NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2011-2012

CV2102 – STRUCTURES I

April – May 2012 **Time Allowed: 2½ hours**

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FOUR (4)** pages.
2. Answer **ALL FOUR (4)** questions.
3. All questions carry equal marks.
4. An Appendix of **ONE (1)** page is attached together with this paper.

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1. A pin-jointed truss is subjected to two-point loads as shown in Figure Q1. The truss is pinned at G and supported by a roller at F.

   (a) Determine its determinacy and stability. (3 marks)

   (b) Calculate all the reactions. (2 marks)

   (c) Calculate the internal forces for members AD and BD. (20 marks)

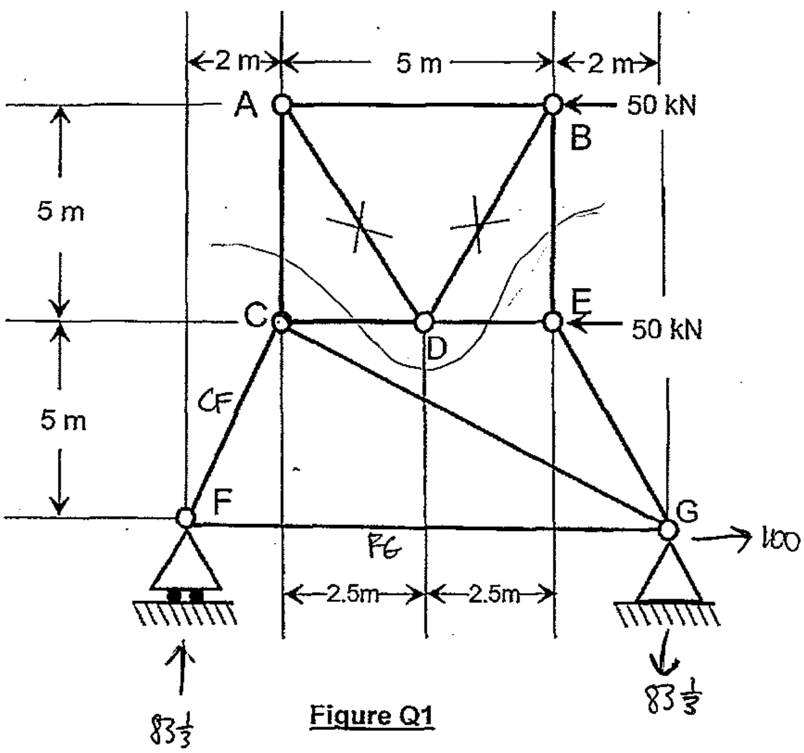


Figure Q1

2. The frame ABCDEF shown in Figure Q2 is pin-supported at A and F, and hinged at C. The column AB is subjected to a uniformly distributed load of 4 kN/m. A point load of 12 kN is applied at D, as shown in Figure Q2.

(a) Determine its determinacy and stability. (3 marks)

(b) Calculate all the reactions. (8 marks)

(c) Draw the bending moment and shear force diagrams. (7 marks)

(d) Calculate the bending moments and shear forces at the points A, B, C, D, E and F. (7 marks)

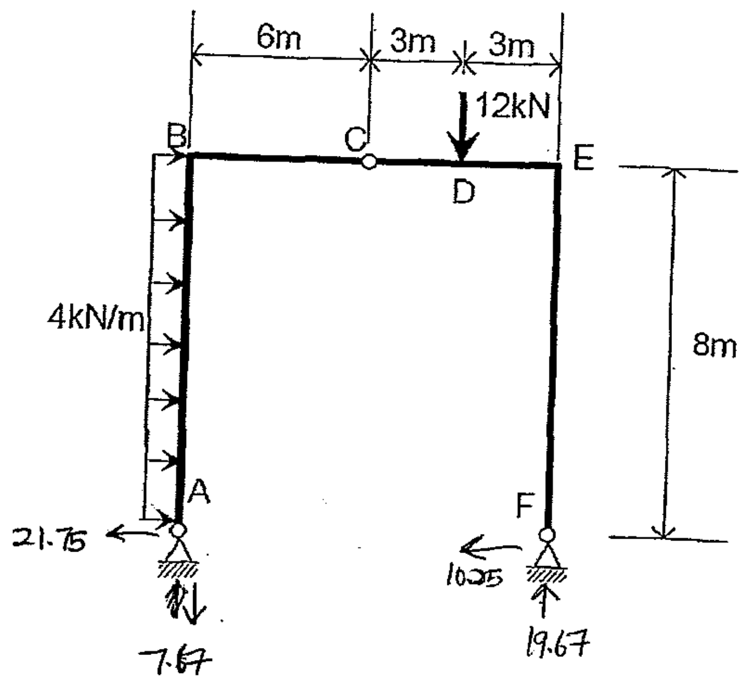


Figure Q2

3. (a) As shown in Figure Q3(a), a beam ABCD with constant EI is simply supported at A and pinned at D. A uniformly distributed load of 10 kN/m is applied along the span AB. A 10 kN downward point load is applied at B while a 20 kN upward point load is applied at C. A clockwise point moment equal to $18M_0$ kNm is applied at D.

   (i) Calculate the reaction at A in terms of M_0.

   (ii) By using the **Macaulay's Method**, derive the deflection curve for the beam in terms of x, M_0 and EI.

   (iii) Determine the value of M_0 so that the rotation of the beam at B is zero.

(15 marks)

EI constant for the whole beam

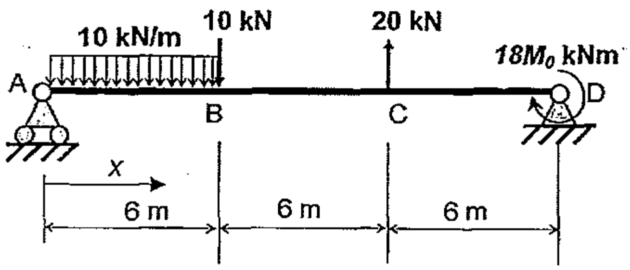


Figure Q3(a)

(b) A simply supported beam ABC with constant EI = 9,000 kNm^2 is shown in Figure Q3(b). An anti-clockwise point moment of 20 kNm is applied at end A while an anti-clockwise point moment of 10 kNm is applied at end C. By using the **Moment Area Method**, compute the rotation at A and the deflection at B.

(10 marks)

EI = 9,000 kNm^2 for the whole beam

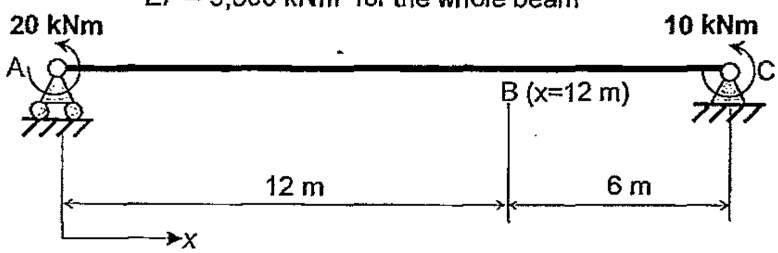


Figure Q3(b)

4. As shown in Figure Q4, a cantilever beam AB is connected to a pin-jointed truss at B. The pin-jointed truss is simply supported at B and pinned at G. It is subjected to a 30 kN point load at joint C and joint F. The flexural rigidity of the beam AB is equal to 200,000 kNm^2. The EA values of all the truss members are equal to 50,000 kN.

   (a) Calculation the reactions at B and G for the truss.

   (3 marks)

   (b) Use the **Method of Virtual Work** to calculate the vertical deflection at B for the beam.

   (7 marks)

   (c) Use the **Method of Virtual Work** and the **Method of Superposition** to calculate the vertical deflection at C.

   (15 marks)

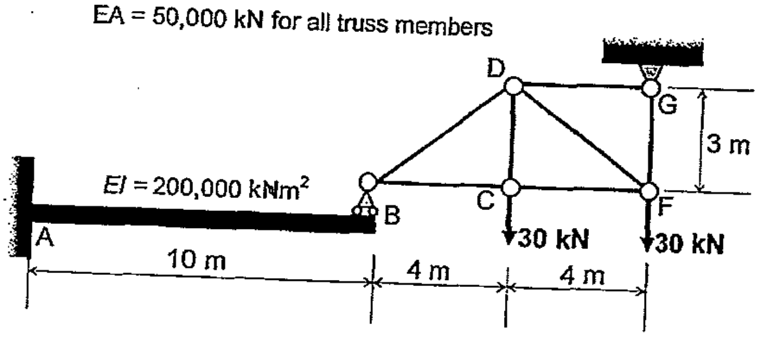
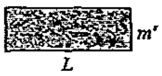
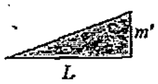
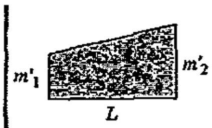
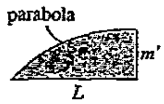
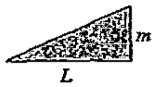
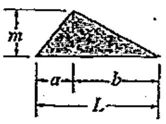
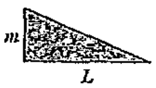


Figure Q4

END OF PAPER

Appendix: Values of Product Integrals $\int_0^L mm' dx$

$\int_0^L m m' dx$				
	$mm'L$	$\frac{1}{2}mm'L$	$\frac{1}{2}m(m'_1 + m'_2)L$	$\frac{2}{3}mm'L$
	$\frac{1}{2}mm'L$	$\frac{1}{3}mm'L$	$\frac{1}{6}m(m'_1 + 2m'_2)L$	$\frac{5}{12}mm'L$
	$\frac{1}{2}m'(m_1 + m_2)L$	$\frac{1}{6}m'(m_1 + 2m_2)L$	$\frac{1}{6}[m'_1(2m_1 + m_2) + m'_2(m_1 + 2m_2)]L$	$\frac{1}{12}[m'(3m_1 + 5m_2)]L$
	$\frac{1}{2}mm'L$	$\frac{1}{6}mm'(L + a)$	$\frac{1}{6}m_1[m'_1(L + b) + m'_2(L + a)]$	$\frac{1}{12}mm'\left(3 + \frac{3a}{L} - \frac{a^2}{L^2}\right)L$
	$\frac{1}{2}mm'L$	$\frac{1}{6}mm'L$	$\frac{1}{6}m(2m'_1 + m'_2)L$	$\frac{1}{4}mm'L$

Fong Kah Jun.
Yes, U Can!

CV2102 - Structures I (2011-2012) S2

1\. a) $r = 3$, $r_a = 3$

$\therefore r = r_a$, static determine

$m = 11$, $r = 3$, $j = 7$

$m + r = 2j$, internally static determine

$m_a = 11$ $r_a = 3$, $2j = 14$

$m_a + r_a = 2j$, overall static determine

b) $\sum M_G = 0$,

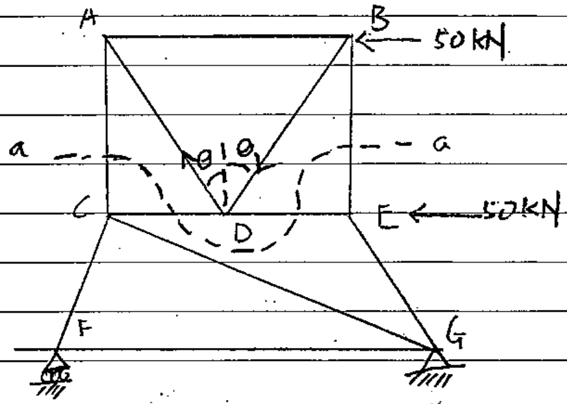
$F_y(9) = 50(5) + 50(10)$

$F_y = 83.33\,kN$

$\sum F_y = 0$ $\qquad$ $\sum F_x = 0$

$G_y = F_y$ $\qquad$ $G_x = 50 + 50$

$= 83.33\,kN$ $\qquad$ $= 100\,kN$

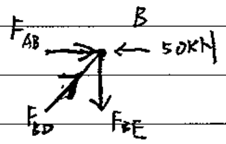


c) $\sum M_C = 0$

$F_{BE}(5) = 5(50)$

$F_{BE} = 50\,kN$

Joint B:



$\sum F_y = 0$

$F_{BD}\left(\frac{5}{\sqrt{5^2 + 2.5^2}}\right) = F_{BE}$

$F_{BD} = 50\left(\frac{\sqrt{5^2+2.5^2}}{5}\right)$

$= 55.9\,kN$

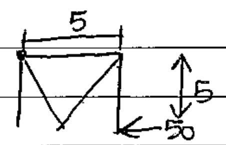
Joint D:

$\sum F_y = 0$

$F_{AD}\cos\theta = F_{BD}\cos\theta$

$F_{AD} = 55.9\,kN$

Taking moment abt A?



$M_A = 0$

(+ve $5(50) = BE(5)$

$BE_y = 50$

2\. a) $r = 3 + 1$ $\qquad$ $r_a = 4$

$= 4$

$r = r_a$, static determine.

Yes, U Can!

b) $\sum M_A = 0$

$4\times8\times4 + 9\times12 - 12\times F_y = 0$

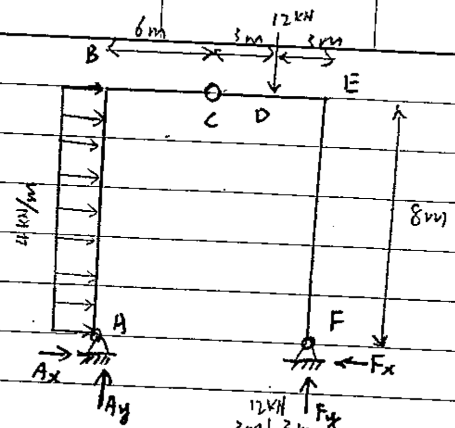
$F_y = 19.67\,kN$

$\sum M_c = 0$

$12\times3 + 8F_x - 6\times F_y = 0$

$8F_x = 6(19.67) - 12\times3$

$F_x = 10.25\,kN$



$\sum F_y = 0$

$A_y + F_y = 12$

$A_y = 12 - 19.67$

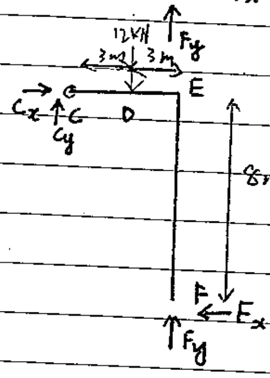
$= -7.67\,kN$

$\sum F_x = 0$

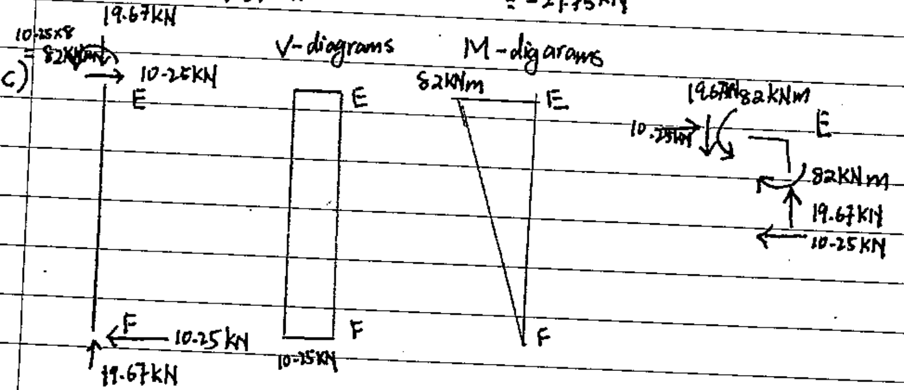
$A_x + 4\times8 = F_x$

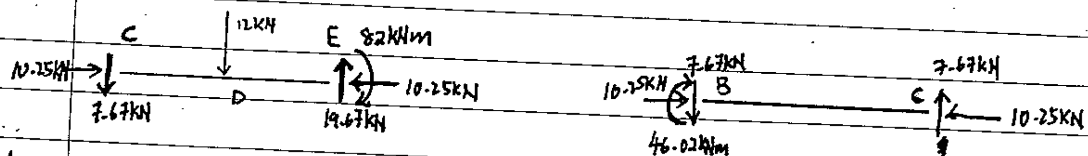
$A_x = 10.25 - 32$

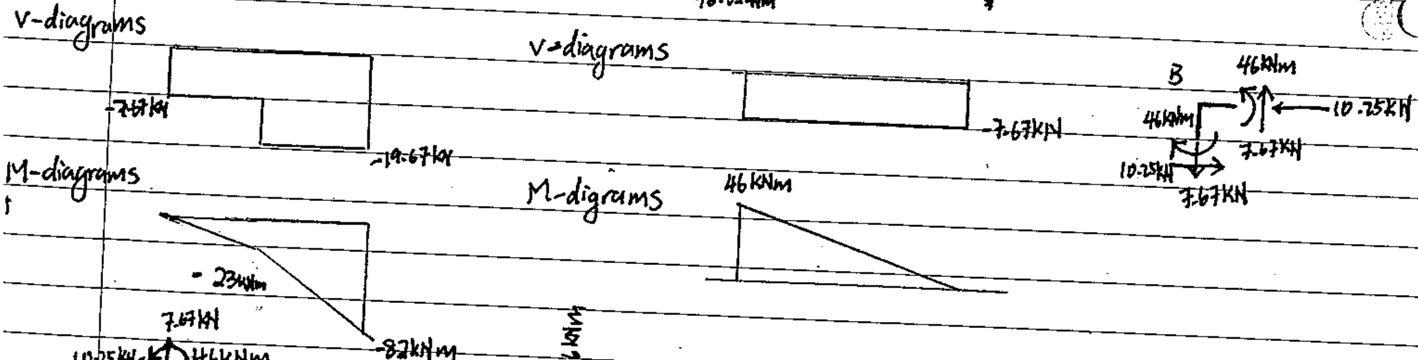
$= -21.75\,kN$

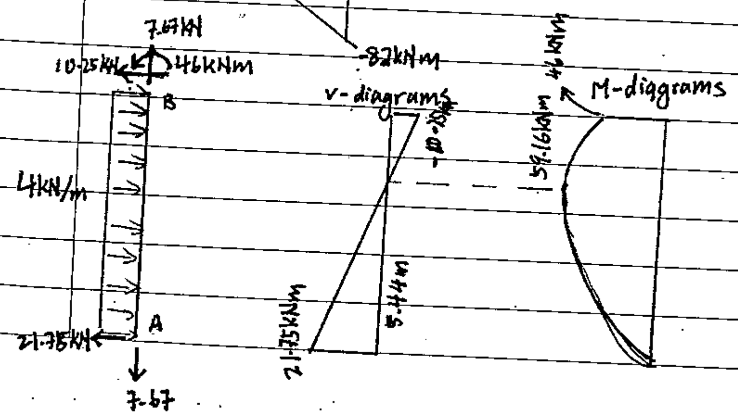


c)









Yes, U Can!

d) $M_A = 0$, $V_A = 21.75$ kN

$M_B = 46$ kNm, $V_B = -10.25$ kN or $V_B = -7.67$ kN

$M_C = 0$, $V_C = -7.67$ kN

$M_D = -23$ kNm, $V_{DL} = -7.67$ kN, $V_{DR} = -19.67$ kN

$M_E = -82$ kNm, $V_E = -19.67$ kN or $V_E = 10.25$ kN

$M_F = 0$; $V_F = 10.25$ kN.

3. i) $\Sigma M_D = 0$

$18A_y + 18M_0 + 20\times 6 = 6\times 10\times 15 + 10\times 12$

$18A_y = 900 - 18M_0$

$A_y = 50 - M_0$ $\qquad D_y$?

ii) $M = A_y x - \frac{10}{2}\langle x\rangle^2 - 10\langle x-6\rangle + \frac{10}{2}\langle x-6\rangle^2 + 20\langle x-12\rangle + 18M_0\langle x-18\rangle^0$

$= (50-M_0)x - 5x^2 - 10\langle x-6\rangle + 5\langle x-6\rangle^2 + 20\langle x-12\rangle + 18M_0\langle x-18\rangle^0$

$\frac{d^2v}{dx^2} = \frac{M}{EI}$

$= \frac{1}{EI}[(50-M_0)x - 5x^2 - 10\langle x-6\rangle + 5\langle x-6\rangle^2 + 20\langle x-12\rangle + 18M_0\langle x-18\rangle^0$

$EI\frac{dv}{dx} = \frac{(50-M_0)}{2}x^2 - \frac{5}{3}x^3 - \frac{10}{2}\langle x-6\rangle^2 + \frac{5}{3}\langle x-6\rangle^3 + \frac{20}{2}\langle x-12\rangle^2 + 18M_0\langle x-18\rangle^1 + C_1$

$= \frac{50-M_0}{2}x^2 - \frac{5}{3}x^3 - 5\langle x-6\rangle^2 + \frac{5}{3}\langle x-6\rangle^3 + 10\langle x-12\rangle^2 + 18M_0\langle x-18\rangle + C_1$

$EIV = \frac{50-M_0}{6}x^3 - \frac{5}{12}x^4 - \frac{5}{3}\langle x-6\rangle^3 + \frac{5}{12}\langle x-6\rangle^4 + \frac{10}{3}\langle x-12\rangle^3 + 9M_0\langle x-18\rangle^2 + C_1x + C_2$

when $x=0$, $v=0$

$C_2 = 0$,

when $x=18$, $v=0$

$0 = \frac{50-M_0}{6}(18)^3 - \frac{5}{12}(18)^4 - \frac{5}{3}(12)^3 + \frac{5}{12}(12)^4 + \frac{10}{3}(6)^3 + 0 + 18C_1$

$0 = 48600 - 972M_0 - 43740 - 2880 + 8640 + 720 + 18C_1$

$0 = 11340 - 972M_0 + 18C_1$

$C_1 = 54M_0 - 630$

$\therefore EIV = \frac{50-M_0}{6}x^3 - \frac{5}{12}x^4 - \frac{5}{3}\langle x-6\rangle^3 + \frac{5}{12}\langle x-6\rangle^4 + \frac{10}{3}\langle x-12\rangle^3 + 9M_0\langle x-18\rangle^2 + (54M_0 - 630)x$

$V = \frac{1}{EI}[\frac{50-M_0}{6}x^3 - \frac{5}{12}x^4 - \frac{5}{3}\langle x-6\rangle^3 + \frac{5}{12}\langle x-6\rangle^4 + \frac{10}{3}\langle x-12\rangle^3 + 9M_0\langle x-18\rangle^2 + (54M_0-630)x]$

iii) $EI\theta = \frac{50-M_0}{2}x^2 - \frac{5}{3}x^3 - 5\langle x-6\rangle^2 + \frac{5}{3}\langle x-6\rangle^3 + 10\langle x-12\rangle^2 + 18M_0\langle x-18\rangle + 54M_0 - 630$

when $x=6$, $\theta=0$

$0 = \frac{50-M_0}{2}(6)^2 - \frac{5}{3}(6)^3 - 0 + 0 + 0 + 0 + 54M_0 - 630$

$= 900 - 18M_0 - 360 + 54M_0 - 630$

$36M_0 = 90$

$M_0 = 2.5$ kNm

b) $\sum M_c = 0$

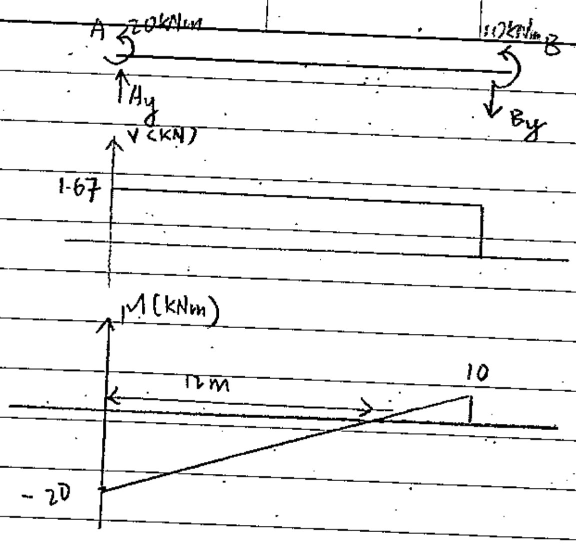
$18A_y = 30$

$A_y = 1.67$ kN

$\sum F_y = 0$

$B_y = A_y$

$= 1.67$ kN



$t_{c/A} = \frac{1}{EI}[(-20)(12)(\frac{1}{2})(4) + 6(10)(\frac{1}{2})(16)]$

$= 0$

$\theta_A = \frac{t_{c/A}}{18}$

$= 0$

$t_{B/A} = \frac{1}{EI}[(-20)(12)(\frac{1}{2})(4)]$

$= \frac{480}{9000}$

$= 0.053$ m

$= 53.33$ mm

4a) $\sum M_G = 0$

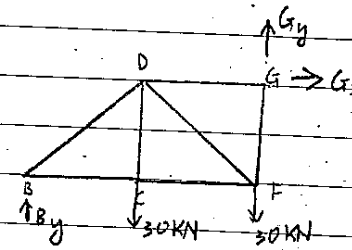
$8B_y = 4 \times 30$

$B_y = 15$ kN

$\sum F_y = 0$ $\quad\quad$ $\sum F_x = 0,$

$B_y + G_y = 60$ $\quad\quad$ $G_x = 0$

$G_y = 45$ kN



b) $\sum M_A = 0,$

$M_A = 15 \times 10$

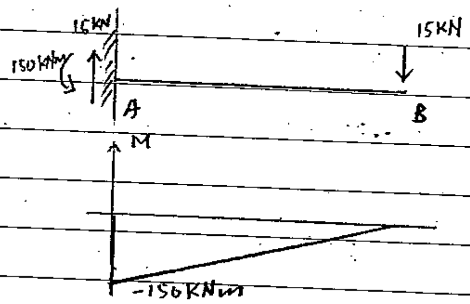
$= 150$ kNm

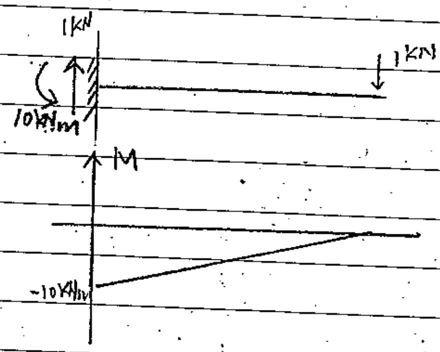
$\Delta_B = \int \frac{Mm}{EI} dx$

$= \frac{1}{EI}(\frac{1}{3})(-150)(10)(10)$

$= \frac{5000}{200{,}000}$

$= 0.025$ m





real

Joint G:

4(c) $F_{DG} = 0$

$F_{GF} = 45\text{ kN}$ (T)

Joint B:

$F_{BD}\sin\theta = 15$

$F_{BD} = 15\left(\frac{5}{3}\right)$

$= 25\text{ kN}$ (C)

$F_{BC} = F_{BD}\cos\theta$

$= 25\left(\frac{4}{5}\right)$

$= 20\text{ kN}$ (T)

Joint D:

$F_{BD}\cos\theta = F_{DF}\cos\theta$

$F_{BD} = F_{DF}$

$F_{DF} = 25\text{ kN}$ (C)

Joint C:

$F_{DC} = 30\text{ kN}$ (T)

$F_{FC} = F_{BC}$

$= 20\text{ kN}$ (T)

virtual

$\Sigma M_G = 0$,

$8B_y = 4(1)$

$B_y = 0.5\text{ kN}$

$\Sigma F_y = 0$ $\quad\Sigma F_x = 0$,

$G_y = 1 - 0.5$ $\quad G_x = 0$

$= 0.5\text{ kN}$

Joint G:

$F_{GF} = 0.5\text{ kN}$ (T)

$F_{DG} = 0$

Joint B:

$F_{BD}\sin\theta = 0.5$

$F_{BD} = 0.5\left(\frac{5}{3}\right)$

$= 0.83\text{ kN}$ (C)

$F_{BC} = F_{BD}\cos\theta$

$= 0.83\left(\frac{4}{5}\right)$

$= 0.66\text{ kN}$

Joint C:

$F_{DC} = 1\text{ kN}$ (T)

$F_{CF} = F_{BC} = 0.66\text{ kN}$ (T)

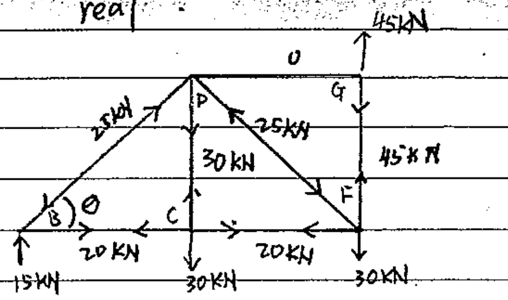
Joint D:

$F_{BD}\cos\theta = F_{DF}\cos\theta$

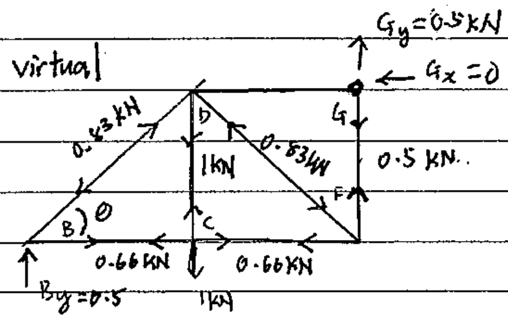
$F_{DF} = F_{BD}$

$= 0.83\text{ kN}$ (C)

real



virtual



Beams	N	n	L	nNL
BC	20	0.66	4	52.8
BD	-25	-0.83	5	103.75
DC	30	1	3	90
CF	20	0.66	4	52.8
DF	-25	-0.83	5	103.75
DG	0	0	4	0
GF	45	0.5	3	67.5
			Σ	470.6

$\Delta_c' = \frac{nNL}{EA}$

$= \frac{470.6}{50000}$

$= 0.0094\text{ m}$

$\Delta_c = \frac{1}{2}\Delta_B + \Delta_c'$

$= \frac{1}{2}(0.025) + 0.0094$

$= 0.0219\text{ m}$