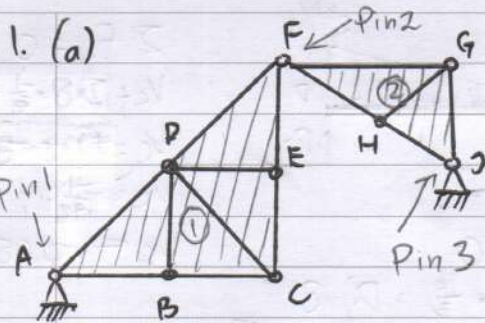


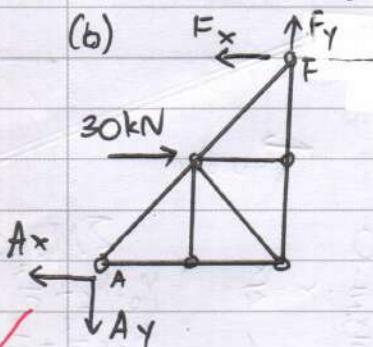
CV2011  
2018/2019 Sem II



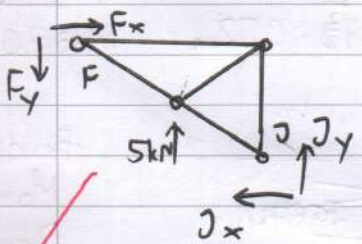
$n=2$      $P=3$   
 $DOF = 3n - 3R - 2P - r$   
 $= 3 \cdot 2 - 3 \cdot 3 - 2 \cdot 3 - 0$   
 $= 0 //$

Overall: Statically determinate  
Stable Structure //

External: Simply-supported //



$\sum M_A = 0$   
 $F_x \cdot 6 + F_y \cdot 6 = 30 \cdot 3$   
 $F_x + F_y = 15$   
 $F_x = 15 - F_y$



$\sum M_J = 0$   
 $F_x \cdot 3 + 5 \cdot 2 = F_y \cdot 4$   
 $3(15 - F_y) + 10 = 4F_y$   
 $55 - 3F_y = 4F_y$   
 $7F_y = 55$   
 $F_y = \frac{55}{7} \text{ kN}$

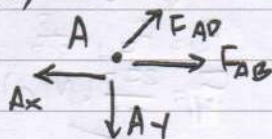
$\sum F_x = 0$   
 $A_x + F_x = 30$   
 $A_x = \frac{160}{7} \text{ kN} (\leftarrow)$

$\sum F_y = 0$   
 $A_y = F_y$   
 $A_y = \frac{55}{7} \text{ kN} (\downarrow)$

$\sum F_x = 0$   
 $J_x = F_x$   
 $J_x = \frac{50}{7} \text{ kN} (\leftarrow)$

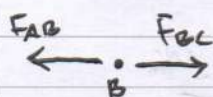
$\sum F_y = 0$   
 $J_y + 5 = F_y$   
 $J_y = \frac{20}{7} \text{ kN} (\uparrow)$

(c) BD and DE are zero force members

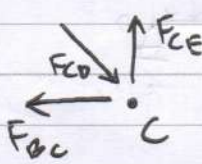


$\sum F_y = 0$   
 $F_{AD} \cdot \frac{1}{\sqrt{2}} = A_y$   
 $F_{AD} = \frac{55}{7} \sqrt{2} \text{ kN (T)}$

$\sum F_x = 0$   
 $A_x = F_{AB} + F_{AD} \cdot \frac{1}{\sqrt{2}}$   
 $F_{AB} = \frac{160}{7} - \frac{55}{7} = 15 \text{ kN (T)}$

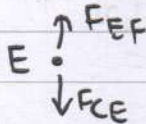


$F_{BC} = 15 \text{ kN (T)}$

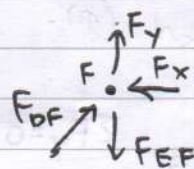


$\sum F_x = 0$   
 $F_{CD} \cdot \frac{1}{\sqrt{2}} = F_{BC}$   
 $F_{CD} = 15 \sqrt{2} \text{ kN (C)}$

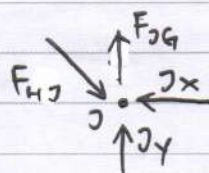
$\sum F_y = 0$   
 $F_{CE} = F_{CD} \cdot \frac{1}{\sqrt{2}}$   
 $F_{CE} = 15 \text{ kN (T)}$



$F_{EF} = 15 \text{ kN (T)}$

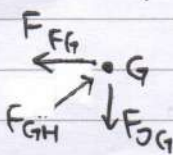


$\sum F_x = 0$   
 $F_{DF} \cdot \frac{1}{\sqrt{2}} = F_x$   
 $F_{DF} = \frac{50}{7} \sqrt{2} \text{ kN (C)}$



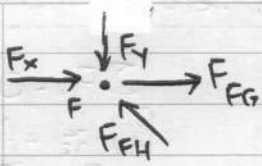
$\sum F_x = 0$   
 $F_{HG} \cdot \frac{2}{2.5} = J_x$   
 $F_{HG} = \frac{125}{14} \text{ kN (C)}$

$\sum F_y = 0$   
 $F_{JG} + J_y = F_{HG} \cdot \frac{1.5}{2.5}$   
 $F_{JG} = 2.5 \text{ kN (T)}$



$\sum F_y = 0$   
 $F_{GH} \cdot \frac{1.5}{2.5} = F_{JG}$   
 $F_{GH} = \frac{25}{6} \text{ kN (C)}$

$\sum F_x = 0$   
 $F_{FG} = F_{GH} \cdot \frac{2}{2.5}$   
 $F_{FG} = 2 \text{ kN (T)}$

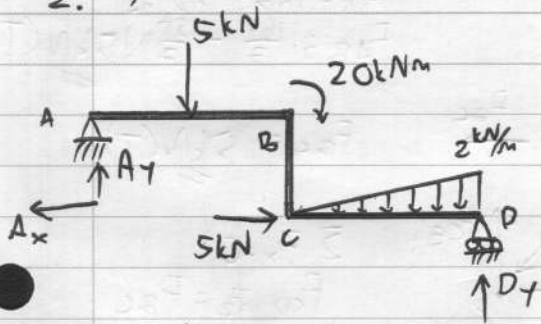


$$\sum F_y = 0$$

$$F_y = F_{FH} \cdot \frac{1.5}{2.5}$$

$$F_{FH} = \frac{27.5}{21} \text{ kN (C)}$$

2. (a)



$$\sum F_x = 0$$

$$A_x = 5 \text{ kN (←)}$$

$$\sum M_A = 0$$

$$-5 \cdot 4 - 20 + 5 \cdot 4 - \frac{1}{2} \cdot 2 \cdot 8 \cdot (8 + \frac{16}{3}) + 16D_y = 0$$

$$-20 - \frac{320}{3} + 16D_y = 0$$

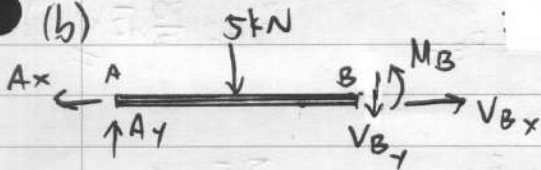
$$D_y = \frac{95}{12} \approx 7.92 \text{ kN (↑)}$$

$$\sum F_y = 0$$

$$A_y + D_y = 5 + \frac{1}{2} \cdot 2 \cdot 8$$

$$A_y = \frac{61}{12} \approx 5.08 \text{ kN (↑)}$$

(b)



$$\sum F_y = 0$$

$$A_y = 5 + V_{By}$$

$$V_{By} = -5 + \frac{61}{12} = \frac{1}{12} \text{ kN}$$

$$V_{By} \approx 0.083 \text{ kN}$$

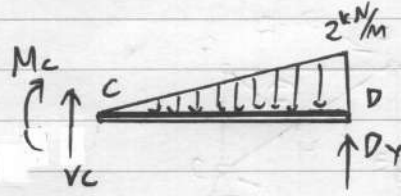
$$\sum M_B = 0$$

$$A_y \cdot 8 = 5 \cdot 4 + M_B$$

$$M_B = \frac{61}{12} \cdot 8 - 20$$

$$M_B = \frac{62}{3} \approx 20.67 \text{ kNm}$$

$$V_{Bx} = A_x = 5 \text{ kN}$$



$$\sum F_y = 0$$

$$V_c + D_y = \frac{1}{2} \cdot 2 \cdot 8$$

$$V_c = 8 - \frac{95}{12}$$

$$V_c = \frac{1}{12} \text{ kN}$$

$$V_c \approx 0.083 \text{ kN}$$

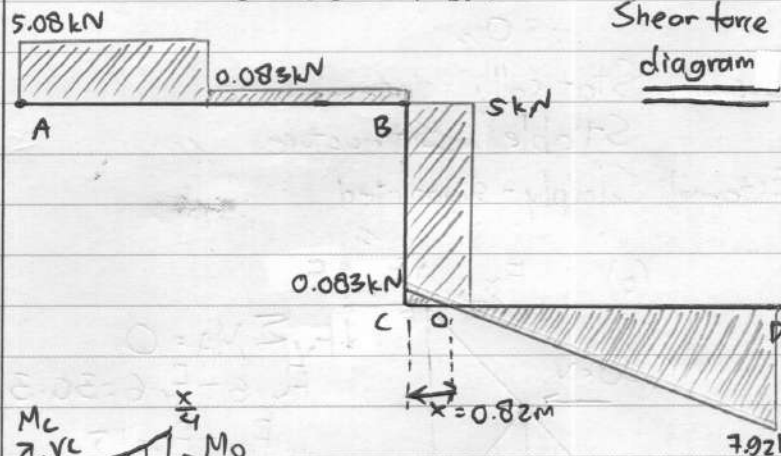
$$\sum M_c = 0$$

$$M_c + \frac{1}{2} \cdot 2 \cdot 8 \cdot \frac{16}{3} = D_y \cdot 8$$

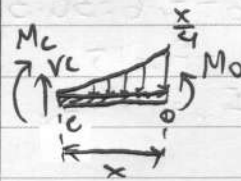
$$M_c = 8 \cdot \frac{95}{12} - \frac{128}{3}$$

$$M_c = \frac{62}{3} \text{ kNm}$$

$$M_c \approx 20.67 \text{ kNm}$$



Shear force diagram



$$\sum F_y = 0$$

$$V_c = \frac{1}{2} \cdot x \cdot \frac{x}{4}$$

$$\frac{1}{12} = \frac{x^2}{8}$$

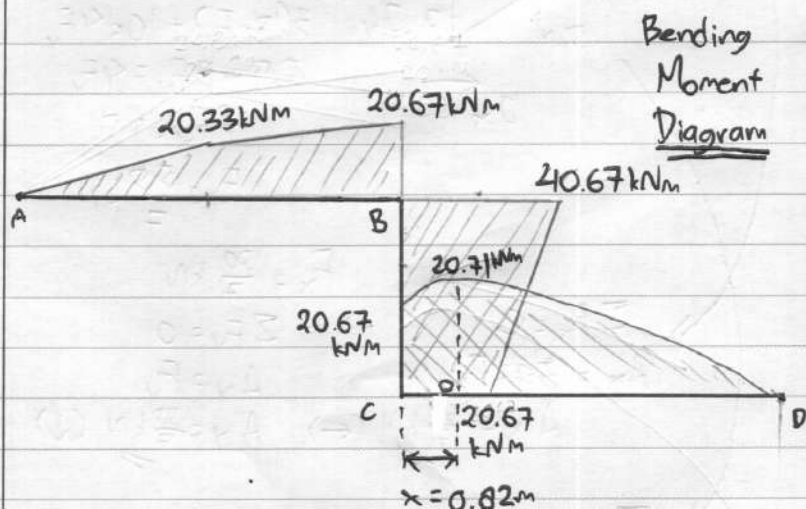
$$x = \sqrt{\frac{8}{3}} = 0.82 \text{ m}$$

$$\sum M_c = 0$$

$$M_0 = M_c + \frac{x^2}{8} \cdot \frac{2}{3} \cdot x$$

$$M_0 = \frac{62}{3} + \frac{1}{12} x^3$$

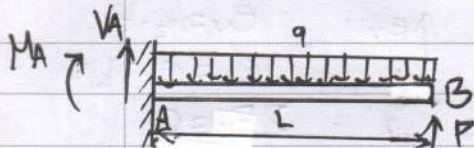
$$M_0 = 20.71 \text{ kNm}$$



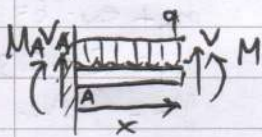
Bending Moment Diagram



3. (a)



$$\begin{aligned} \sum F_y = 0 & \quad \sum M_A = 0 \\ V_A + P = qL & \quad M_A + \frac{1}{2}qL^2 = PL \\ V_A = qL - P & \quad M_A = PL - \frac{1}{2}qL^2 \end{aligned}$$



$$\sum M = 0$$

$$\begin{aligned} M + \frac{1}{2}qx^2 &= M_A + V_A \cdot x \\ M &= PL - \frac{1}{2}qL^2 + (qL - P)x - \frac{1}{2}qx^2 \end{aligned}$$

$$EI \cdot v'' = M$$

$$\begin{aligned} EI \cdot v' &= \int (PL - \frac{1}{2}qL^2 + (qL - P)x - \frac{1}{2}qx^2) dx \\ EI \cdot v' &= (PL - \frac{1}{2}qL^2)x + \frac{(qL - P)^2}{2}x^2 - \frac{qx^3}{6} + C_1 \end{aligned}$$

$$x = 0 \rightarrow v' = 0$$

$$C_1 = 0$$

$$EI \cdot v = \int [(PL - \frac{1}{2}qL^2)x + \frac{(qL - P)^2}{2}x^2 - \frac{qx^3}{6}] dx$$

$$EI \cdot v = \left(\frac{PL - qL^2}{2} - \frac{qL^2}{4}\right)x^2 + \frac{(qL - P)^2}{6}x^3 - \frac{q}{24}x^4 + C_2$$

$$x = 0 \rightarrow v = 0$$

$$C_2 = 0$$

$$EI \cdot v = \left(\frac{PL - qL^2}{2} - \frac{qL^2}{4}\right)x^2 + \frac{(qL - P)^2}{6}x^3 - \frac{q}{24}x^4$$

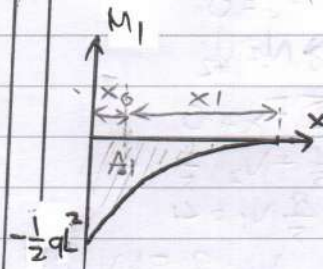
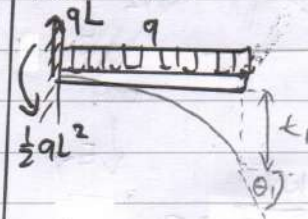
$$x = L \rightarrow v = 0$$

$$0 = \frac{PL^3}{2} - \frac{qL^4}{4} + \frac{qL^4 - PL^3}{6} - \frac{q}{24}L^4$$

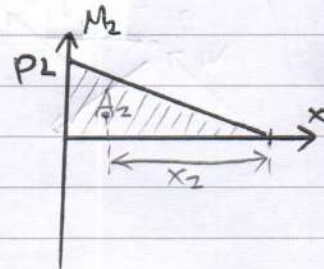
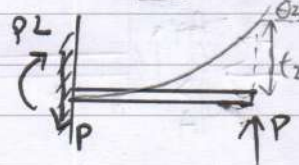
$$0 = \frac{PL^3}{3} - \frac{1}{8}qL^4$$

$$P = \frac{3}{8}qL$$

(b) ①



②



$$A_1 = \frac{1}{3} \cdot \left(-\frac{1}{2}qL^2\right) \cdot L$$

$$A_2 = \frac{1}{2} \cdot PL \cdot L$$

$$A_1 = -\frac{1}{6}qL^3$$

$$A_2 = \frac{1}{2}PL^2$$

Rotation ( $\theta = 0$ )

$$\theta = \theta_1 + \theta_2$$

$$\theta = \frac{A_1}{EI} + \frac{A_2}{EI}$$

$$\theta = 0 \rightarrow A_1 = -A_2$$

$$-\frac{1}{6}qL^3 = -\frac{1}{2}PL^2$$

$$P = \frac{1}{3}qL$$

Deflection

$$EI \cdot \epsilon_1 = A_1 x_1$$

$$EI \cdot \epsilon_2 = A_2 x_2$$

$$EI \cdot \epsilon_1 = A_1 \cdot (L - x_0)$$

$$EI \cdot \epsilon_2 = \frac{1}{2}PL^2 \cdot \frac{2}{3}L$$

$$EI \cdot \epsilon_1 = -\frac{1}{6}qL^3 \left(L - \frac{L}{3}\right)$$

$$EI \cdot \epsilon_2 = \frac{1}{3}PL^3$$

$$\epsilon_1 = -\frac{1}{8} \frac{qL^4}{EI}$$

$$EI \cdot \epsilon_2 = \frac{1}{3} \cdot \frac{1}{3}qL \cdot L^3$$

$$\epsilon_2 = \frac{1}{9} \frac{qL^4}{EI}$$

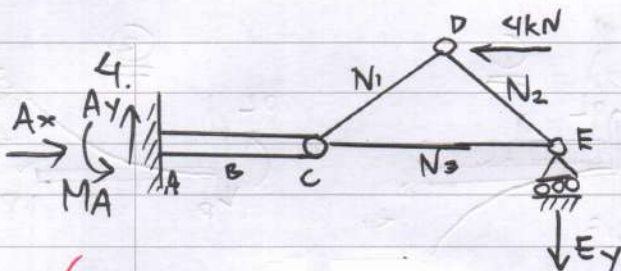
$$\delta_B = \epsilon_2 + \epsilon_1$$

$$\delta_B = \frac{1}{9} \frac{qL^4}{EI} + \left(-\frac{1}{8} \frac{qL^4}{EI}\right)$$

$$\delta_B = -\frac{1}{72} \frac{qL^4}{EI}$$

$$\delta_B = \frac{1}{72} \frac{qL^4}{EI} \text{ (downward)}$$



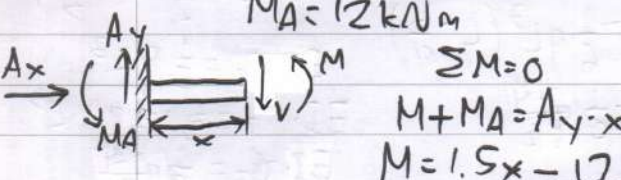


$\sum F_y = 0$   
 $N_1 = N_2$   
 $\sum F_x = 0$   
 $N_1 \cdot \frac{4}{5} + N_2 \cdot \frac{4}{5} = 4$   
 $\frac{8}{5} N_1 = 4$   
 $N_1 = 2.5 \text{ kN (C)}$   
 $N_2 = 2.5 \text{ kN (T)}$

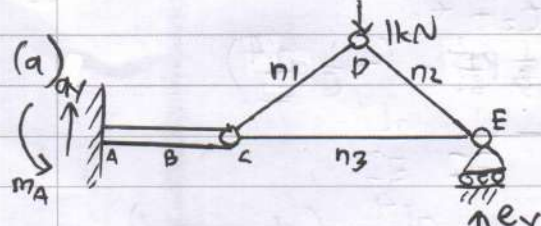
$\sum F_y = 0$   
 $E_y = N_2 \cdot \frac{3}{5}$   
 $E_y = 1.5 \text{ kN}$   
 $\sum F_x = 0$   
 $N_3 = N_2 \cdot \frac{4}{5}$   
 $N_3 = 2 \text{ kN (C)}$

$\sum F_x = 0$   
 $A_x = 4 \text{ kN}$   
 $\sum F_y = 0$   
 $A_y = 1.5 \text{ kN}$

$\sum M_A = 0$   
 $M_A + 4 \cdot 3 = E_y \cdot 16$   
 $M_A = 1.5 \cdot 16 - 12$   
 $M_A = 12 \text{ kNm}$



$\sum M = 0$   
 $M + M_A = A_y \cdot x$   
 $M = 1.5x - 12$



$\sum F_x = 0$   
 $n_1 = n_2$   
 $\frac{3}{5} (n_1 + n_2) = 1$   
 $n_1 = \frac{5}{6} \text{ kN (C)}$   
 $n_2 = \frac{5}{6} \text{ kN (C)}$

$\sum F_y = 0$   
 $E_y = n_2 \cdot \frac{3}{5}$   
 $E_y = 0.5 \text{ kN}$   
 $\sum F_x = 0$   
 $n_3 = n_2 \cdot \frac{4}{5}$   
 $n_3 = \frac{2}{3} \text{ kN (T)}$

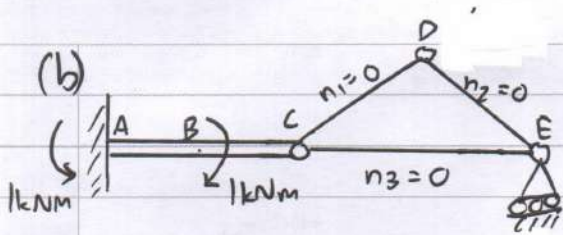
$\sum F_y = 0$   
 $a_y = 0.5 \text{ kN}$   
 $\sum M_A = 0$   
 $m_A + e_y \cdot 16 = 1 \cdot 12$   
 $m_A = 12 - 0.5 \cdot 16$   
 $m_A = 4 \text{ kNm}$

$\sum M = 0$   
 $m + m_A = a_y \cdot x$   
 $m = 0.5x - 4$

$n_1 = -\frac{5}{6} \text{ kN}$	$N_1 = -2.5 \text{ kN}$	$L_1 = 5 \text{ m}$
$n_2 = -\frac{5}{6} \text{ kN}$	$N_2 = 2.5 \text{ kN}$	$L_2 = 5 \text{ m}$
$n_3 = \frac{2}{3} \text{ kN}$	$N_3 = -2 \text{ kN}$	$L_3 = 8 \text{ m}$

$n_1 N_1 L_1 = \frac{125}{12}$   
 $n_2 N_2 L_2 = -\frac{125}{12}$   
 $n_3 N_3 L_3 = -\frac{32}{3}$   
 $\sum nNL = -\frac{32}{3}$

$\delta_D = \int \frac{Mm}{EI} dx + \sum n \cdot \frac{NL}{AE}$   
 $\delta_D = \frac{1}{EI} \int_0^8 [(1.5x - 12) \cdot (0.5x - 4)] dx - \frac{32}{3AE}$   
 $\delta_D = \frac{1}{EI} \int_0^8 (\frac{3}{4}x^2 - 12x + 48) dx - \frac{32}{3AE}$   
 $\delta_D = \frac{1}{EI} [\frac{1}{4}x^3 - 6x^2 + 48x]_0^8 - \frac{32}{3AE}$   
 $\delta_D = \frac{128}{EI} - \frac{32}{AE}$  (downward)



$$m = -1 \text{ kNm for } 0 \leq x \leq 4 \text{ m}$$

$$1 \cdot \theta_B = \int_0^4 M m \, dx \cdot \frac{1}{EI}$$

$$\theta_B = \int_0^4 (1.5x - 12) \cdot (-1) \, dx \cdot \frac{1}{EI}$$

$$\theta_B = \int_0^4 (12 - 1.5x) \, dx \cdot \frac{1}{EI}$$

$$\theta_B = \left[ 12x - \frac{3}{4}x^2 \right]_0^4 \cdot \frac{1}{EI}$$

$$\theta_B = \frac{36}{EI} \text{ (clockwise)}$$