NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2010-2011

Hendric

CV2102 – STRUCTURES I

May 2011

Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FIVE (5)** pages.
2. Answer **ALL FOUR (4)** questions.
3. All questions carry equal marks.
4. An Appendix of **ONE (1)** page is attached together with this paper.
5. This is an Open Book Examination with restriction to only **ONE (1)** sheet of A4 size paper containing any reference materials.

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1. A pin-jointed truss is subjected to three point loads as shown in Figure Q1.

   (a) Determine its external and overall determinacy. (3 marks)

   (b) Calculate the reactions at all supports. (2 marks)

   (c) Calculate the internal forces of members CJ and BD. (20 marks)

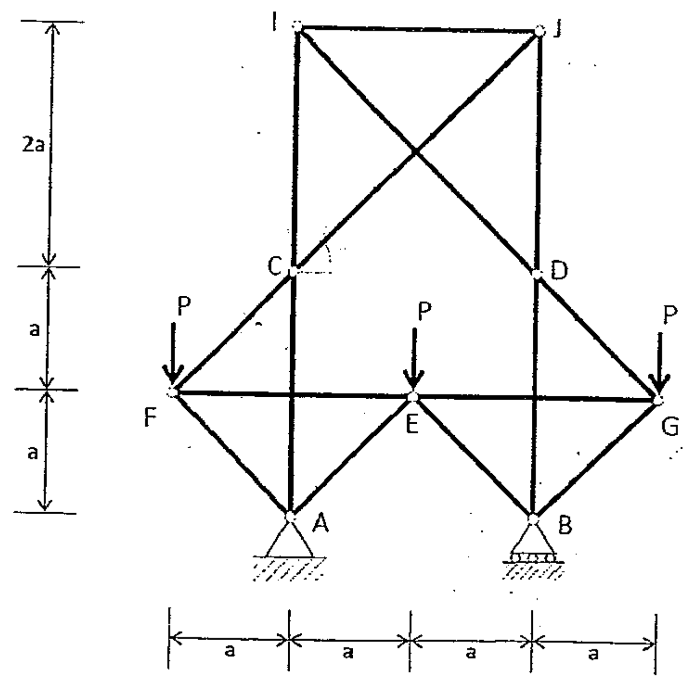


Figure Q1

2. A pin-jointed beam as shown in Figure Q2 is pin-supported at B, and supported by rollers at D,H,I and K. The beam is subjected to two sets of uniformly distributed loading of 20 kN/m at sections ABC and JKL. Two point loads of 20 kN are applied at E and F respectively, as shown in Figure Q2.

(a) Determine its overall determinacy.

(2 marks)

(b) Calculate the reactions at all supports.

(8 marks)

(c) Draw the bending moment in the **TENSION SIDE** of the beam. Indicate the values of the bending moments at points A, B, C, D, E ,F, G, H, I, J, K and L.

(15 marks)

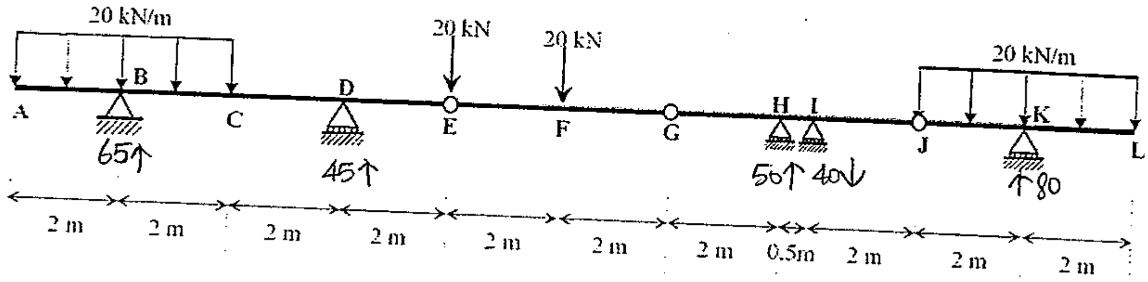


Figure Q2

3. (a) As shown in Figure Q3(a), a beam *ABCD* with constant *EI* is simply supported at *A* and pinned at *C*. A 24 kN downward point load is applied at *B*. An anti-clockwise point moment equal to $8M_0$ kNm is applied at *C*. A downward uniformly distributed loading of 24 kN/m is applied between *C* and *D*.

(i) Using the **Macaulay's Method** to derive the deflection curve for the beam in terms of *x*, M_0 and *EI*.

(ii) Determine the value of M_0 so that the rotation of the beam at *A* is zero.

(15 marks)

EI constant for the whole beam

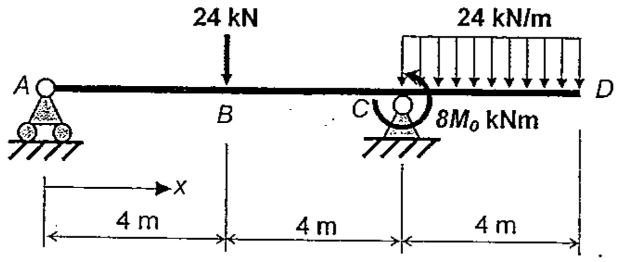


Figure Q3(a)

(b) A beam *ABC* with constant *EI*=100,000 kNm^2 is simply supported at the end *A* and pinned at the end *C* is shown in Figure Q3(b). A 100 kNm anti-clockwise point moment is applied at the end *A*. A 100 kNm clockwise point moment is applied at the point *B* and the end *C*. Use the **Moment Area Method** to compute the rotation at *A* and the deflection at *B*.

(10 marks)

EI =100,000 kNm^2

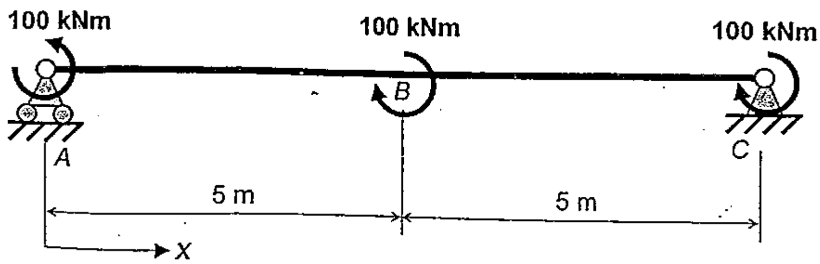


Figure Q3(b)

4. (a) A pin-jointed truss is subjected to a 120 kN point load at joint *F* and joint *G* as shown in Figure Q4(a). The *EA* values of all members are equal to 200,000 kN. Use the **Method of Virtual Work** to compute the vertical deflection at *C*.

(12 marks)

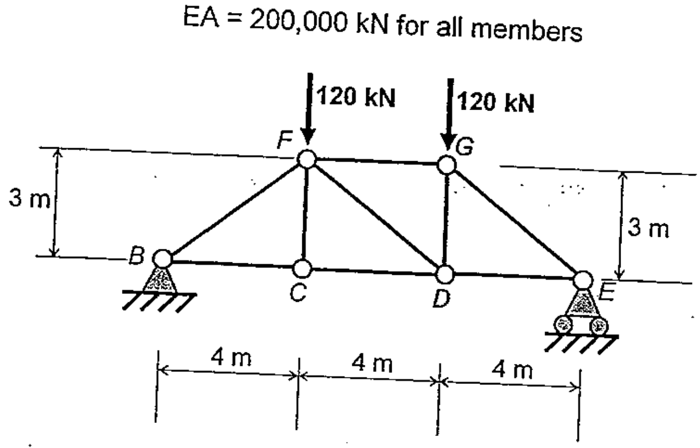


Figure Q4(a)

(b) A cantilever beam *ABC* of length *2L* is subjected to a point load *P* at the free end *C* as shown in Figure Q4(b). The flexural rigidity of the beam is equal to *2EI* and *EI* for the span *AB* and the span *BC*, respectively. Use the **Method of Virtual Work** to determine the vertical deflection at the free end *C* in terms of *L*, *EI* and *P*.

(7 marks)

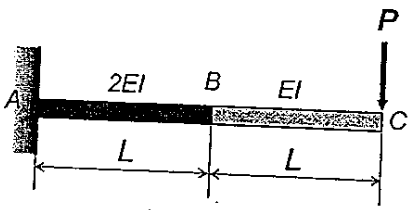


Figure Q4(b)

Note: Question No. 4 continues on page 5

(c) By using the results from parts (a) and (b) and the **Method of Superposition**, determine the vertical deflection at the joint H for the structure shown in Figure Q4(c).

(6 marks)

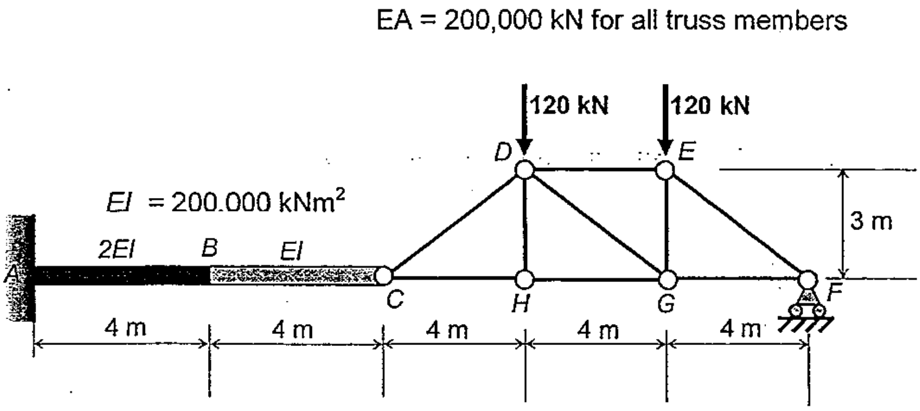
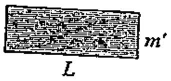
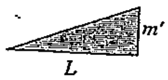
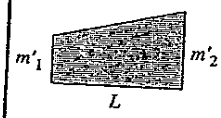
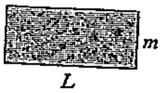
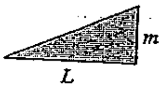
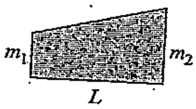
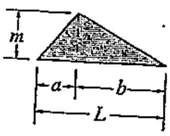
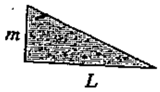


Figure Q4(c)

END OF PAPER

Appendix: Values of Product Integrals $\int_0^L mm'dx$

$\int_0^L m\,m'\,dx$				
	$mm'L$	$\frac{1}{2}mm'L$	$\frac{1}{2}m(m_1' + m_2')L$	$\frac{2}{3}mm'L$
	$\frac{1}{2}mm'L$	$\frac{1}{3}mm'L$	$\frac{1}{6}m(m_1' + 2m_2')L$	$\frac{5}{12}mm'L$
	$\frac{1}{2}m'(m_1 + m_2)L$	$\frac{1}{6}m'(m_1 + 2m_2)L$	$\frac{1}{6}[m_1'(2m_1 + m_2)$ $+ m_2'(m_1 + 2m_2)]L$	$\frac{1}{12}[m'(3m_1 + 5m_2)]L$
	$\frac{1}{2}mm'L$	$\frac{1}{6}mm'(L + a)$	$\frac{1}{6}m_1[m_1'(L + b) +$ $m_2(L + a)]$	$\frac{1}{12}mm'\left(3 + \frac{3a}{L} - \frac{a^2}{L^2}\right)L$
	$\frac{1}{2}mm'L$	$\frac{1}{6}mm'L$	$\frac{1}{6}m(2m_1' + m_2')L$	$\frac{1}{4}mm'L$

1a External determinacy: $r = 3$, $n = 3$

$r = n$ (statically determinate)

Overall determinacy: $b = 15$, $j = 9$, $r = 3$

$b + r = 2j$

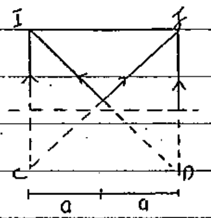
$15 + 3 = 2 \times 9$

$18 = 18$ (Statically determinate)

b $+\circlearrowleft \Sigma M_A = 0$ $\quad +\uparrow \Sigma F_y = 0$ $\quad \xrightarrow{+} \Sigma F_x = 0$

$2a B_y - \cancel{Pa} - 3Pa + \cancel{Pa} = 0$ $\quad A_y + B_y - 3P = 0$ $\quad A_x = 0$

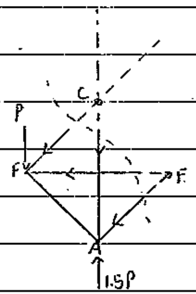
$B_y = \frac{3}{2}P$ $\quad A_y = 1.5P$

c 

$+\circlearrowleft \Sigma M_D = 0$

$-2aCI - \frac{1}{2}\sqrt{2}\,CJ \cdot 2a = 0$

$CI + \frac{1}{2}\sqrt{2}\,CJ = 0 \ldots (1)$



Joint C: $\xrightarrow{+} \Sigma F_x = 0$ $\quad +\uparrow \Sigma F_y = 0$

$CJ = CF$ $\quad CI = CA$

$+\circlearrowleft \Sigma M_E = 0$

$2aP + aCA + 2aCF \cdot \frac{1}{2}\sqrt{2} - 1.5Pa = 0$

$CA + \sqrt{2}\,CF = -0.5P$

$CI + \sqrt{2}\,CJ = -0.5P \ldots (2)$

$(1) - (2)$: $-\frac{1}{2}\sqrt{2}\,CJ = \frac{1}{2}P$

$CJ = -\frac{1}{2}\sqrt{2}\,P$

$CJ = \frac{1}{2}\sqrt{2}\,P$ (T)

Since the system is symmetrical, $BD = CA = CI$

$CI + \sqrt{2}(-\frac{1}{2}\sqrt{2}P) = -0.5P$

$CC = \frac{1}{2}P$ (C)

$BD = \frac{1}{2}P$ (C)

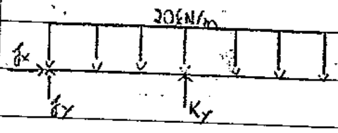
Yes, U can!

(Prof Li Bing's method)

2a $r:6$, $j:3$, $n:3$

$n+j = r$ (Statically determinate)

b Beam JKL

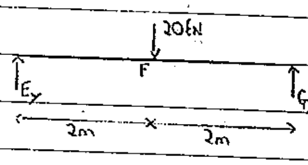


$\xrightarrow{+}\Sigma F_x = 0$ $\quad$ $+\circlearrowleft\Sigma M_J = 0$ $\quad$ $+\uparrow\Sigma F_y = 0$

$J_x = 0$ $\quad$ $2K_y - 20\times4\times2 = 0$ $\quad$ $J_y + K_y - 20\times4 = 0$

$K_y = 80kN$ $\quad$ $J_y = 0$

Beam EFG

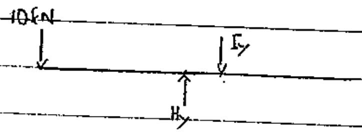


$E_x = G_x = 0$ $\quad$ $+\circlearrowleft\Sigma M_E = 0$ $\quad$ $F_y = G_y = 10kN$

$4G_y - 2\times20 = 0$

$G_y = 10kN$

Beam GHIJ

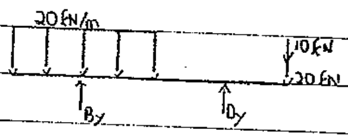


$+\circlearrowleft\Sigma M_H = 0$ $\quad$ $+\uparrow\Sigma F_y = 0$

$-0.5I_y + 2\times10 = 0$ $\quad$ $-10-40+H_y = 0$

$I_y = 40kN$ $\quad$ $H_y = 50kN$

Beam ABCDE

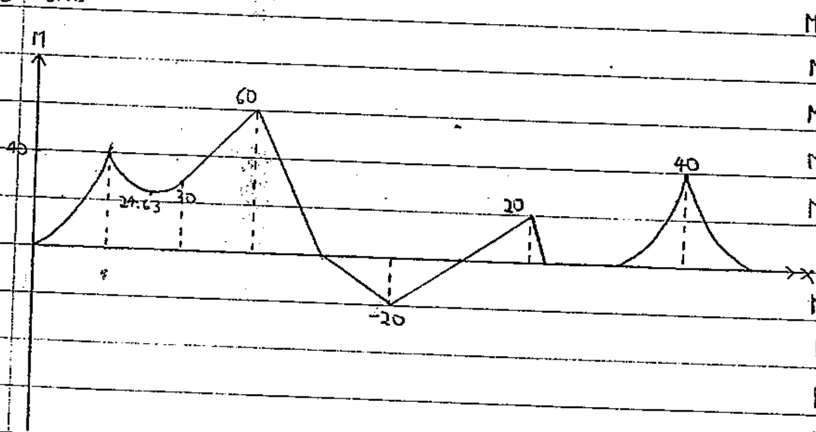


$+\circlearrowleft\Sigma M_B = 0$ $\quad$ $+\uparrow\Sigma F_y = 0$ $\quad$ $B_x = 0$

$4D_y - 6\times30 = 0$ $\quad$ $B_y + 45 - 10 - 20 - 20\times4 = 0$

$D_y = 45kN$ $\quad$ $B_y = 65kN$

c BMD



$M_A = 0$

$M_B = 40kNm$

$M_C = 30kN.m$

$M_D = 60kNm$

$M_E = 0$

$M_F = -20kNm$

$M_G = 0$

$M_H = 20kNm$

$M_I = 0$

$M_J = 0$

$M_K = 40kNm$

$M_L = 0$

3a(i) $+\circlearrowleft\Sigma M_C = 0$ $\qquad$ $+\uparrow\Sigma F_y = 0$

$-8A_y + 4\times24 + 8M_0 - 4\times24\times2 = 0$ $\qquad$ $M_0 - 12 - 24 + C_y - 24\times4 = 0$

$A_y = M_0 - 12$ $\qquad$ $C_y = 132 - M_0$

$EIM = (M_0-12)x - 24\langle x-4\rangle + (132-M_0)\langle x-8\rangle - 8M_0\langle x-8\rangle^0 - 12\langle x-8\rangle^2$

$EIv' = \frac{1}{2}(M_0-12)x^2 - 12\langle x-4\rangle^2 + \frac{1}{2}(132-M_0)\langle x-8\rangle^2 - 8M_0\langle x-8\rangle - 4\langle x-8\rangle^3 + C_1$

$EIv = \frac{1}{6}(M_0-12)x^3 - 4\langle x-4\rangle^3 + \frac{1}{6}(132-M_0)\langle x-8\rangle^3 - 4M_0\langle x-8\rangle^2 - \langle x-8\rangle^4 + C_1x + C_2$

$v(0) = 0$ $\qquad$ $EIv(8) = \frac{1}{6}(M_0-12)(8)^3 - 4(4)^3 + C_1\cdot 8$

$C_2 = 0$ $\qquad$ $0 = \frac{512}{6}(M_0-12) - 256 + C_1\cdot 8$

$C_1 = 160 - \frac{32}{3}M_0$

$\therefore\ v(x) = \frac{1}{EI}\left[\frac{1}{6}(M_0-12)x^3 - 4(x-4)^3 + \frac{1}{6}(132-M_0)(x-8)^3 - 4M_0(x-8)^2 - (x-8)^4 + 160 - \frac{32}{3}M_0\right]$

(ii) $EIv'(0) = 160 - \frac{32}{3}M_0 = 0$

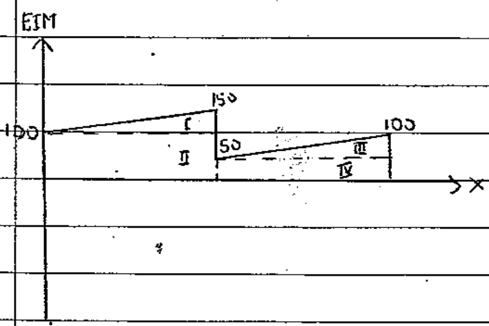
$M_0 = 15\,\text{kNm}$

b $+\circlearrowleft\Sigma M_A = 0$ $\qquad$ $+\uparrow\Sigma F_y = 0$

$100 - 100 - 100 + 10C_y = 0$ $\qquad$ $A_y = -10\,\text{kN}\ (\downarrow)$

$C_y = 10\,\text{kN}$

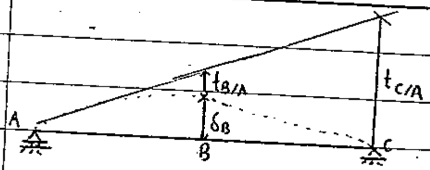
BMD



$M_1 = M_3 = \frac{1}{EI}\cdot\frac{1}{2}\times5\times50 = \frac{125}{EI}$

$M_2 = \frac{1}{EI}\times5\times100 = \frac{500}{EI}$

$M_4 = \frac{1}{EI}\times5\times50 = \frac{250}{EI}$



$t_{C/A} : \frac{125}{EI}(5+\frac{5}{3}) + \frac{125}{EI}(\frac{5}{3}) + \frac{500}{EI} \times 7.5 + \frac{250}{EI}(2.5) \ldots$

$: \frac{5416.67}{EI}$

$\theta_A : \frac{t_{C/A}}{10} , \frac{5416.67}{10EI}$ ~~$: 0.0054 \text{ rad}$~~

$t_{B/A} : \frac{125}{EI}(\frac{5}{3}) + \frac{500}{EI}(2.5) : \frac{1458.33}{EI}$

$\theta_A \cdot 5 : t_{B/A} + \delta_B$

$0.027 : \frac{1458.33}{EI} + \delta_B$

$\delta_B : 12.42 \text{mm}$

8

4a $B_y : E_y : 120 \text{ kN}$

$B_x : 0$

Zero force member : $CF : 0$

Joint B : $+\uparrow \Sigma F_y : 0$ $\quad \xrightarrow{+} \Sigma F_x : 0$ $\quad CD : BC : 160 \text{ kN (T)}$

$120 - BF(\frac{3}{5}) : 0$ $\quad BC - \frac{4}{5} BF : 0$

$BF : 200 \text{ kN (C)}$ $\quad BC : 160 \text{ kN (T)}$

Joint E : $+\uparrow \Sigma F_y : 0$ $\quad \xrightarrow{+} \Sigma F_x : 0$

$120 - GE(\frac{3}{5}) : 0$ $\quad -DE + GE(\frac{4}{5}) : 0$

$GE : 200 \text{ kN (C)}$ $\quad DE : 160 \text{ kN (T)}$

Joint D : $\xrightarrow{+} \Sigma F_x : 0$ $\quad +\uparrow \Sigma F_y : 0$

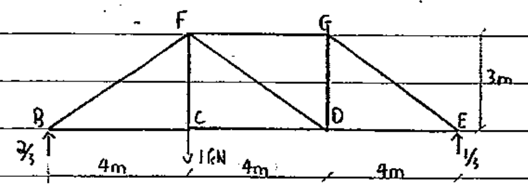
$DE - CD + FD(\frac{4}{5}) : 0$ $\quad DG : 0$

$FD : 0$

Joint G : $\xrightarrow{+} \Sigma F_x : 0$

$FG - GE(\frac{4}{5}) : 0$

$FG : 160 \text{ kN (C)}$



CF = 1kN

Joint B: $+\uparrow \Sigma F_y = 0$ $\quad \overset{+}{\rightarrow} \Sigma F_x = 0$ $\quad CD = BC = \frac{8}{9}$ kN (T)

$\frac{3}{5}BF = \frac{2}{3}$ $\quad BC = \frac{10}{9} \times \frac{4}{5}$

$BF = \frac{10}{9}$ kN (C) $\quad BC = \frac{8}{9}$ kN (T)

Joint E: $+\uparrow \Sigma F_y = 0$ $\quad \overset{+}{\rightarrow} \Sigma F_x = 0$

$\frac{3}{5}GE = \frac{1}{3}$ $\quad DE = \frac{5}{9} \times \frac{4}{5}$

$GE = \frac{5}{9}$ kN (C) $\quad DE = \frac{4}{9}$ kN (T)

Joint D: $\overset{+}{\rightarrow} \Sigma F_x = 0$ $\quad +\uparrow \Sigma F_y = 0$

$FD(\frac{4}{5}) + \frac{4}{9} - \frac{8}{9} = 0$ $\quad GD = \frac{5}{9} \times \frac{3}{5}$

$FD = \frac{5}{9}$ kN (C) $\quad GD = \frac{1}{3}$ kN (T)

Joint G: $\overset{+}{\rightarrow} \Sigma F_x = 0$

$FG = \frac{5}{9} \times \frac{4}{5}$

$FG = \frac{4}{9}$ kN (C)

Beam	n (kN)	N (kN)	L (m)	nNL (kN²m)
BC	8/9	160	4	568.89
BF	-10/9	-200	5	1111.11
CD	8/9	160	4	568.89
FD	-5/9	0	5	0
FG	-4/9	-160	4	284.44
DG	1/3	0	3	0
DE	4/9	160	4	284.44
GE	-5/9	-200	5	555.56
CF	1	0	3	0
			Σ =	3373.33

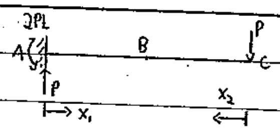
$1 \cdot \Delta_{y_c} = \frac{\Sigma nNL}{AE}$

$\Delta_{y_c} = \frac{3373.33}{200{,}000}$

$\Delta_{y_c} = 0.01687$ m

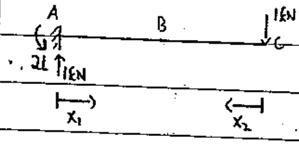
$\Delta_{y_c} = 16.87$ mm

b



$M_1 = Px - 2PL$

$M_2 = -Px$



$m_1 = x - 2L$

$m_2 = -x$

$$1\cdot\Delta_{y_C} = \int_0^L \frac{(Px-2PL)(x-2L)\,dx}{2EI} + \int_0^L \frac{(-Px)(-x)\,dx}{EI}$$

$$\Delta_{y_C} = \int_0^L \frac{(Px^2-4PLx+4PL^2)\,dx}{2EI} + \int_0^L \frac{Px^2\,dx}{EI}$$

$$\Delta_{y_C} = \frac{\frac{1}{3}Px^3 - 2PLx^2 + 4PL^2x}{2EI}\Bigg|_0^L + \frac{Px^3}{3EI}\Bigg|_0^L$$

$$\Delta_{y_C} = \frac{3PL^3}{2EI}\ \cancel{m}$$

c For Beam ABC : $P = 120$ kN ; $L = 4$ m

$$\Delta_H = 16.87 + \frac{3PL^3}{2EI}$$

$$= 16.87 + \frac{3(120)(4)^3}{2\times 200{,}000}\ \cancel{\times 10^3\ mm}$$

$= 16.87 + 57.6$

$= 74.47$ mm