CV2011 2014-2015 S2

FINISH STRONG!

1. a. $r+m=2j$

$r=3$ $\quad 25+3=14(2)$

$m=25$ $\quad 28=28 \rightarrow$ statically determinate

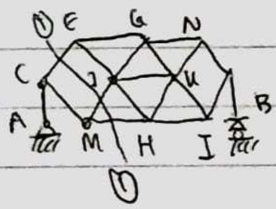
$j=14$

b. $\Sigma M_A=0$ $\qquad \Sigma F_y=0$

$2Pa = B_Y \times 6a$ $\qquad A_Y+B_Y=2P$

$B_Y = P/3$ $\qquad A_Y = 5P/3$

c. Eliminating zero force members:



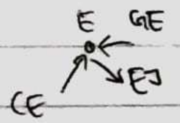
cut 1-1

$\Sigma M_m=0$

$F_{CE}(a\sqrt{2}) = (-\frac{5}{3}P)\times a$

$CE = -\frac{5}{3\sqrt{2}}P = -1.179P\ (C)$

PME



$EJ = CE = 1.179P\ (T)$

$GE = 2\times\frac{a}{a\sqrt{2}}\times 1.179P$

$= 1.667P\ (C)$

PMD

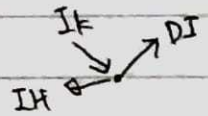
$BD = B_y = \frac{P}{3}$

$DN = DJ = 2\times\frac{a}{a\sqrt{2}}BD$

$DN = \frac{P}{3\sqrt{2}} = 0.236P\ (C)$

$DJ = 0.236P\ (T)$

PMI



$DJ = IK \Rightarrow IH = 2\times\frac{a}{a\sqrt{2}}\times\frac{P}{3\sqrt{2}}$

$= \frac{P}{3}\ (T)$

cut (2)-(2)

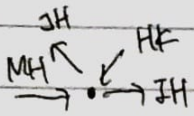
$\Sigma M_J = 0$

$MH \times a = Ay \times \cancel{a}\ 2a - (EG \times a + 2P \times a)$

$MH\ a = (\frac{5}{3}P \times 2a) - (1.667 P \times a + 2P \times a)$

$MH = \frac{1}{a}(\frac{10}{3}Pa - \frac{11}{3}Pa) = -\frac{P}{3}$ (C)

Pin H



$JH = HK$

$2 \times \frac{a}{a\sqrt{2}} \times JH = \frac{2}{3}P$

$JH = \frac{\sqrt{2}}{3}P$ (T)

$\Sigma M_M = 0$

$-5/3\, P \times a + 1.667\, P \times 2a \cancel{\quad} - \cancel{\quad}\, JK \times a - \frac{\sqrt{2}}{3}Pa\sqrt{2} = 0$

$\cancel{\quad}\ \cancel{\quad}\ a \times JK = -2/3\, Pa$

$JK = -2/3\, P$ (C)

2. a. $r + m = 2j$

$r = 4$ $\quad 4+2 = 3(2)$

$m = 2$ $\quad 6 = 6 \rightarrow$ statically determinate

$j = 3$

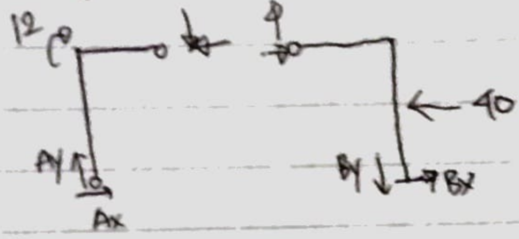
b. $\Sigma F_x = 0$ $\quad \Sigma F_y = 0$ $\quad \Sigma M_A = 0$

$Ax + Bx = 40$ $\quad Ay + By = 0$ $\quad 6By = 40 \times 3 - 12$

$Ay = -By$ $\quad By = 108/6 = 18\,kN$ (↓)

$Ay = 18\,kN$ (↑)

taking out pin C & draw FBD:



LHS $\Sigma M_C = 0$

$A_y \times 3 + 12 = A_x \times 6$

$A_x = \frac{1}{2} \times 18 + \frac{12}{6} = 11 kN$

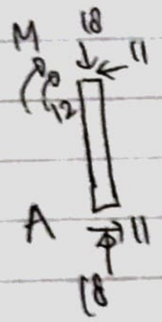
$C_x = A_x = 11 kN$

$C_y = A_y = 18 kN$

RHS $\Sigma M_C = 0$

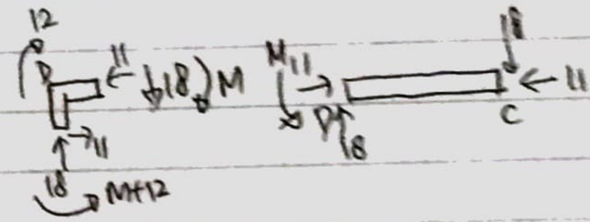
$B_x \times 6 = 54 + 40 \times 3$

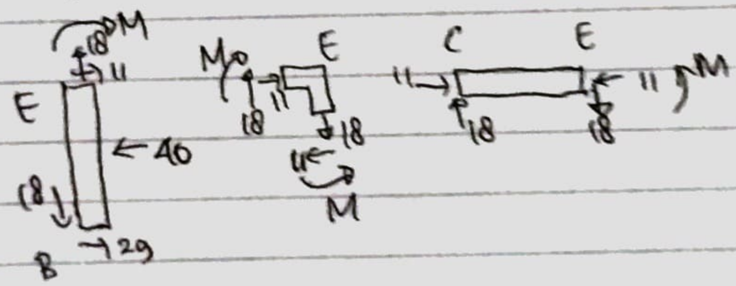
$B_x = 174/6 = 29 kN$



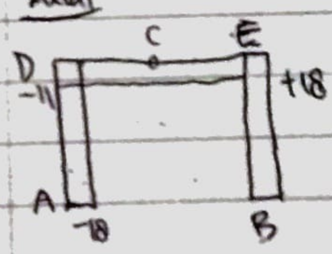
$M = 66 - 12$

$= 54 kNm$

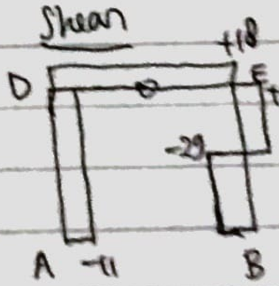




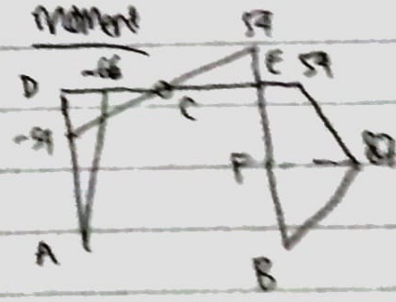
Axial



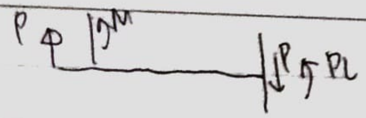
Shear



Moment



3. a.



$M = Px$

$EIv'' = Px$

$EIv' = \frac{Px^2}{2} + C_1$

$EIv = \frac{P}{6}x^3 + C_1x + C_2$

$v'(L) = 0$

$\frac{PL^2}{2} + C_1 = 0$

$C_1 = -\frac{PL^2}{2}$

$v(L) = 0$

$0 = \frac{P}{6}L^3 + C_1L + C_2$

$0 = \frac{P}{6}L^3 - \frac{PL^3}{2} + C_2$

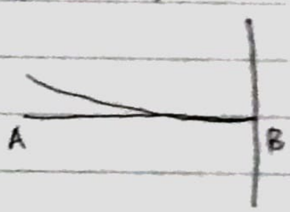
$C_2 = \frac{1}{3}PL^3$

$v = \frac{1}{EI}\left(\frac{Px^3}{6} - \frac{PL^2}{2}x + \frac{1}{3}PL^3\right)$

$v = \frac{P}{6EI}\left(x^3 - 3L^2x + 2L^3\right)$

$v' = \left(\frac{Px^2}{2} - \frac{PL^2}{2}\right)\frac{1}{EI}$

$v' = \frac{P}{2EI}\left(x^2 - L^2\right)$



b. $q = w$

$EIv'''' = w$

$EIv''' = wx + C_1$ $\quad$ V at $x=0 \to 0$

$C_1 = 0 \to EIv''' = wx$

$EIv'' = \frac{w}{2}x^2 + C_2$ $\quad$ M at $x=0 \to 0$

$C_2 = 0 \to EIv'' = \frac{w}{2}x^2$

$EIv' = \frac{w}{6}x^3 + C_3$

$\therefore v'(L) = 0$

$\frac{w}{6}L^3 + C_3 = 0$

$C_3 = \frac{-wL^3}{6}$

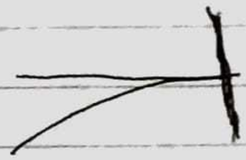
$v' = \frac{w}{6EI}\left(x^3 - L^3\right)$

$EIv = \frac{w}{24}x^4 + C_3x + C_4$

$v(L)=0 \rightarrow \frac{w}{24}L^4 - \frac{wL^4}{6} + C_4 = 0$

$C_4 = \frac{wL^4}{6} - \frac{w}{24}L^4$

$= \frac{1}{8}wL^4$



$v = \frac{w}{24EI}\left(x^4 - 4L^3x + 3L^4\right)$

c. $L = 10m$

$P = 187.5kN$

$w = 50\ kN/m$

deflection at A total : deflection at A from point load + from UDL

from point load:

v at $x=0 \rightarrow \frac{P}{6EI} \times 2L^3$

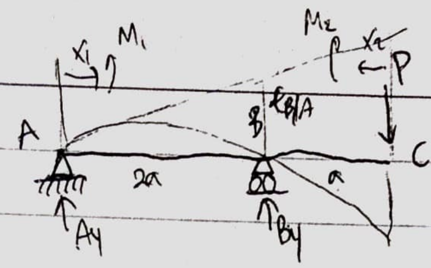
$= \frac{187.5 \times 10^3}{3EI} = \frac{62500}{EI}$ (↑)

from UDL → v at $x=0 \rightarrow \frac{w}{24EI}(3L^4) = \frac{50 \times 10^4}{8EI} = \frac{62500}{EI}$ (↓)

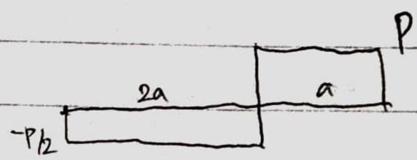
deflection at free end: $\frac{62500}{EI} - \frac{62500}{EI} = 0$

FINISH STRONG!

4.



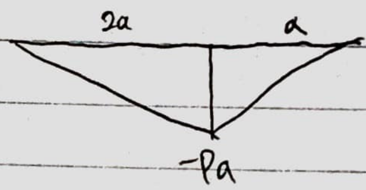
a. $\Sigma M_B = 0$

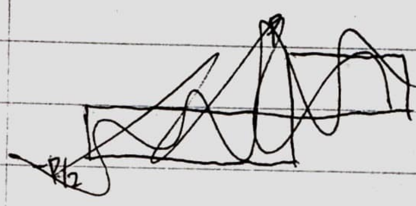


$A_y \times 2a + P \times a = 0$

$A_y = -\frac{Pa}{2a} = -\frac{P}{2}$

$B_y = \frac{3P}{2}$





b. Macaulay method

$M = -\frac{P}{2}x + \frac{3P}{2}\langle x-2a \rangle$

$EIv'' = M$

$EIv' = -\frac{P}{4}x^2 + \frac{3P}{4}\langle x-2a \rangle^2 + C_1$

$EIv = -\frac{P}{12}x^3 + \frac{3P}{12}\langle x-2a \rangle^3 + C_1 x + C_2$

$v(0) = 0 \rightarrow C_2 = 0$

$v(2a) = 0 \rightarrow \quad 0 = -\frac{P}{12}(2a)^3 + \frac{3P}{12} \times 0 + C_1 \times 2a$

$C_1 \times 2a = \frac{P}{12}(2a)^3$

$C_1 = \frac{P}{3}a^2$

$EIv = -\frac{P}{12}x^3 + \frac{3P}{12}\langle x-2a \rangle^3 + \frac{P}{3}a^2 x$

at $C \rightarrow x = 3a$

$EIv = -\frac{P}{12}(3a)^3 + \frac{3P}{12}a^3 + \frac{P}{3}a^2(3a)$

$= \frac{-Pa^3}{3EI}$ $(\downarrow)$

ntusu

c. moment area method θ_A

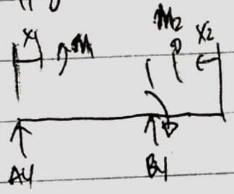
$$t_{B/A} = \left[\frac{1}{2}(2a)\left(\frac{Pa}{EI}\right)\right]\left(\frac{1}{3} \times 2a\right)$$

$$= \frac{2}{3}\frac{Pa^3}{EI}$$

$$\theta_A = \frac{t_{B/A}}{L} = \frac{\frac{2}{3}\frac{Pa^3}{EI}}{2a} = \frac{1}{3}\frac{Pa^2}{EI}$$

d. method of virtual work θ_B

apply moment 1kNm clockwise at B



$$\theta_B = \int_0^{2a} \left(-\frac{P}{2}x_1\right)\left(-\frac{1}{2a}x_1\right)dx_1 + 0$$

$$= \int_0^{2a} \frac{P}{4a}x_1^2\, dx_1$$

$$\frac{P}{12a}x_1^3 \Big|_0^{2a}$$

$$= \frac{2}{3}\frac{Pa^2}{EI} \text{ (clockwise)}$$

$\Sigma M_B = 0$

$2a \times A_y = -1$

$A_y = -\frac{1}{2a}$

$B_y = \frac{1}{2a}$

ALL THE BEST!

Josh Andersen