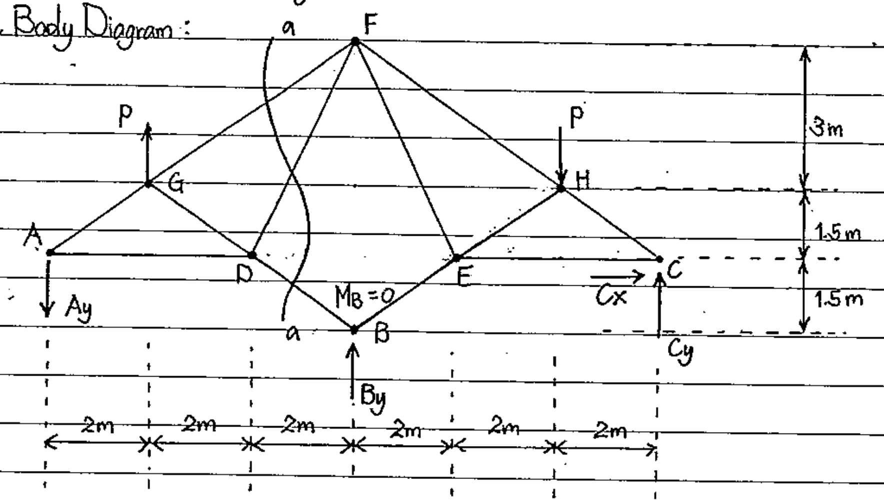
Past Year Paper Solution (AY 2012 - 2013 / 1st semester) Yes, U Can!

CV 2011 - Structural Analysis I

1. Free Body Diagram :



(a) (i) External determinacy :

* $r_a = 1 + 1 + 2$
  $= 4$. (no. of reactions)
* $r = 3$ (no. of equilibrium equations)
* $n = 1$ (no. of conditional equations)

$r_a = r + n$
$4 = 4$

$\therefore$ Since $r_a = r + n$, hence the structure is statically determinate externally.

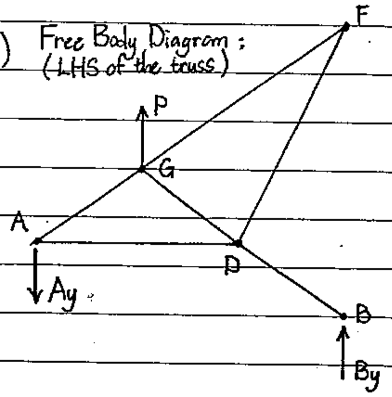
(ii) Overall determinacy :

* $r_a = 1 + 1 + 2$
  $= 4$ (no. of reactions)
* $m_a = 12$. (no. of truss members).
* $j = 8$. (no. of joints)

$r_a + m_a = 2j$
$4 + 12 = 2(8)$
$16 = 16$.

$\therefore$ Since $r_a + m_a = 2j$, hence the structure is statically determinate overall.

(b) Free Body Diagram ; (LHS of the truss)



* $M_B = 0$ (conditional equation) ↺+

$A_y(6) - P(4) = 0$.

$A_y = \frac{2}{3}P (\downarrow)$

* From the Free Body Diagram of the whole truss above, we can deduce that :

* $\sum F_x = 0 \xrightarrow{+}$ $C_x = 0$.

* $\sum M_B = 0$ ↺+

$A_y(6) - P(4) - P(4) + C_y(6) = 0$.

$(\frac{2}{3}P)(6) - 8P = -6C_y$.

$-4P = -6C_y$

$C_y = \frac{2}{3}P (\uparrow)$

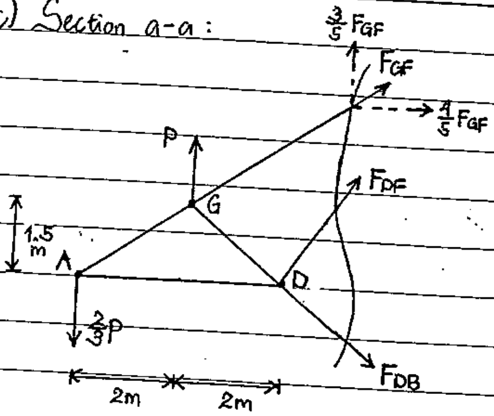
* $\sum F_y = 0$.

$-\frac{2}{3}P + P - P + \frac{2}{3}P + B_y = 0$

$B_y = 0$.

Yes, U can!

1. (c) Section a-a :



$* \sum M_D = 0.$ ↺+

$(\frac{2}{3}P)(4) - (P)(2) - (\frac{3}{5}F_{GF})(2) - (\frac{4}{5}F_{GF})(1.5) = 0$

$\frac{8}{3}P - 2P = \frac{6}{5}F_{GF} + \frac{6}{5}F_{GF}$

$\frac{2}{3}P = \frac{12}{5}F_{GF}$

$F_{GF} = \frac{5}{18}P \ (T)$

$* \sum M_G = 0.$ ↺+

$(\frac{2}{3}P)(2) + (\frac{9}{\sqrt{97}}F_{DF})(2) + (\frac{4}{\sqrt{97}}F_{DF})(1.5) = 0$

$\frac{4}{3}P = -\frac{18}{\sqrt{97}}F_{DF} - \frac{6}{\sqrt{97}}F_{DF}$

$\frac{4}{3}P = -\frac{24}{\sqrt{97}}F_{DF}$

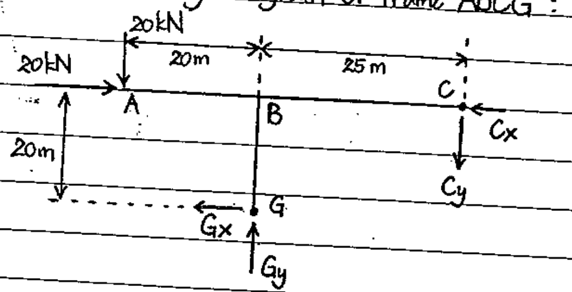
$F_{DF} = -\frac{\sqrt{97}}{18}P$ or $F_{DF} = \frac{\sqrt{97}}{18}P\ (C)$

* Since the truss is symmetric & the loads imposed on the truss are also symmetric about the center but have opposite directions, hence :

$* F_{FE} = -F_{DF}$
$= -(-\frac{\sqrt{97}}{18}P)$
$= \frac{\sqrt{97}}{18}P \ (T)$

$* F_{FH} = -F_{GF}$
$= -\frac{5}{18}P$ or $F_{FH} = \frac{5}{18}P\ (C)$

2. (a) * Free Body Diagram of frame ABCG :



$* \sum M_G = 0.$ ↺+

$-C_y(25) + C_x(20) + (20)(20) - (20)(20) = 0$

$25C_y = 20C_x$

$C_y = 0.8C_x \ldots (1)$

$* \sum F_x = 0$

$-G_x + 20 - C_x = 0$

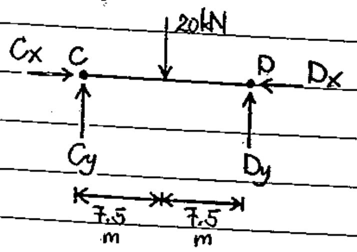
$G_x = 20 - C_x \ldots (3)$

$* \sum F_y = 0$

$G_y - C_y - 20 = 0$

$G_y = 20 + C_y \ldots (2)$

* Free Body Diagram of beam CD :



$* \sum M_D = 0.$ ↺+

$-C_y(15) + (20)(7.5) = 0$

$15C_y = 150$

$C_y = 10\text{ kN}$

$* \sum F_y = 0$

$C_y + D_y - 20 = 0$

$10 + D_y = 20$

$D_y = 10\text{ kN}$

$* \sum F_x = 0 \rightarrow C_x - D_x = 0 \rightarrow C_x = D_x \ldots (4)$

2. (a) * Substitute C_y to eq. (1) : $C_y = 0.8C_x$

$10 = 0.8C_x$

$C_x = 12.5\ kN$

* Substitute C_y to eq. (2) : $G_y = 20 + C_y$

$= 20 + 10$

$G_y = 30\ kN$

* Substitute C_x to eq. (3) : $G_x = 20 - C_x$

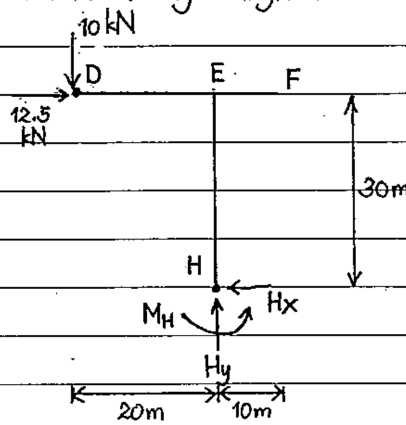
$= 20 - 12.5$

$G_x = 7.5\ kN$

* Substitute C_x to eq. (4) : $C_x = D_x$

$D_x = 12.5\ kN$

* Free Body Diagram of frame DEFH :



* $\sum F_y = 0$

$H_y - 10 = 0$

$H_y = 10\ kN$

* $\sum F_x = 0$

$12.5 - H_x = 0$

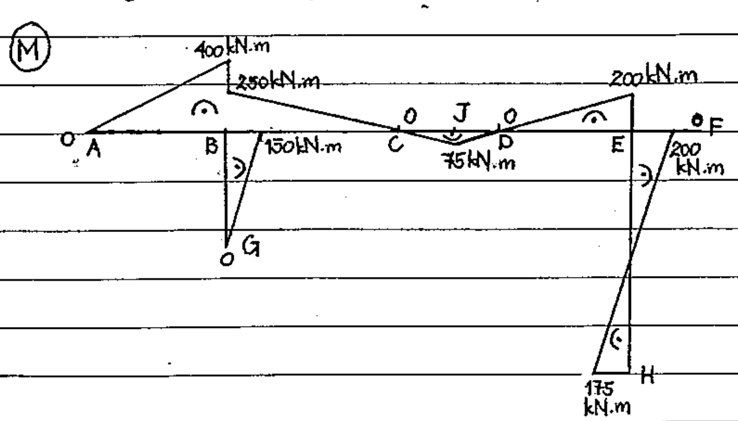
$H_x = 12.5\ kN$

* $\sum M_H = 0$ ↺+

$(10)(20) - (12.5)(30) + M_H = 0$

$M_H = 175\ kN.m$

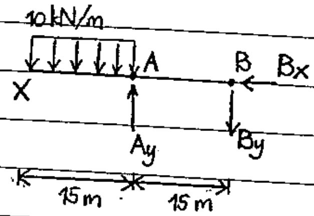
∴ Bending Moment Diagram of the whole frame ABCDEFGH :



* Notes :

all moment diagrams are drawn on the tension sides.

2. (b) * Free Body Diagram of beam XAB :



* $\sum M_B = 0$ ↺+

$(10\times15)(22.5) - A_y(15) = 0$

$A_y = 225\ kN\ (\uparrow)$

* $\sum F_x = 0$

$B_x = 0$

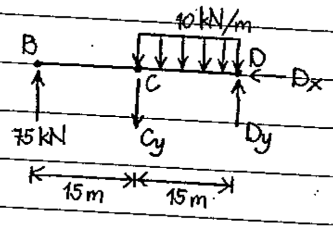
* $\sum F_y = 0$

$A_y - B_y - (10\times15) = 0$

$225 - B_y = 150$

$B_y = 75\ kN\ (\downarrow)$

* Free Body Diagram of beam BCD :



* $\sum M_D = 0$ ↺+

$-(75)(30) + C_y(15) + (10\times15)(7.5) = 0$

$-2250 + 15\,C_y + 1125 = 0$

$15\,C_y = 1125$

$C_y = 75\ kN\ (\downarrow)$

* $\sum F_y = 0$

$75 + D_y - C_y - (10\times15) = 0$

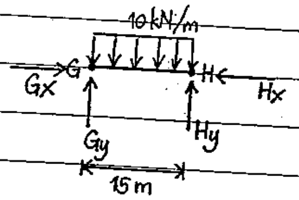
$75 + D_y - 75 - 150 = 0$

$D_y = 150\ kN\ (\uparrow)$

* $\sum F_x = 0$

$D_x = 0$

* Free Body Diagram of beam GH :



* $\sum M_H = 0$ ↺+

$(10\times15)(7.5) - G_y(15) = 0$

$G_y = 75\ kN\ (\uparrow)$

* $\sum F_y = 0$

$G_y + H_y - (10\times15) = 0$

$75 + H_y = 150$

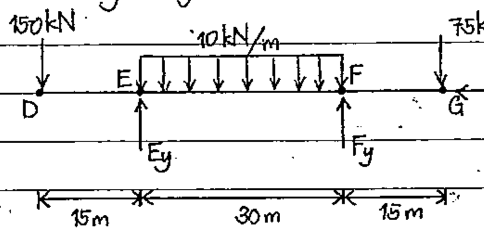
$H_y = 75\ kN\ (\uparrow)$

* $\sum F_x = 0$

$G_x - H_x = 0$

$G_x = H_x \quad \ldots (1)$

2. (b) * Free Body Diagram of beam DEFG :



* $\sum M_F = 0$. ↺+

$(150)(45) - E_y(30) + (10\times30)(15) - (75)(15) = 0$

$6750 - 30E_y + 4500 - 1125 = 0$

$30E_y = 10,125$

$\therefore E_y = 337.5$ kN (↑)

* $\sum F_x = 0$.

$G_x = 0$.

* $\sum F_y = 0$.

$E_y + F_y - 150 - (10\times30) - 75 = 0$

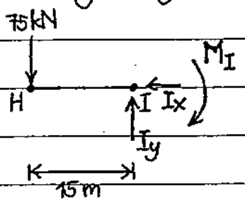
$337.5 + F_y - 525 = 0$

$\therefore F_y = 187.5$ kN (↑)

* Substitute G_x to eq. (1) : $G_x = H_x$.

$H_x = 0$.

* Free Body Diagram of beam HI :



* $\sum M_I = 0$. ↺+

$(75)(15) - M_I = 0$

$M_I = 1,125$ kN.m (↺)

* $\sum F_x = 0$

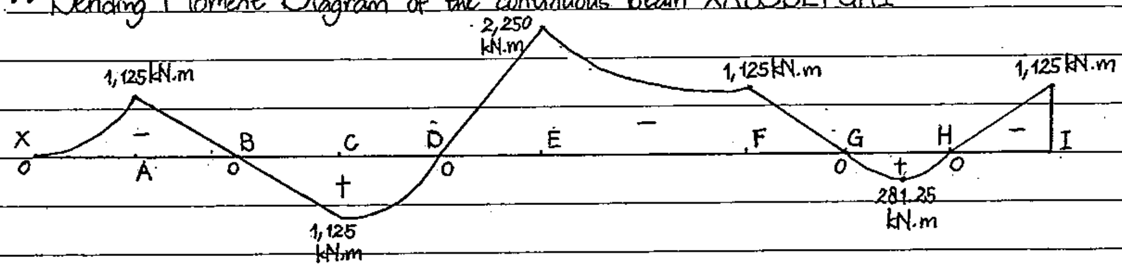
$\therefore I_x = 0$.

* $\sum F_y = 0$.

$I_y - 75 = 0$.

$I_y = 75$ kN (↑)

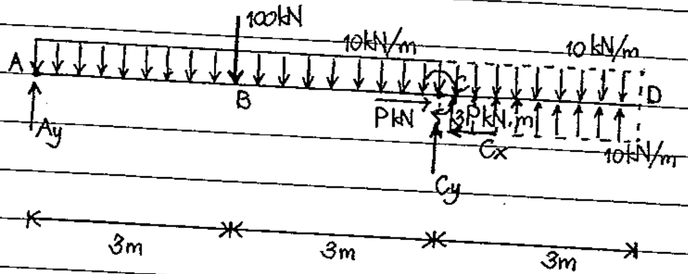
∴ Bending Moment Diagram of the continuous beam XABCDEFGHI :



* Notes :

all moment diagrams are drawn on the tension sides.

3. (a) * Free Body Diagram of beam ABCD :



(i) * $\sum M_c = 0$. ↺+

$$-A_y(6) + (10\times6)(3) + (100)(3) - 3P = 0.$$

$$6A_y = 480 - 3P$$

$$A_y = \left(80 - \tfrac{1}{2}P\right)\text{ kN } (\uparrow)$$

* $\sum F_y = 0$.

$$A_y + C_y - 100 - (10\times6) = 0.$$

$$\left(80 - \tfrac{1}{2}P\right) + C_y - 160 = 0.$$

$$C_y = \left(\tfrac{1}{2}P + 80\right)\text{ kN } (\uparrow)$$

* $\sum F_x = 0$.

$$P - C_x = 0$$

$$C_x = P\text{ kN } (\leftarrow)$$

(ii) Macaulay's Method:

* $M = \left(80 - \tfrac{1}{2}P\right)x - \tfrac{1}{2}(10)x^2 - (100)\langle x-3\rangle + \left(\tfrac{1}{2}P + 80\right)\langle x-6\rangle + 3P\langle x-6\rangle^0 + \tfrac{1}{2}(10)\langle x-6\rangle^2$.

* Differential equation of the deflection curve:

$$v'' = \frac{1}{EI}.M$$

$$= \frac{1}{EI}\left[\left(80 - \tfrac{1}{2}P\right)x - 5x^2 - 100\langle x-3\rangle + \left(\tfrac{1}{2}P + 80\right)\langle x-6\rangle + 3P\langle x-6\rangle^0 + 5\langle x-6\rangle^2\right]$$

* Slope of deflection:

$$v' = \frac{1}{EI}\int M\,dx$$

$$= \frac{1}{EI}\left[\left(40 - \tfrac{1}{4}P\right)x^2 - \tfrac{5}{3}x^3 - 50\langle x-3\rangle^2 + \left(40 + \tfrac{1}{4}P\right)\langle x-6\rangle^2 + 3P\langle x-6\rangle + \tfrac{5}{3}\langle x-6\rangle^3 + C_1\right]$$

* Deflection :

$$v = \frac{1}{EI}\int v'\,dx.$$

$$= \frac{1}{EI}\left[\left(\tfrac{40}{3} - \tfrac{1}{12}P\right)x^3 - \tfrac{5}{12}x^4 - \tfrac{50}{3}\langle x-3\rangle^3 + \left(\tfrac{40}{3} + \tfrac{1}{12}P\right)\langle x-6\rangle^3 + \tfrac{3}{2}P\langle x-6\rangle^2 + \tfrac{5}{12}\langle x-6\rangle^4 + C_1.x + C_2\right].$$

Yes, U can!

3. (a)(ii) * B.C. 1 : $v(0) = 0. \rightarrow C_2 = 0.$

* B.C. 2 : $v(6) = 0. \rightarrow \frac{1}{EI}\left[\left(\frac{40}{3} - \frac{1}{12}P\right)(6)^3 - \frac{5}{12}(6)^4 - \frac{50}{3}(3)^3 + 6C_1\right] = 0.$

$$(2880 - 18P) - 540 - 450 + 6C_1 = 0.$$

$$6C_1 = 18P - 1890$$

$$C_1 = 3P - 315.$$

$\therefore$ The deflection curve :

$$v = \frac{1}{EI}\left[\left(\frac{40}{3} - \frac{1}{12}P\right)x^3 - \frac{5}{12}x^4 - \frac{50}{3}\langle x-3\rangle^3 + \left(\frac{40}{3} + \frac{1}{12}P\right)\langle x-6\rangle^3 + \frac{3}{2}P\langle x-6\rangle^2 + \frac{5}{12}\langle x-6\rangle^4 + (3P - 315)x\right]$$

(iii) * The deflection at D is zero ; i.e. :

$v(9) = 0.$

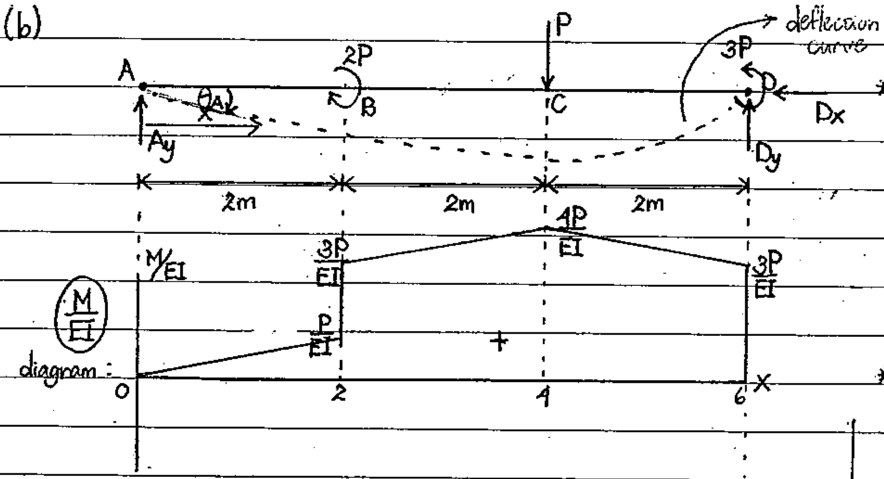
$$\frac{1}{EI}\left[\left(\frac{40}{3} - \frac{1}{12}P\right)(9)^3 - \frac{5}{12}(9)^4 - \frac{50}{3}(6)^3 + \left(\frac{40}{3} + \frac{1}{12}P\right)(3)^3 + \frac{3}{2}P(3)^2 + \frac{5}{12}(3)^4 + (3P - 315)(9)\right] = 0.$$

$$(9{,}720 - 60.75P) - 2{,}733.75 - 90 + (360 + 2.25P) + 13.5P + 33.75 + (27P - 2{,}835) = 0.$$

$$4{,}455 - 18P = 0.$$

$$P = 247.5 \text{ kN}$$

(b)



* Support Reaction :

* $\sum M_D = 0. \circlearrowleft^{+}$

$$-A_y(6) - 2P + P(2) + 3P = 0.$$

$$-6A_y + 3P = 0.$$

$$A_y = \frac{1}{2}P\ (\uparrow)$$

* $\sum F_x = 0.$

$$D_x = 0.$$

* $\sum F_y = 0$

$$A_y + D_y - P = 0.$$

$$\frac{1}{2}P + D_y - P = 0.$$

$$D_y = \frac{1}{2}P\ (\uparrow).$$

* $t_{D/A} = A_{\triangle}.\left(\frac{2}{3} + 4\right) + A_{\square}.(2)$

$$= \left(\frac{1}{2} \times 2 \times \frac{P}{EI}\right)\left(\frac{14}{3}\right) + 2\left(\frac{1}{2} \times \left(\frac{3P}{EI} + \frac{4P}{EI}\right) \times 2\right)(2).$$

$$= \frac{14P}{3EI} + \frac{28P}{EI}$$

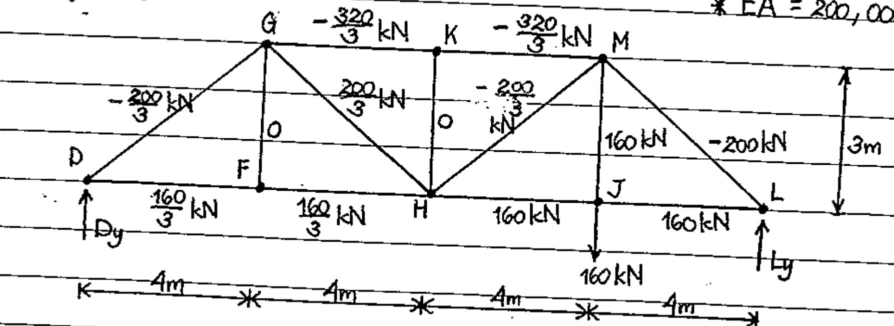
$$= \frac{98P}{3EI}$$

$\therefore \theta_A = \frac{t_{D/A}}{6} = \frac{49P}{9EI}$ (clockwise)

Yes, U Can!

4. * Free Body Diagram of the truss :

* EA = 200,000 kN for all truss members.



(a) * Support Reactions :

$* \sum M_L = 0.$ (↺+)

$-D_y(16) + (160)(4) = 0$

$D_y = 40\ kN\ (\uparrow)$.

$* \sum F_y = 0.$

$D_y + L_y - 160 = 0.$

$40 + L_y = 160.$

$L_y = 120\ kN\ (\uparrow)$.

(b) * At D :

$D_y = 40kN$, ΔX_D, $k\Delta X_D$

($k = 20 kN/mm$)

$* \sum F = 0.$

$-D_y + k.\Delta X_D = 0$

$40 = (20)\Delta X_D$

$\Delta X_D = 2\ mm$

* At L :

$L_y = 120 kN$, ΔX_L, $k.\Delta X_L$

($k = 30 kN/mm$)

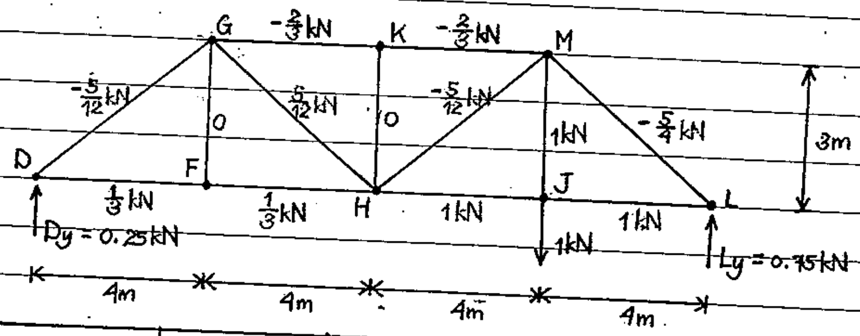
$* \sum F = 0.$

$-L_y + k.\Delta X_L = 0.$

$120 = (30).\Delta X_L$

$\Delta X_L = 4\ mm.$

(c) * Method of Virtual Work :



Member	n (kN)	N (kN)	L (m)	n.N.L ($kN^2.m$)
GK	-2/3	-320/3	4	284.44
KM	-2/3	-320/3	4	284.44
DF	1/3	160/3	4	71.11
FH	1/3	160/3	4	71.11
HJ	1	160	4	640
JL	1	160	4	640
GF	0	0	3	0
KH	0	0	3	0
MJ	1	160	3	480
DG	-5/12	-200/3	5	138.89
GH	5/12	200/3	5	138.89
MH	-5/12	-200/3	5	138.89
ML	-5/4	-200	5	1250
				Σ = 4,137.77

* Deflection on the truss only :

$1kN.\ \Delta J_1 = \sum \frac{n.N.L}{AE}.$

$\Delta J_1 = \frac{1}{200,000}(4,137.77)$

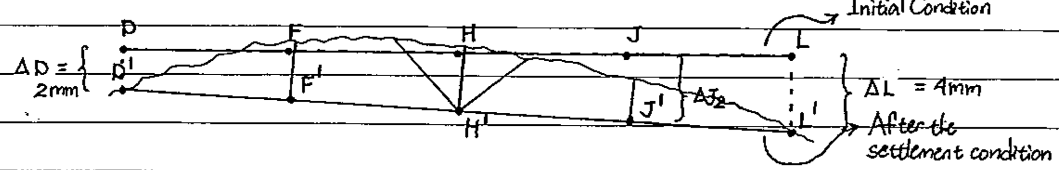
$= 0.0207\ m$

$= 20.7\ mm$

(downward)

4. (c) * Method of Super position :

* Due to the settlement of the spring at D and L :



$$\Delta J_2 = \Delta D + \left(\frac{12}{16} \times (\Delta L - \Delta D)\right).$$

$$= 2\,mm + \left(\frac{3}{4}(4mm - 2mm)\right).$$

$$= 3.5\,mm. \text{ (downward)}$$

∴ Total vertical deflection at J :

$$\Delta J = \Delta J_1 + \Delta J_2.$$

$$= 20.7 + 3.5.$$

$$= \underline{24.2\ mm} \text{ (downward)}$$