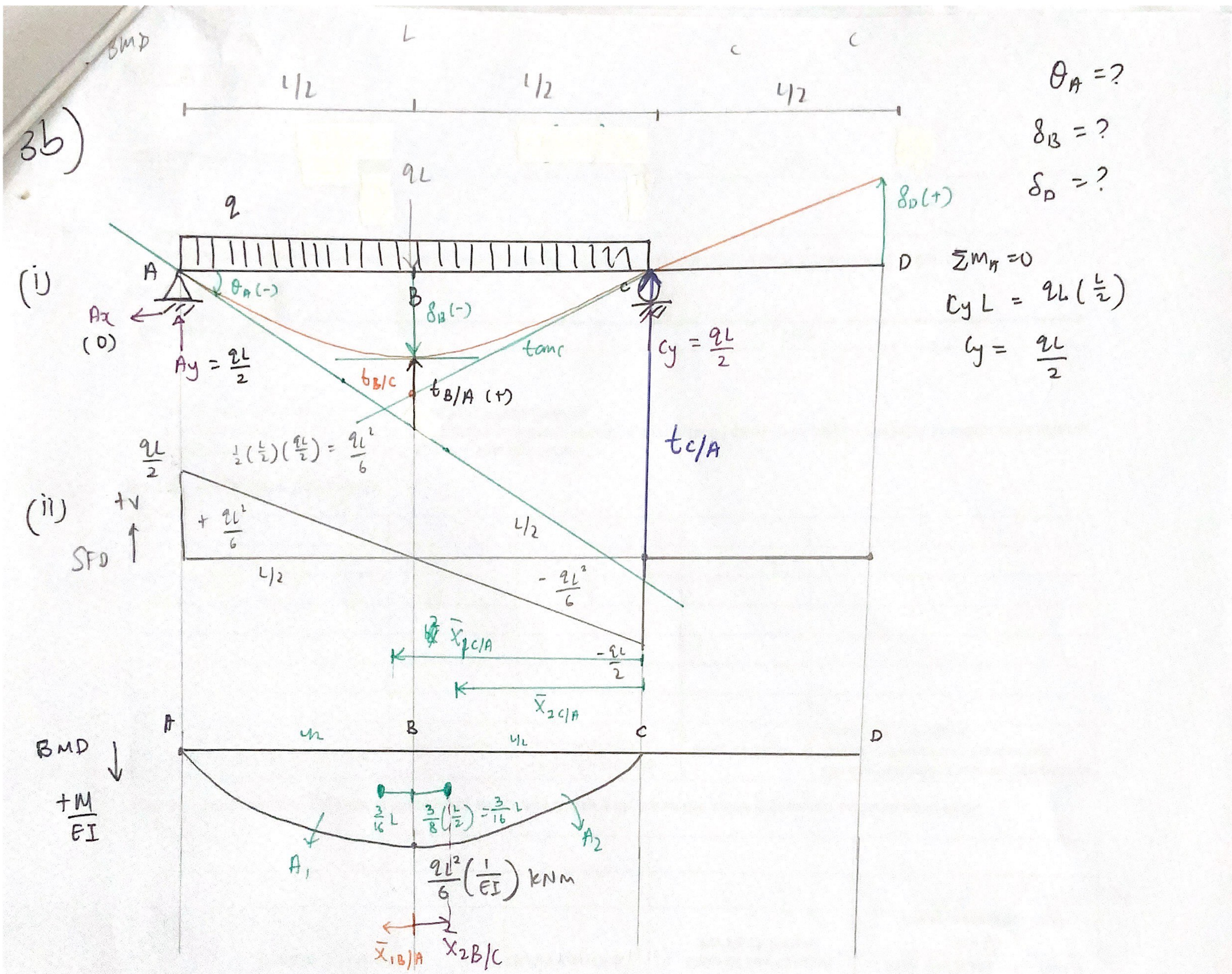
$= \left(-\frac{1}{6} - \frac{5}{24}\right)\frac{qL^3}{EI}$

$= -\frac{3}{8}\frac{qL^3}{EI}$

$= \left(\frac{3}{8}\frac{qL^3}{EI}\right)$ rad clockwise

#

3b)



3b)
L/2
L/2
L/2
$\theta_A = ?$
$\delta_B = ?$
$\delta_D = ?$
qL
q
$\delta_D (+)$
(i)
A
B
C
D
$\theta_A (-)$
$\delta_B (-)$
Ax
(0)
$A_y = \frac{qL}{2}$
$C_y = \frac{qL}{2}$
$\Sigma M_A = 0$
$C_y L = qL(\frac{L}{2})$
$C_y = \frac{qL}{2}$
tanc
$t_{B/C}$
$t_{B/A}$ (+)
$t_{C/A}$
$\frac{qL}{2}$
$\frac{1}{2}(\frac{L}{2})(\frac{qL}{2}) = \frac{qL^2}{6}$
(ii)
+V
SFD
$+ \frac{qL^2}{6}$
L/2
L/2
$- \frac{qL^2}{6}$
$- \frac{qL}{2}$
$\bar{x}_{1C/A}$
$\bar{x}_{2C/A}$
BMD
$\frac{+M}{EI}$
$\frac{3}{16}L$
$\frac{3}{8}(\frac{L}{2}) = \frac{3}{16}L$
A_1
A_2
$\frac{qL^2}{6}(\frac{1}{EI})$ kNm
$\bar{x}_{1B/A}$
$x_{2B/C}$

3b) continue

Find $t_{C/A}$

$$t_{C/A} = A_1 \bar{x}_{1C/A} + A_2 \bar{x}_{2C/A}$$

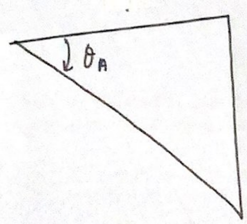
$$A_1 = A_2 = \frac{2}{3}bh = \frac{2}{3}\left(\frac{L}{2}\right)\left(\frac{1}{6}\frac{qL^2}{EI}\right) = \frac{2}{9}\left(\frac{qL^3}{EI}\right) \; m^2$$

$$\bar{x}_{1C/A} = \frac{L}{2} + \frac{3}{16}L = \frac{11}{16}L$$

$$\bar{x}_{2C/A} = \frac{L}{2} - \frac{3}{16}L = \frac{5}{16}L$$

$$\therefore t_{C/A} = \frac{2}{9}\left(\frac{qL^3}{EI}\right)\left(\frac{11}{16}L + \frac{5}{16}L\right) = \left(\frac{2}{9}\frac{qL^4}{EI}\right) m \uparrow$$

To find θ_A



$$\tan\theta_A = \theta_A = \frac{2}{9}\frac{qL^4}{EI} \div L$$

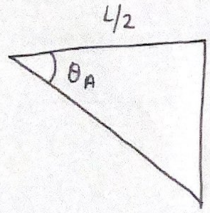
$$= \frac{2}{9}\frac{qL^4}{EI} \times \frac{1}{L}$$

$$= \frac{2}{9}\left(\frac{qL^3}{EI}\right) \text{rad clockwise}$$

#.

Find $t_{B/A}$

$$t_{B/A} = A_1 \bar{x}_{1B/A} = \frac{2}{9}\left(\frac{qL^3}{EI}\right)\left[\frac{3}{16}L\right] = \frac{1}{24}\frac{qL^4}{EI}$$



$$\theta_A = \frac{H}{L/2}$$

$$\frac{2}{9}\left(\frac{qL^3}{EI}\right) = \frac{2}{L}H$$

$$H = \frac{1}{9}\frac{qL^4}{EI}$$

$$H = t_{B/A} + \delta_B$$

$$\delta_B = H - t_{B/A}$$

$$= \left(\frac{1}{9} - \frac{1}{24}\right)\frac{qL^4}{EI}$$

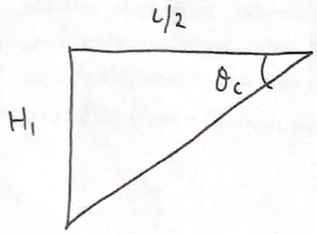
$$\delta_B = \cancel{\frac{5}{72}}\left(\frac{5}{72}\frac{qL^4}{EI}\right) m \text{ downward.}$$

#.

3b) continue

3b)

$t_{B/C} = A_2 \bar{X}_{2B/C} = \frac{2}{9}\left(\frac{qL^3}{EI}\right)\left(\frac{3}{16}L\right) = \frac{1}{24}\left(\frac{qL^4}{EI}\right)$ m



$H_1 = |\delta_B| + t_{B/C}$

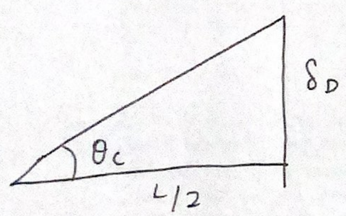
$= \left[\frac{5}{72} + \frac{1}{24}\right]\left(\frac{qL^4}{EI}\right)$

$= \left[\frac{1}{9}\ \frac{qL^4}{EI}\right]$ m

$\theta_C = \frac{H_1}{L/2} = \frac{2}{L}\left[\frac{1}{9}\right]\left[\frac{qL^4}{EI}\right] = \frac{2}{9}\frac{qL^3}{EI}$

$\theta_C = \theta_A$

Find δ_D



$\theta_C = \frac{\delta_D}{L/2}$

$\theta_C = \frac{2}{L}\delta_D$

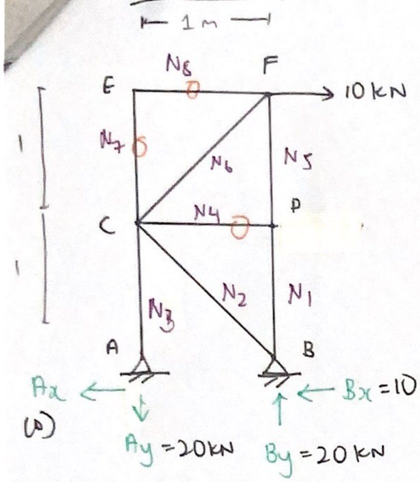
$\frac{2}{9}\frac{qL^3}{EI} \times \frac{L}{2} = \delta_D$

$\delta_D = \left[\frac{1}{9}\ \frac{qL^4}{EI}\right]$ m upwards

#

4a)

Real loads (N_i)

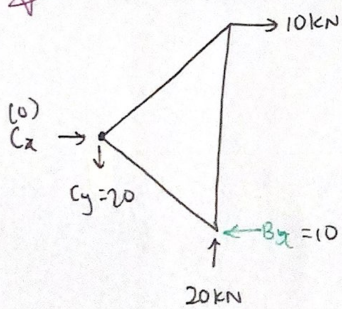


$\Sigma M_A=0$, $B_y(1)=10(2)$

$B_y=20$ kN

$\Sigma M_B=0$, $A_y(1)=10(2)$

$A_y=20$ kN



$\Sigma M_C=0$

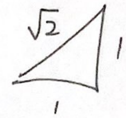
$B_x(1)+10(1)=20(1)$

$B_x=10$ kN

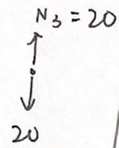
$N_7=0$ $N_5=-10$ kN $N_2=-10\sqrt{2}$

$N_8=0$ $N_4=0$ $N_1=-10$ kN

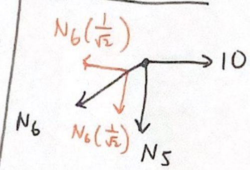
$N_6=10\sqrt{2}$ $N_3=20$ kN



Joint A



Joint F



$N_6\left(\frac{1}{\sqrt{2}}\right)=10$

$N_6=10\sqrt{2}$ kN

$N_5+N_6\left(\frac{1}{\sqrt{2}}\right)=0$

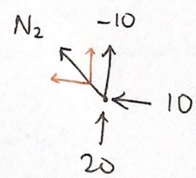
$N_5=-10\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$

$=-10$

Joint D

$N_1=-10$ kN

Joint B



$N_2\left(\frac{1}{\sqrt{2}}\right)+10=0$

$N_2=-10\sqrt{2}$

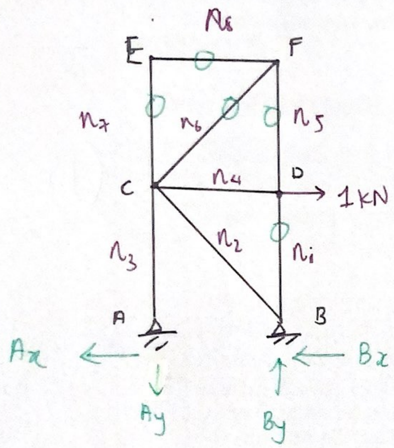
~~$L_7=1$~~

$L_2=L_6=\sqrt{2}$ m

others $=1$ m

4a)

virtual load



$n_{5/6/7/8} = 0$

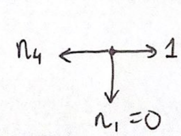
$n_1 = 0$

$n_2 = -\sqrt{2}$

$n_3 = 1$

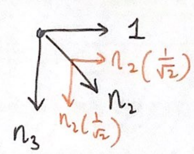
$n_4 = 1$

Joint D



$n_4 = 1$

Joint C



$1 + n_2\left(\frac{1}{\sqrt{2}}\right) = 0$

$n_2 = -\sqrt{2}$

$n_3 + n_2\left(\frac{1}{\sqrt{2}}\right) = 0$

$n_3 = -(-\sqrt{2})\left(\frac{1}{\sqrt{2}}\right)$

$n_3 = 1$

$$1 \cdot \Delta_D = \sum_i n_i \left(\frac{N_i L_i}{A_i E_i}\right) = \frac{1}{AE} \sum_i n_i N_i L_i$$

	n_i	N_i	L_i	$n_i N_i L_i$
1	0	-	-	
2	$-\sqrt{2}$	$-10\sqrt{2}$	$\sqrt{2}$	$20\sqrt{2}$
3	1	20	1	20
4	1	0	-	
5	0	-	-	
6	0	-	-	
7	0	0	-	
8	0	0	-	

$20\sqrt{2} + 20 = 48.3$

$$\therefore \Delta_D = \frac{48.3}{AE} \text{ m right.}$$

#

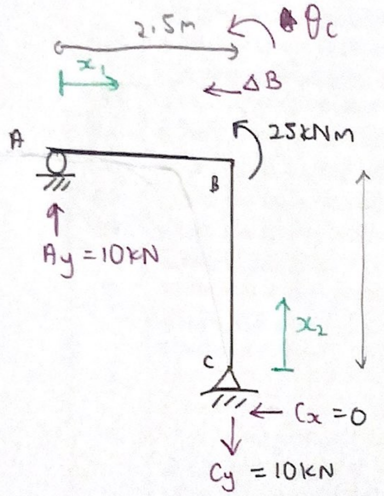
4b)

$E = (200 \times 10^6)\ kN/m^2$ $\qquad \delta_B = ?$

$I = (235 \times 10^{-6})\ m^4$ $\qquad \theta_B = ?$

~~Displacement at B~~

Real work M_i



$\Sigma M_C = 0$

$A_y(2.5) = 25$

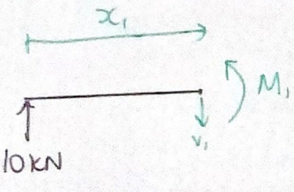
$A_y = 10kN$

$\Sigma M_B = 0$

$C_x(3) + 10(2.5) = 25$

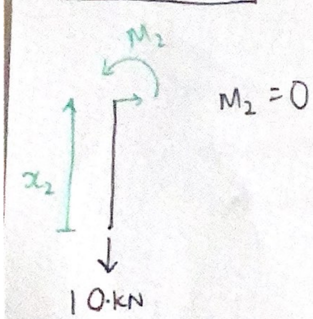
$C_x = 0$

$\boxed{0 \le x_1 \le 2.5}$



$M_1 = 10x_1$

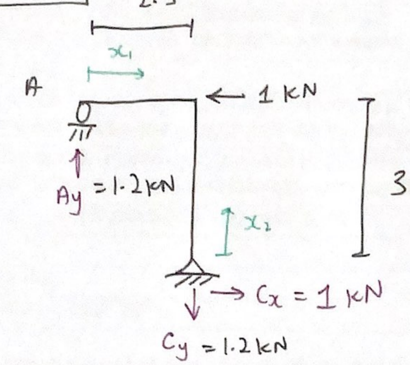
$\boxed{0 \le x_2 \le 3}$



$M_2 = 0$

Displacement at B

Virtual m_i

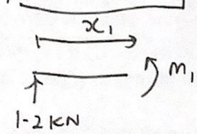


$\Sigma M_C = 0$

$A_y(2.5) = 1(3)$

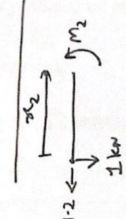
$A_y = 1.2\ kN$

$\boxed{0 \le x_1 \le 2.5}$



$m_1 = 1.2x_1$

$\boxed{0 \le x_2 \le 3}$



$m_2 = -x_2$

$$\therefore 1 \cdot \Delta_B = \int_0^L \frac{mM}{EI}\,dx = \frac{1}{EI}\int_0^L mM\,dx$$

$$\Delta_B = \frac{1}{EI}\left[\int_0^{2.5} 10x_1(1.2x_1)\,dx_1 + 0\right]$$

$$= \frac{62.5}{200 \times 235}$$

$$= (1.33 \times 10^{-3})\ m \quad \text{left}$$

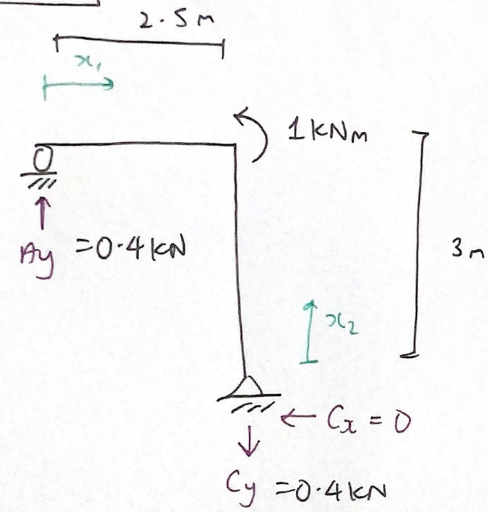
#.

4b)

4b)

rotation at B

$\boxed{\text{virtual}}$ m_θ

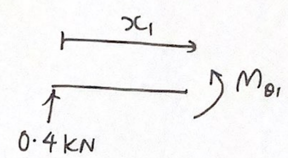


$$\Sigma M_c = 0$$

$$A_y(2.5) = 1$$

$$A_y = 0.4$$

$$\boxed{0 \le x_1 \le 2.5}$$



$$M_{\theta 1} = 0.4x_1$$

$$\boxed{0 \le x_2 \le 3}$$

$$- \quad \cos \quad M_2 = 0$$

$$\therefore 1 \cdot \theta_c = \frac{1}{EI}\int_0^L m_\theta M \, dx$$

$$\theta_c = \frac{1}{EI}\left[\int_0^{2.5} 0.4x_1(10x_1)\,dx_1 + 0\right]$$

$$= \frac{20.8333}{200 \times 235}$$

$$= (4.43 \times 10^{-4}) \text{ rad} \quad \text{ccw}$$

#