1) (a) $DOF = 3\times25 - 3\times0 - 2\times(4\times2 + 3\times8 + 1\times4 + 1) - 1\times1$

$= 0$

forces are not parallel and not concurrent,

and there are no collapsible structures

$\therefore$ statically determinate

(b) $\circlearrowleft + \sum M_B = 0$

$-A_y \times 6a + 2P \times 5a = 0$

$A_y = \frac{5P}{3}\ (\uparrow)$,

$\circlearrowleft + \sum M_A = 0$

$-2P \times a + B_y \times 6a = 0$

$B_y = \frac{P}{3}\ (\uparrow)$,

(c) zero-force members: CL, LE, NF, FD

MOJ at A:

$\xrightarrow{+} \sum F_x = 0$

$F_{AM} = 0$

$+\uparrow \sum F_y = 0$

$-F_{AC} + \frac{5P}{3} = 0$

$F_{AC} = \frac{5P}{3}\ (C)$

MOJ at C:

$\xrightarrow{+} \sum F_x = 0$

$-F_{CE}\cos45^\circ + F_{CM}\cos45^\circ = 0$

$+\uparrow \sum F_y = 0$

$\frac{5P}{3} - F_{CE}\sin 45^\circ - F_{CM}\sin 45^\circ = 0$

$F_{CE} = \frac{5\sqrt{2}}{6} P \text{ (C)}$

$F_{CM} = \frac{5\sqrt{2}}{6} \text{ (T)}$

MOJ at E:

$\xrightarrow{+} \sum F_x = 0$

$\frac{5\sqrt{2}}{6} P\cos 45^\circ - F_{EG} + F_{EJ}\cos 45^\circ = 0$

$+\uparrow \sum F_y = 0$

$\frac{5\sqrt{2}}{6} P\sin 45^\circ - F_{EJ}\sin 45^\circ = 0$

$F_{EJ} = \frac{5\sqrt{2}}{6} P \text{ (T)}$

$F_{EG} = \frac{5}{3} P \text{ (C)}$

MOJ at M:

$\xrightarrow{+} \sum F_x = 0$

$-\frac{5\sqrt{2}}{6} P\cos 45^\circ + F_{MJ}\cos 45^\circ - F_{MH} = 0$

$+\uparrow \sum F_y = 0$

$\frac{5\sqrt{2}}{6} P\sin 45^\circ - 2P + F_{MJ}\sin 45^\circ = 0$

$F_{MJ} = \frac{7\sqrt{2}}{6} P \text{ (T)}$

$F_{MH} = \frac{P}{3} \text{ (C)}$

MOJ at B:

$\xrightarrow{+} \sum F_x = 0$

$F_{IB} = 0$

$+\uparrow \sum F_y = 0$

$-F_{BD} + \frac{P}{3} = 0$

$F_{BD} = \frac{P}{3}\ (C)$

MOJ at D:

$\overset{+}{\rightarrow} \sum F_x = 0$

$F_{ND} \cos 45^\circ - F_{ID} \cos 45^\circ = 0$

$+\uparrow \sum F_x = 0$

$\frac{P}{3} - F_{ND} \sin 45^\circ - F_{ID} \sin 45^\circ = 0$

$F_{ND} = \frac{\sqrt{2}}{6} P\ (C)$

$F_{ID} = \frac{\sqrt{2}}{6} P\ (T)$

MOJ at I:

$\overset{+}{\rightarrow} \sum F_x = 0$

$-F_{HI} + F_{KI} \cos 45^\circ + \frac{\sqrt{2}}{6} P \cos 45^\circ = 0$

$+\uparrow \sum F_y = 0$

$-F_{KI} \sin 45^\circ + \frac{\sqrt{2}}{6} P \sin 45^\circ = 0$

$F_{KI} = \frac{\sqrt{2}}{6} P\ (C)$

$F_{HI} = \frac{P}{3}\ (T)$

MOJ at H:

$\overset{+}{\rightarrow} \sum F_x = 0$

$\frac{P}{3} - F_{JH} \cos 45^\circ - F_{HK} \cos 45^\circ + \frac{P}{3} = 0$

$+\uparrow \sum F_y = 0$

$F_{JH} \sin 45^\circ - F_{HK} \sin 45^\circ = 0$

$F_{JH} = \frac{\sqrt{2}}{3} P\ (T)$

$F_{HK} = \frac{\sqrt{2}}{3} P\ (C)$

MOJ at J:

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$$-\frac{5\sqrt{2}}{6}P\cos45^\circ - \frac{7\sqrt{2}}{6}P\cos45^\circ + F_{JG}\cos45^\circ + F_{JK} + \frac{\sqrt{2}}{3}P\cos45^\circ = 0$$

$+\uparrow \Sigma F_y = 0$

$$\frac{5\sqrt{2}}{6}P\sin45^\circ - \frac{7\sqrt{2}}{6}P\sin45^\circ + F_{JG}\sin45^\circ - \frac{\sqrt{2}}{3}P\sin45^\circ = 0$$

$F_{JG} = \frac{2\sqrt{2}}{3}P \ (C)$

$F_{JK} = P \ (T)$,

2) (a) AE and DG are primary structures

FCG is a link

EBF is a secondary structure with virtual roller at F and pin at E

$\therefore$ statically determinate

(b) for EBF,

$\circlearrowright + \sum M_E = 0$

$-8 \times 1 - 25 \times 2 - 60 \times 2.5 + F_y \times 5 + F_x \times 2 = 0$

for FCG,

$\circlearrowright + \sum M_G = 0$

$F_y \times 5 - F_x \times 3 + 60 \times 2.5 = 0$

$F_x = 71.6\,kN$

$F_y = 12.96\,kN$

$+\uparrow \sum F_y = 0$

$-12.96 - 60 + G_y = 0$

$G_y = 72.96\,kN$

$\xrightarrow{+} \sum F_x = 0$

$71.6 - G_x = 0$

$G_x = 71.6\,kN$

for GD,

$\circlearrowright + \sum M_D = 0$

$-71.6 \times 3 + M_D = 0$

$M_D = 214.8\,kNm$ ($\circlearrowright$)$_{//}$

$\circlearrowright + \sum M_G = 0$

$-D_x \times 3 + 214.8 = 0$

$D_x = 71.6\,kN\ (\leftarrow)_{,,}$

$+\uparrow \sum F_y = 0$

$-72.96 + D_y = 0$

$D_y = 72.96\,kN\ (\uparrow)_{,,}$

for EBF,

$\overset{+}{\rightarrow} \sum F_x = 0$

$25 + 8 + E_x - 71.6 = 0$

$E_x = 38.6\,kN$

$+\uparrow \sum F_y = 0$

$E_y - 60 + 12.96$

$E_y = 47.04\,kN$

for EA,

$\circlearrowright + \sum M_A = 0$

$-M_A - 8 \times 1 + 38.6 \times 2 = 0$

$M_A = 69.2\,kNm\ (\circlearrowright)_{,,}$

$\circlearrowright + \sum M_E = 0$

$A_x \times 2 + 8 \times 1 - 69.2 = 0$

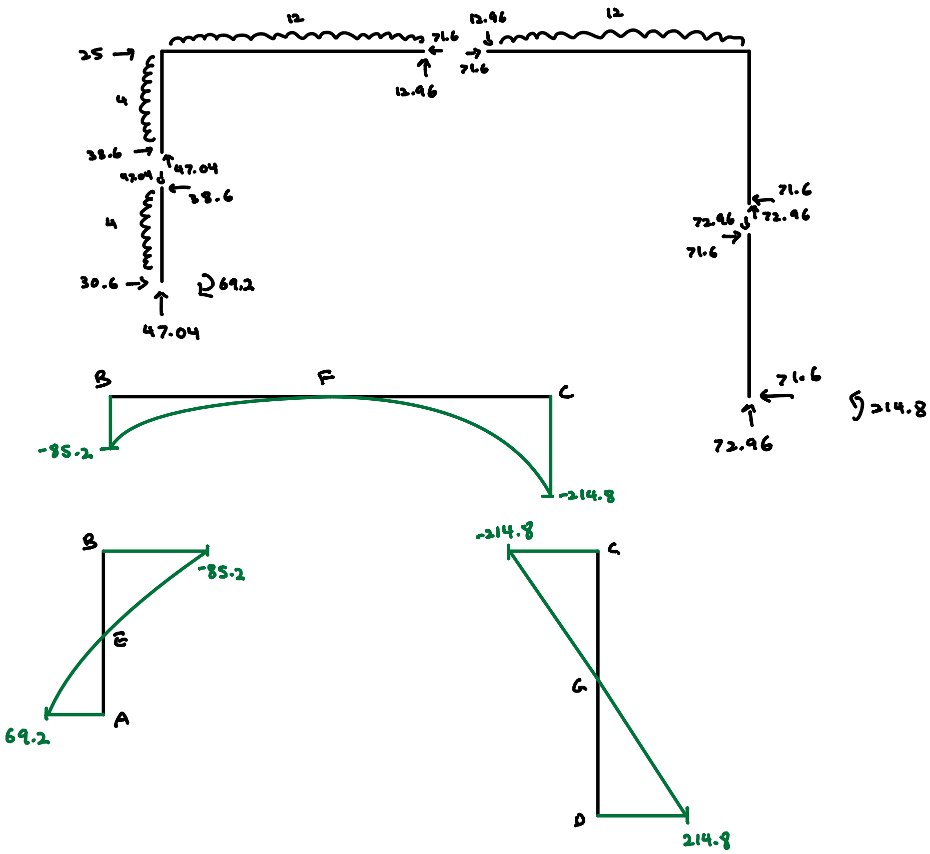
$A_x = 30.6\,kN\ (\rightarrow)_{,,}$

$+\uparrow \sum F_y = 0$

$-47.04 + A_y = 0$

$A_y = 47.04\,kN\ (\uparrow)_{,,}$

(c)



12
12.96
12
25
71.6
4
38.6
47.04
4
30.6
69.2
12.96
71.6
72.96
71.6
214.8
72.96
B
F
C
-85.2
-214.8
B
-85.2
E
A
69.2
-214.8
C
G
D
214.8

3) (a) $\circlearrowright + \sum M_A = 0$

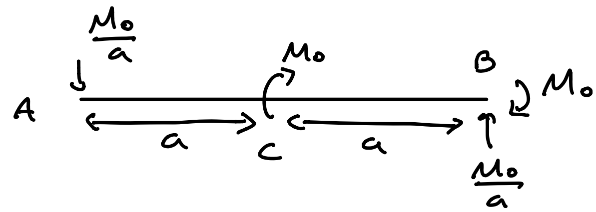
$-M_0 - M_0 + B_y \times 2a = 0$

$B_y = \frac{M_0}{a}\ (\uparrow)$

$\circlearrowright + \sum M_B = 0$

$A_y \times 2a - M_0 - M_0 = 0$

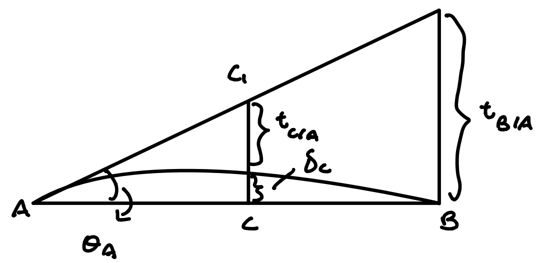
$A_y = \frac{M_0}{a}\ (\downarrow)$



$A_1 = A_2$

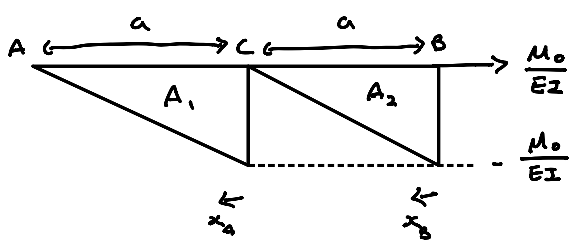
$= \frac{1}{2} \times a \times \left(-\frac{M_0}{EI}\right)$

$= -\frac{M_0 a}{2EI}$



$\overline{x_{B1}} = a + \frac{a}{3}$

$= \frac{4a}{3}$



$\overline{x_{B2}} = \frac{a}{3}$

$t_{B/A} = A_1 \times \overline{x_{B1}} + A_2 \times \overline{x_{B2}}$

$= -\frac{M_0 a}{2EI} \times \frac{4a}{3} + \left(-\frac{M_0 a}{2EI}\right) \times \frac{a}{3}$

$= -\frac{5M_0 a^2}{6EI}$

$\theta_A = \frac{t_{B/A}}{AB}$

$= \frac{-\frac{5M_0 a^2}{6EI}}{2a}$

$= -\frac{5M_0 a}{12EI}$

$= \frac{5M_0 a}{12EI}\ (\circlearrowright)$

$CC_1 = \theta_A \times AC$

$= \frac{5M_0 a}{12EI} \times a$

$= \frac{5M_0 a^2}{12EI}\ (\uparrow)$

$\overline{x_{C1}} = \frac{a}{3}$

$t_{C_1/A} = A_1 \times \overline{x_{c1}}$

$= -\frac{M_0 a}{2EI} \times \frac{a}{3}$

$= -\frac{M_0 a^2}{6EI}$

$= \frac{M_0 a^2}{6EI}\ (\uparrow)$

$\delta_c = CC_1 - t_{C_1/A}$

$= \frac{5M_0 a^2}{12EI} - \frac{M_0 a^2}{6EI}$

$= \frac{M_0 a^2}{4EI}\ (\uparrow)_{/\!/}$

(b) for $0 \le x_1 \le a$

$\circlearrowright + \sum M_{x_1} = 0$

$\frac{M_0}{a} x_1 + M_1 = 0$

$M_1 = -\frac{M_0}{a} x_1$

$= EIv_1''$

$EIv_1' = -\frac{M_0}{2a} x^2 + C_1$

$EIv_1 = -\frac{M_0}{6a} x^3 + C_1 x$

for $0 \le x_2 \le a$

$\circlearrowright + \sum M_{x_2} = 0$

$-M_2 - M_0 + \frac{M_0}{a} x_2 = 0$

$M_2 = \frac{M_0}{a} x_2 - M_0$

$= EIv_2''$

$EIv_2' = \frac{M_0}{2a} x_2^2 - M_0 x_2 + C_3$

$EIv_2 = \frac{M_0}{6a} x_2^3 - \frac{M_0}{2} x_2^2 + C_3 x$

$CC1 : v_1(a) = v_2(a)$

$$-\frac{M_0}{6}a^2 + C_1 a = \frac{M_0}{6}a^2 - \frac{M_0}{2}a^2 + C_3 a$$

$CC2 : v_1'(a) = -v_2'(a)$

$$-\frac{M_0}{2}a + C_1 = -\left(\frac{M_0}{2}a - M_0 a + C_3\right)$$

$$C_1 = \frac{5M_0 a}{12}$$

$$C_3 = \frac{7M_0 a}{12}$$

$$EIv_2' = \frac{M_0}{2a}x_2^2 - M_0 x_2 + \frac{7M_0 a}{12}$$

$$EIv_2'(0) = \frac{7M_0 a}{12}$$

$$\therefore \theta_B = \frac{7M_0 a}{12EI} \ (D)_{//}$$

4) (a) for BDC,

$\circlearrowleft + \Sigma M_B = 0$

$-120 \times 1 + C_y \times 2 = 0$

$C_y = 60\ kN\ (\uparrow)$

$\circlearrowleft + \Sigma M_c = 0$

$-B_y \times 2 + 120 \times 1 = 0$

$B_y = 60\ kN\ (\uparrow)$

$\xrightarrow{+} \Sigma F_x = 0$

$B_x = 0$

for AB,

$\circlearrowleft + \Sigma M_A = 0$

$M_A - 40 \times 1 - 60 \times 2 = 0$

$M_A = 160\ kNm\ (\circlearrowleft)$

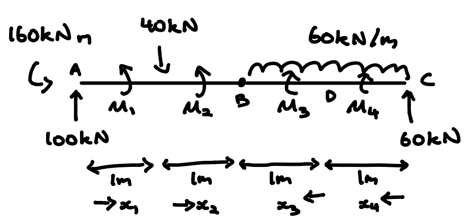
$\circlearrowleft + \Sigma M_B = 0$

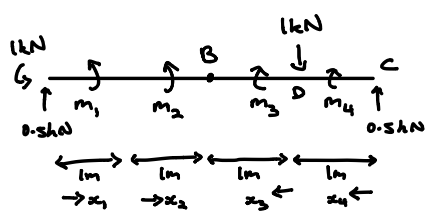
$160 - A_y \times 2 + 40 \times 1 = 0$

$A_y = 100\ kN\ (\uparrow)$

$\xrightarrow{+} \Sigma F_x = 0$

$A_x = 0\ kN$





$\circlearrowleft + \Sigma M_{x_1} = 0$

$160 - 100x_1 + M_1 = 0$ $\qquad$ $1 - 0.5 \times x_1 + m_1 = 0$

$M_1 = 100x_1 - 160$ $\qquad$ $m_1 = 0.5x_1 - 1$

$\circlearrowleft + \Sigma M_{x_2} = 0$

$160 - 100(x_2 + 1) + 40x_2 + M_2 = 0$ $\qquad$ $1 - 0.5 \times (x_2 + 1) + m_2 = 0$

$M_2 = 60x_2 - 60$ $\qquad$ $m_2 = 0.5x_2 - 0.5$

$\circlearrowleft + \Sigma M_{x_3} = 0$

$-M_3 - 60(x_3 + 1) \times \frac{x_3 + 1}{2} + 60 \times (x_3 + 1) = 0$ $\qquad$ $-m_3 - 1 \times x_3 + 0.5 \times (x_3 + 1) = 0$

$M_3 = -30x_3^2 + 30$ $\qquad$ $m_3 = -0.5x_3 + 0.5$

$\circlearrowleft + \Sigma M_{x_4} = 0$

$-M_4 - 60x_4 \times \frac{x_4}{2} + 60 \times x_4 = 0$ $\qquad$ $-m_4 + 0.5 \times x_4 = 0$

$M_4 = -30x_4^2 + 60x_4$ $\qquad$ $m_4 = 0.5x_4$

$$1.\ \Delta_D = \int_0^L \frac{mM}{EI}\,dx$$

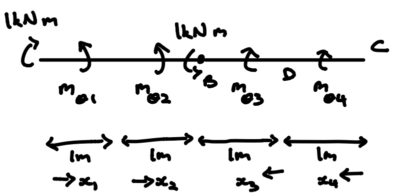
$$= \int_0^1 \frac{(0.5x_1 - 1) \times (100x_1 - 160)}{EI}\,dx_1 + \int_0^1 \frac{(0.5x_2 - 0.5) \times (60x_2 - 60)}{EI}\,dx_2$$

$$+ \int_0^1 \frac{(-0.5x_3 + 0.5) \times (-30x_3^2 + 30)}{EI}\,dx_3 + \int_0^1 \frac{(0.5x_4) \times (-30x_4^2 + 60x_4)}{EI}\,dx_4$$

$$= \frac{655}{6EI}$$

$$\Delta_D = \frac{655}{6EI}\ (\downarrow)$$

(b)



$C_s + \sum M_{x_1} = 0$

$-1 + m_{\theta 1} = 0$

$m_{\theta 1} = 1$

$C_s + \sum M_{x_2} = 0$

$-1 + m_{\theta 2} = 0$

$m_{\theta 2} = 1$

$C_s + \sum M_{x_3} = 0$

$m_{\theta 3} = 0$

$C_s + \sum M_{x_4} = 0$

$m_{\theta 4} = 0$

$$1 \cdot \theta_B = \int_0^L \frac{m_\theta M}{EI} dx$$

$$= \int_0^1 \frac{1 \times (100x_1 - 160)}{EI} dx_1 + \int_0^1 \frac{1 \times (60x_2 - 60)}{EI} dx_2$$

$$+ \int_0^1 \frac{0 \times (-30x_3^2 + 30)}{EI} dx_3 + \int_0^1 \frac{0 \times (-30x_4^2 + 60x_4)}{EI} dx_4$$

$$= -\frac{140}{EI}$$

$$\theta_B = \frac{140}{EI} \; (\circlearrowright)_{//}$$

ONG JING WEI, AUGUSTIN

24/01/2021