CV2011 - Structural Analysis I Semester 2 AY 2015 - 2016

① ⓐ External: $r=3$, $n=1$ } $r=3n$ ok.

Internal: $m=29$, $r=3$, $j=16$ } $m+r=2j$ ok.

∴ Truss is stable and determinate.

ⓑ $\Sigma M_A = 0$ $\quad 30 \cdot 2.5 = N_B \cdot 2.5$

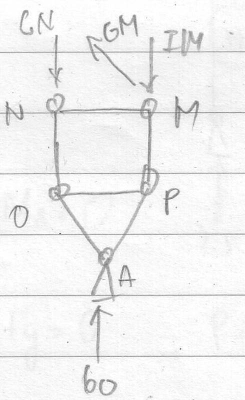
$N_B = 30 \downarrow$

$\Sigma F_y = 0$ $\quad -30 - 30 + A_y = 0$

$A_y = 60\text{ kN}$

$\Sigma F_x = 0$ $\quad A_x = 0\text{ kN}$

ⓒ

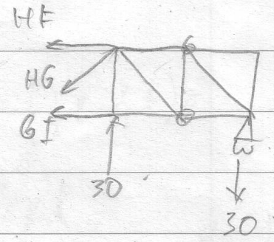


Take moment at M

$GN \cdot 1 = 60 \cdot 0.5$

$GN = 30\text{ kN (C)}$

$IM = 30\text{ kN}$, $GM = 0\text{ kN}$

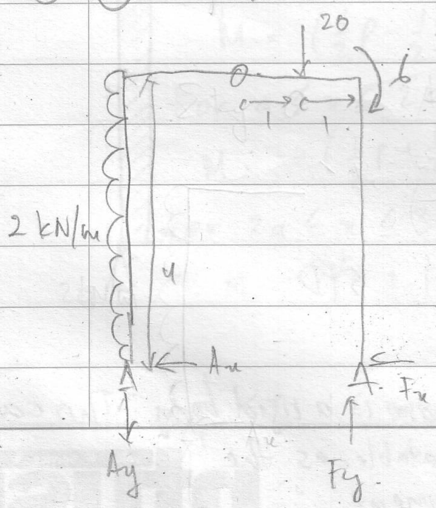


$HG = 0\text{ kN}$

Take joint I, $HI = 30\text{ kN (C)}$

② ⓐ $\Sigma M_A = 0$ $\quad F_y \cdot 4 = 8 \cdot 2 + 20 \cdot 3 + 6 \Rightarrow F_y = 20.5\text{ kN}$

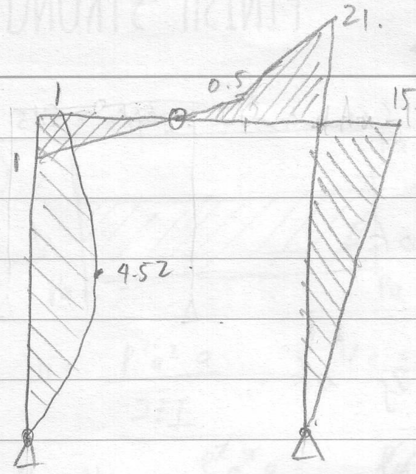
$A_y = -0.5\text{ kN}$



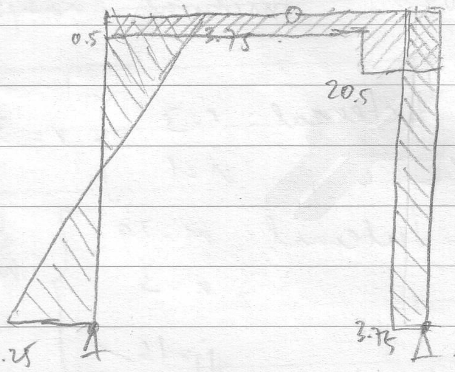
M at pin $\quad 20 \cdot 1 + 6 + 4F_x = 20.5 \cdot 2$

$F_x = 3.75\text{ kN}$

$\Sigma F_x = 0$ $\quad A_x = 8 - 3.75 = 4.25\text{ kN}$

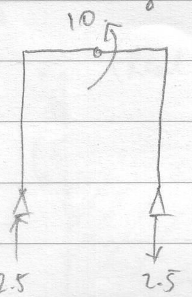


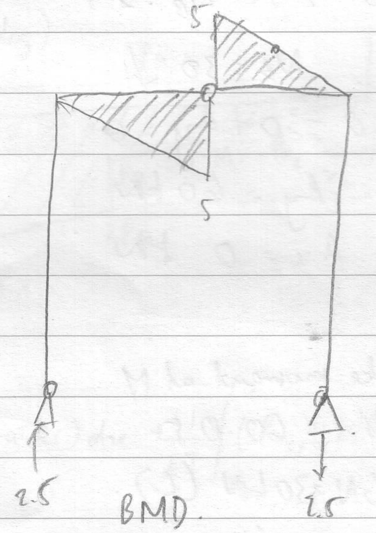
BMD.



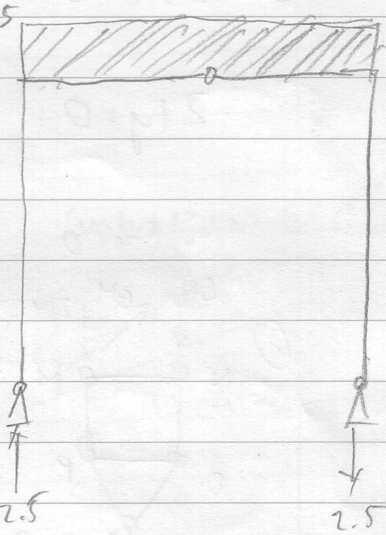
SFD

Extra loading due to the 10kNm moment at the pin joint

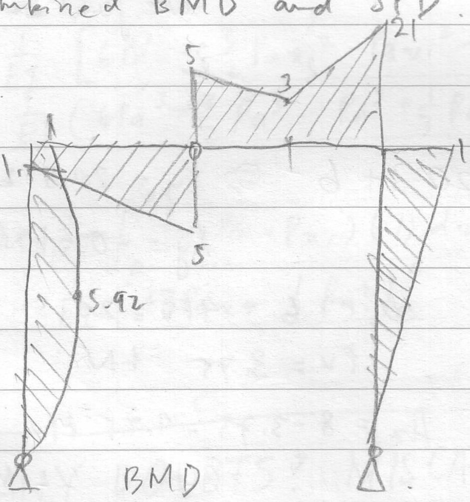




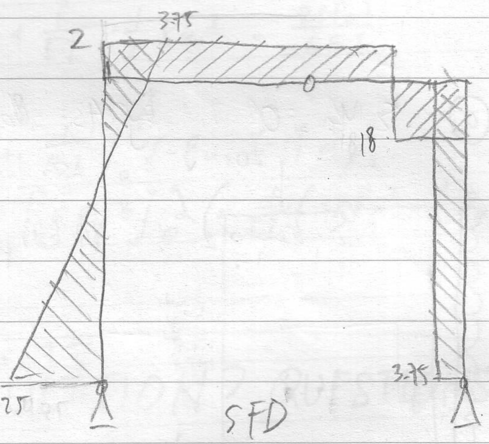
BMD.



Combined BMD and SFD.



BMD



SFD

* The combined is assuming that the frame is a rigid body. This case where the moment is applied at the pin is debatable, as the pin is sometimes defined unable to transfer moment. Personally I am not sure which one to use. That's why I'll separate the diagrams here.

② ⑥ $\Sigma M_A = 0 \quad 12 \cdot 2 + 10 \cdot 4 + 20 = 8 N_E$

$N_E = 10.5\ \text{kN}$

$\Sigma F_y = 0 \quad 10 + 12 = 10.5 + A_y$

$A_y = 11.5\ \text{kN}$

$\Sigma F_x = 0 \quad A_x = 0$

$M_A = 0$

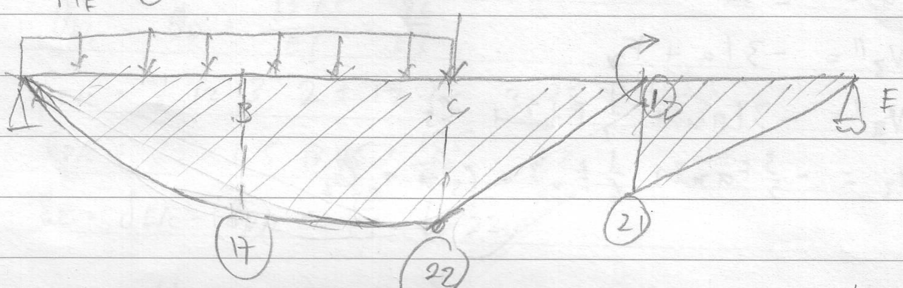
$M_B = 11.5 \cdot 2 - 6 = 17\ \text{kNm}$

$M_C = 11.5 \cdot 4 - 12 \cdot 2 = 22\ \text{kNm}$

$M_D^- = 11.5 \cdot 6 - 12 \cdot 4 - 10 \cdot 2 = 1\ \text{kNm}$

$M_D^+ = 10.5 \cdot 2 = 21\ \text{kNm}$

$M_E = 0$

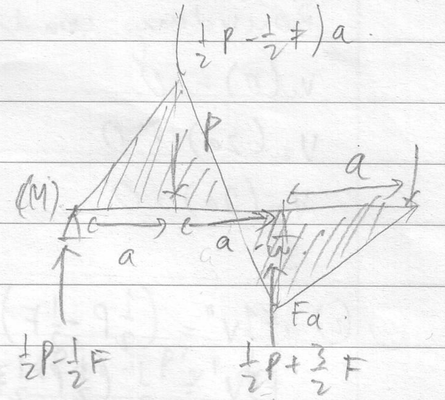


③ $\Sigma M_A = 0 \quad P\,a + F\,3a = N_c\,2a$

$N_c = \frac{1}{2}P + \frac{3}{2}F$

$\Sigma F_y = 0 \quad P + F - \frac{1}{2}P - \frac{3}{2}F = A_y$

$A_y = \frac{1}{2}P - \frac{1}{2}F$



ⓐ For $0 \le x \le a$

$M = (\frac{1}{2}P - \frac{1}{2}F)\,x$

For $a \le x \le 2a$

$M = -(\frac{1}{2}P + \frac{1}{2}F)\,x + Pa$

For $2a \le x \le 3a$

$M = -3Fa + Fx$

FINISH STRONG!

(b) For $0 \le x \le a$

$$EIv_1'' = \left(\tfrac{1}{2}P - \tfrac{1}{2}F\right)x$$

$$EIv_1' = \tfrac{1}{2}\left(\tfrac{1}{2}P - \tfrac{1}{2}F\right)x^2 + C_1$$

$$EIv_1 = \tfrac{1}{6}\left(\tfrac{1}{2}P - \tfrac{1}{2}F\right)x^3 + C_1x + C_2$$

For $a \le x \le 2a$

$$EIv_2'' = -\left(\tfrac{1}{2}P + \tfrac{1}{2}F\right)x + Pa$$

$$EIv_2' = -\tfrac{1}{2}\left(\tfrac{1}{2}P + \tfrac{1}{2}F\right)x^2 + Pax + C_3$$

$$EIV_2 = -\tfrac{1}{6}\left(\tfrac{1}{2}P + \tfrac{1}{2}F\right)x^3 + \tfrac{1}{2}Pax^2 + C_3x + C_4$$

For $2a \le x \le 3a$

$$EIV_3'' = -3Fa + Fx$$

$$EIV_3' = -3Fax + \tfrac{1}{2}Fx^2 + C_5$$

$$EIV_3 = -\tfrac{3}{2}Fax^2 + \tfrac{1}{6}Fx^3 + C_5x + C_6$$

Boundary conditions

$v_1(0) = 0$

$V_2(2a) = 0$

$V_3(2a) = 0$

Continuity conditions

$V_2'(2a) = V_3'(2a)$

(c) $$EIv'' = \left(\tfrac{1}{2}P - \tfrac{1}{2}F\right)x - P\langle x-a\rangle + \left(\tfrac{1}{2}P + \tfrac{3}{2}F\right)\langle x-2a\rangle$$

$$EIv' = \tfrac{1}{2}\left(\tfrac{1}{2}P - \tfrac{1}{2}F\right)x^2 - \tfrac{1}{2}P\langle x-a\rangle^2 + \tfrac{1}{2}\left(\tfrac{1}{2}P + \tfrac{3}{2}F\right)\langle x-2a\rangle^2 + C_1$$

$$EIv = \tfrac{1}{6}\left(\tfrac{1}{2}P - \tfrac{1}{2}F\right)x^3 - \tfrac{1}{6}P\langle x-a\rangle^3 + \tfrac{1}{6}\left(\tfrac{1}{2}P + \tfrac{3}{2}F\right)\langle x-2a\rangle^3 + C_1x + C_2$$

at $x = 0,\ v = 0 \Rightarrow C_2 = 0$

at $x = 2a,\ v = 0$

$$0 = \tfrac{1}{6}\left(\tfrac{1}{2}P - \tfrac{1}{2}F\right)\cdot 8a^3 - \tfrac{1}{6}Pa^3 + 2aC_1$$

$$C_1 = \frac{-\frac{1}{6}\left(4Pa^3 - 4Fa^3 - Pa^3\right)}{2a} = \frac{4Fa^3 - 3Pa^3}{12a} = \frac{4Fa^2 - 3Pa^2}{12}$$

$$EIv = \tfrac{1}{6}\left(\tfrac{1}{2}P - \tfrac{1}{2}F\right)x^3 - \tfrac{1}{6}P\langle x-a\rangle^3 + \tfrac{1}{6}\left(\tfrac{1}{2}P + \tfrac{3}{2}F\right)\langle x-2a\rangle^3 + \frac{4Fa^2 - 3Pa^2}{12}$$

$\rightarrow$

For $v=0$ at $x = 3a$

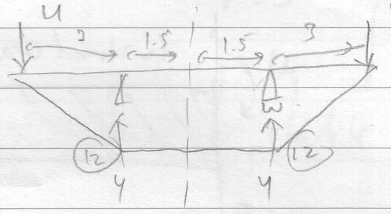
$$0 = \frac{1}{6}\left(\frac{1}{2}P - \frac{1}{2}F\right)27a^3 - \frac{1}{6}P \cdot 8a^3 + \frac{1}{12}\left(4Fa^2 - 3Pa^2\right)3a$$

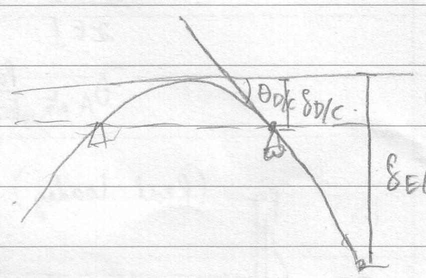
$$0 = \frac{27}{12}P\cancel{a^3} - \frac{27}{12}F\cancel{a^3} - \frac{16}{12}P\cancel{a^3} + F\cancel{a^3} - \frac{9}{12}P\cancel{a^3}$$

$$\frac{15}{12}F = \frac{1}{12}P$$

$$F = \frac{12}{15}\cdot\frac{1}{12}P = \frac{1}{15}P$$

4 (a)





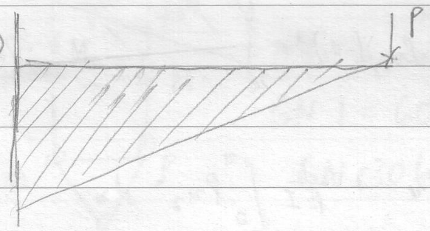
$$\theta_{D/C} = \theta_D = \frac{12 \cdot 1.5}{EI} = \frac{36}{EI}$$

$$\delta_{E/C} = \frac{1}{2}\cdot 12 \cdot 3 \cdot 2 + 12 \cdot 1.5 \cdot 3.75 = \frac{103.5}{EI}$$

$$\delta_{D/C} = 12 \cdot 1.5 \cdot 0.75 = \frac{13.5}{EI}$$

$$\delta_E = \delta_{E/C} - \delta_{D/C} = \frac{90}{EI} \downarrow$$

(b) (i)



$$M = 2Pa - Px$$

$$U_i = \int_0^{2a} \frac{(2Pa - Px)^2}{2EI}\,dx = \frac{1}{2EI}\int_0^{2a} 4P^2a^2 - 4P^2ax + P^2x^2\,dx$$

$$= \frac{1}{2EI}\left[4P^2a^2x - 2P^2ax^2 + \frac{1}{3}P^2x^3\right]_0^{2a} = \frac{1}{2EI}\left(4P^2a^2\cdot 2a - 2P^2a\,4a^2 + \frac{8}{3}P^2a^3\right)$$

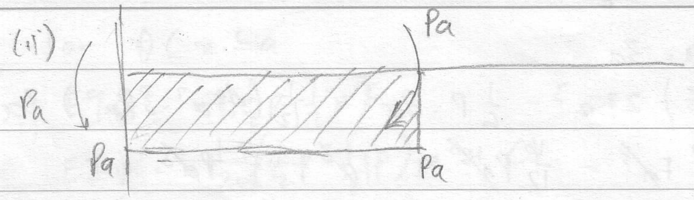
$$= \frac{1}{EI}\left(\cancel{4P^2a^3} - \cancel{4P^2a^3} + \frac{4}{3}P^2a^3\right) = \frac{4P^2a^3}{3EI}$$

$$U_e = \frac{P\delta_A}{2}$$

$$U_i = U_e \Rightarrow \frac{\cancel{P}\delta_A}{2} = \frac{4P^{\cancel{2}}a^3}{3EI}$$

$$\delta_A = \frac{8Pa^3}{3EI}$$

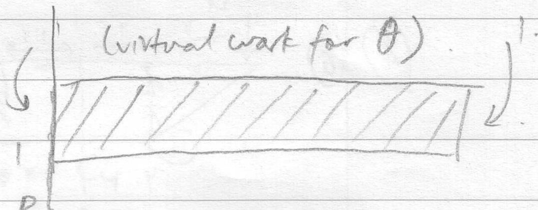
(ii)



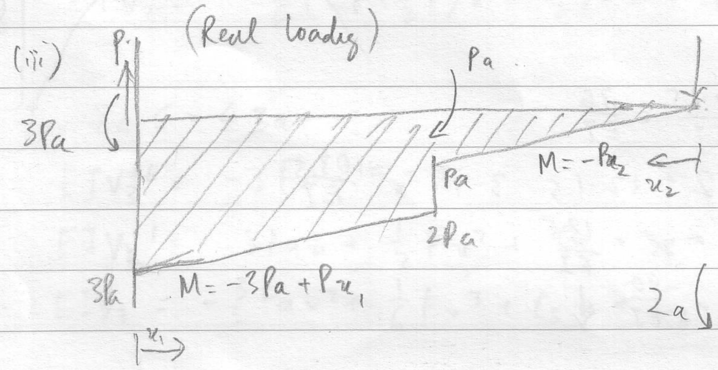
$$U_i = \frac{P^2a^2a}{2EI} \qquad U_e = \frac{Pa\,\theta_A}{2}$$

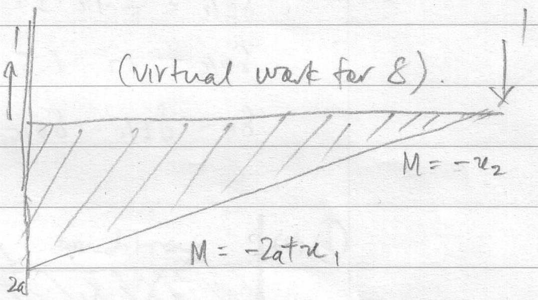
$$U_i = U_e \qquad \frac{P^{\cancel{2}}a^{\cancel{2}}a}{2EI} = \frac{\cancel{P}\cancel{a}\,\theta_A}{\cancel{2}}$$

$$\theta_A = \frac{Pa^2}{EI}$$



(iii)





$$\delta = \frac{1}{EI}\int_0^a (-3Pa + Px_1)(-2a + x_1)\,dx + \frac{1}{EI}\int_0^a (-Px_2)(-x_2)\,dx$$

$$= \frac{1}{EI}\int_0^a (6Pa^2 - 3Pax_1 - 2Px_1a + Px_1^2)\,dx + \frac{1}{EI}\int_0^a Px_2^2\,dx$$

$$= \frac{1}{EI}\left[6Pa^2x - \frac{3}{2}Pax_1^2 - Pax_1^2 + \frac{1}{3}Px_1^3\right]_0^a + \frac{1}{EI}\left[\frac{1}{3}Px_2^3\right]_0^a$$

$$= \frac{1}{EI}\left(6Pa^3 - \frac{3}{2}Pa^3 - Pa^3 + \frac{1}{3}Pa^3\right) + \frac{1}{EI}\cdot\frac{1}{3}Pa^3 = \frac{25Pa^3}{6EI}$$

$$\theta = \frac{1}{EI}\int_0^a (-3Pa + Px_1)(-1)\,dx_1 + \frac{1}{EI}\int_0^a (-Px_2)(-1)\,dx_2$$

$$= \frac{1}{EI}\left[+3Pax - \frac{1}{2}Px^2\right]_0^a + \frac{1}{EI}\left[\frac{1}{2}Px^2\right]_0^a = \frac{1}{EI}\left(3Pa^2 - \frac{1}{2}Pa^2 + \frac{1}{2}Pa^2\right) = \frac{3Pa^2}{EI}$$

ANY DOUBTS? MISCALCULATION? QUESTIONS?

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-Dicky Djayadi Sugeng

Errata for CV2011 Structural Analysis 1 AY2015-16 Semester 2

Question 4(a)(i)

There is a calculation error in this question. It should be as shown below:

$$\theta_D = \theta_{D/C} = \frac{12 \times 1.5}{EI} = \frac{\mathbf{18}}{EI}$$

Notice that the 36 is now replaced with 18. My apologies for the error.