NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2009-2010

CV2102 – STRUCTURES I

April – May 2010 **Time Allowed: 2½ hours**

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **FOUR (4)** pages.
2. Answer **ALL FOUR (4)** questions.
3. All questions carry equal marks.
4. An Appendix of **ONE (1)** page is attached together with this paper.
5. This is an open book examination restricted to **ONE (1)** sheet of A4 size paper containing any reference materials.

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1. A pin-jointed truss is subjected to two-point loads as shown in Figure Q1. The truss is pinned at A and supported by a roller at B.

   (a) Determine its determinacy and stability. (3 marks)

   (b) Identify all zero force members. (6 marks)

   (c) Calculate all the reactions. (6 marks)

   (d) Calculate internal forces for members CD, HJ, DH, AH and DE. (10 marks)

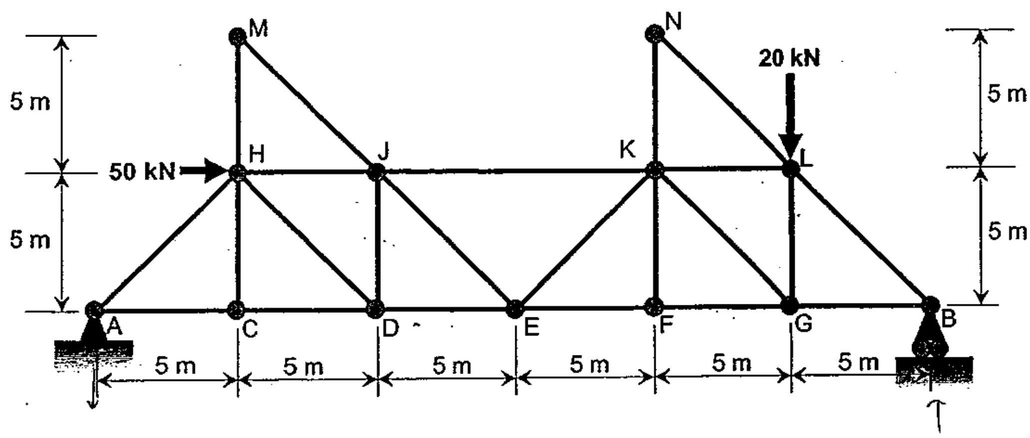


Figure Q1

2. (a) The beam ABCDE shown in Figure Q2(a) is supported by a roller at C and pinned at E. An anti-clockwise point moment **M** is applied at B. Determine the value of **M** so that the bending moment at D is zero. Draw the corresponding shear force and bending moment diagrams for the beam, indicating all critical values.

(10 marks)

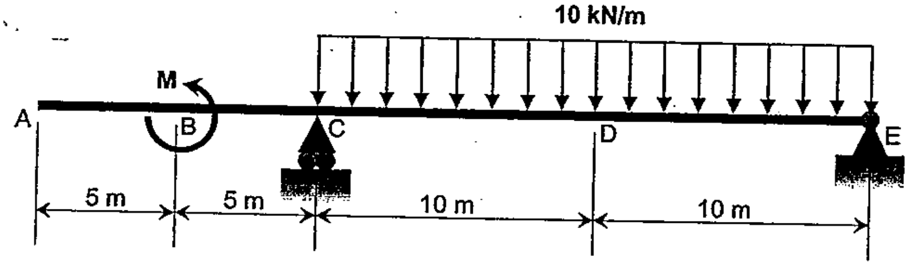


Figure Q2(a)

(b) A frame ABCDEF is fixed at A, supported by a roller at F and has an internal hinge at D as shown in Figure Q2(b). A horizontal point load and a vertical point load are applied at B and D, respectively. A clockwise moment is applied at E. Determine the reactions at the supports A and F. Draw the axial force, shear force and bending moment diagrams of the frame, indicating all critical values.

(15 marks)

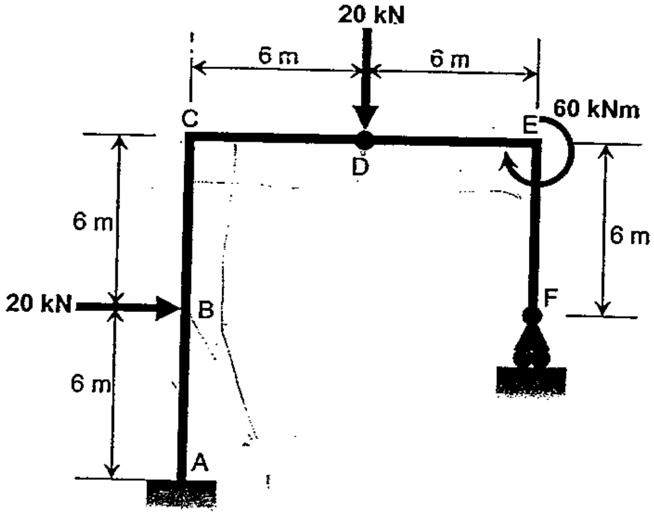
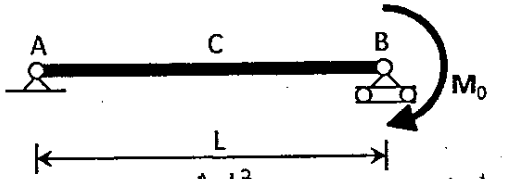


Figure Q2(b)

3. (a) Derive the equation of the deflection curve for the simple beam AB loaded by a moment $\mathbf{M_0}$ at the right-hand support, as shown in Figure Q3(a). The beam has constant flexural rigidity EI. Also, determine the deflection at the midpoint C and the angle of rotation at support B.

(12 marks)

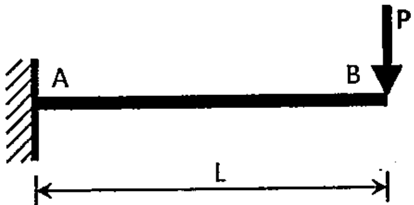


$V = \frac{M_0 L^2}{16EI}$ $\quad V' = -\frac{M_0 L}{3EI}$

Figure Q3(a)

(b) Using the moment-area method, determine the deflection at the free end B of the cantilever beam shown in Figure Q3(b). The beam has constant flexural rigidity EI and is loaded by point load **P** at the free end.

(6 marks)



$V = \frac{PL^3}{3EI}$

Figure Q3(b)

(c) Using the results obtained in part (a) and part (b) above, determine the deflection at point C of the structure shown in Figure Q3(c). The beam ABC has constant flexural rigidity EI and member BD is assumed to be axially rigid.

(7 marks)

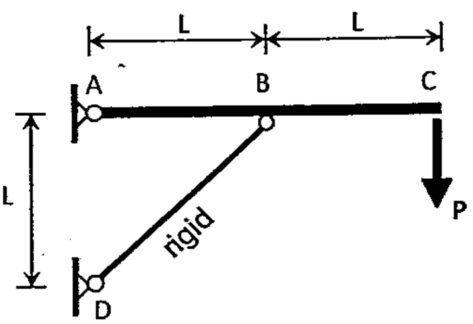


Figure Q3(c)

4. Using the virtual-work method, compute the vertical deflection at point B of the structure shown in Figure Q4. The members are made of steel with a modulus of elasticity of 200×10^9 N/m^2. Beam ABC has a cross sectional area of 2580 mm^2 and a moment of inertia (second moment of area) of 6.656×10^7 mm^4. Truss member CD has a cross sectional area of 1935 mm^2. Consider the strain energy produced by both axial and flexural deformations.

Note: you may use the product integrals table shown in the appendix, but it may be easier to integrate analytically.

(25 marks)

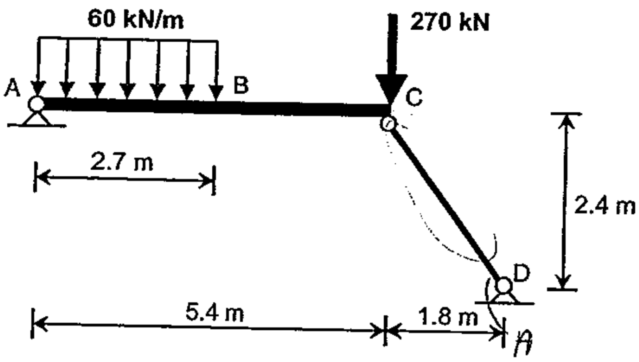


Figure Q4

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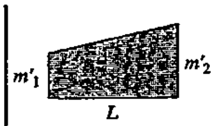
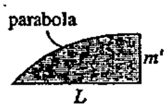
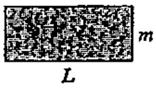
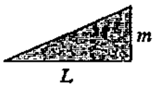
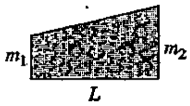
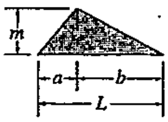
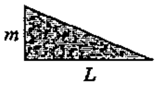
Appendix: Values of Product Integrals $\int_0^L mm'dx$

$\int_0^L m\,m'\,dx$				
	$mm'L$	$\frac{1}{2}mm'L$	$\frac{1}{2}m(m_1' + m_2')L$	$\frac{2}{3}mm'L$
	$\frac{1}{2}mm'L$	$\frac{1}{3}mm'L$	$\frac{1}{6}m(m_1' + 2m_2')L$	$\frac{5}{12}mm'L$
	$\frac{1}{2}m'(m_1 + m_2)L$	$\frac{1}{6}m'(m_1 + 2m_2)L$	$\frac{1}{6}[m_1'(2m_1 + m_2) + m_2'(m_1 + 2m_2)]L$	$\frac{1}{12}[m'(3m_1 + 5m_2)]L$
	$\frac{1}{2}mm'L$	$\frac{1}{6}mm'(L + a)$	$\frac{1}{6}m_1[m_1'(L + b) + m_2(L + a)]$	$\frac{1}{12}mm'\left(3 + \frac{3a}{L} - \frac{a^2}{L^2}\right)L$
	$\frac{1}{2}mm'L$	$\frac{1}{6}mm'L$	$\frac{1}{6}m(2m_1' + m_2')L$	$\frac{1}{4}mm'L$

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0910 sem 2. CV2102. Date No.

1. (a). Determinacy , $b = 23$ $r = 3$ $j = 13$.

stability : $b + r = 2j$

$23 + 3 = 2(13)$

$= 26$ stable and statically determinate

b) zero force members.

NL, NK, MH, MJ, FK, JD, CH,

(c). $\circlearrowleft \Sigma M_A = 0$

$-50(5) - 20(25) + B_y(30) = 0$

$B_y = 25\text{ kN} \uparrow$

$+\uparrow \Sigma F_y = 0$

$-20 + 25 + A_y = 0$

$A_y = -5\text{ kN}$

$= 5\text{kN} \downarrow$

$\xrightarrow{+} \Sigma F_x = 0$

$-50 + A_x = 0$

$A_x = 50\text{kN}$

(d) CD: AC = CD

$\xrightarrow{+} \Sigma F_x = 0$

$-50 + AC = 0$

$AC = 50\,(T)$

$AC = CD = 50\,(T)$

HJ: cut through HJ, HD & CD & take moment about D:

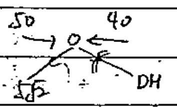
$\circlearrowleft \Sigma M_D = 0$

$-50(5) - HJ(5) + 5(10) = 0$

$HJ = -40\text{ kN}$

$= 40\text{ kN}\,(C)$

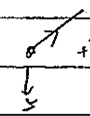
DH:



$50 + DH\left(\frac{1}{\sqrt{2}}\right) - 40 - 5\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = 0.$

$DH = -5\sqrt{2}\text{ kN}\ (C).$

AH:

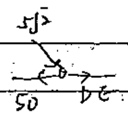


$+\uparrow \Sigma F_y = 0$

$-5 + AH\left(\frac{1}{\sqrt{2}}\right) = 0.$

$AH = 5\sqrt{2}\text{ kN}\ (T)$

At D :



$\xrightarrow{+} \Sigma F_x = 0$

$DE - 50 + 5\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = 0$

$DE = 45\text{ kN}\ (T).$

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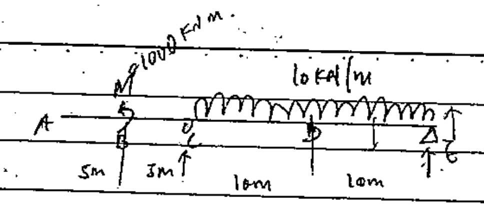
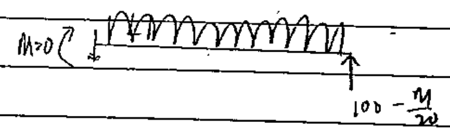
2.(a).





$\circlearrowleft \sum M_E = 0$

$10(20)(10) + M - C_y(20) = 0$

$C_y = 100 + \frac{M}{20}$

$\circlearrowleft \sum M_E = 0$

$M - 10(20)(10) + E_y(20) = 0$

$E_y = 100 - \frac{M}{20}$

M

$10 \times 10(5) - \left(100 - \frac{M}{20}\right)10 = 0$

$500 = 1000 - \frac{M}{2}$

$M = 1000$ kNm.

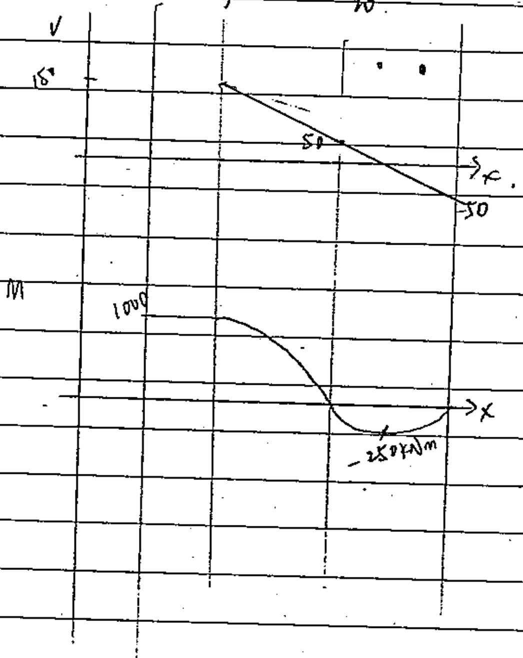
$C_y = 100 + \frac{M}{20}$

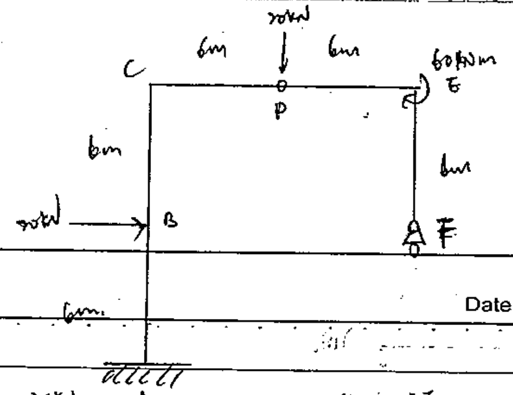
$= 100 + 50$

$= 150$ kN

$E_y = 100 - \frac{1000}{20}$

$= 50$ kN





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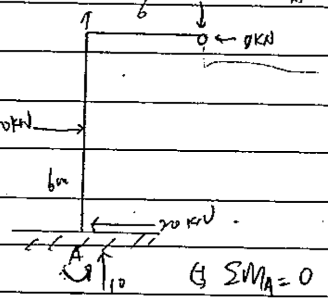
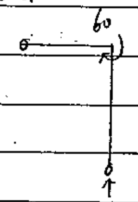
2.(b).

section AB CD



section DEF



$\circlearrowleft \Sigma M_D = 0$

$-60 + F_y(6) = 0$

$F_y = 10\text{ kN}$

$F_x = 0 \quad D_x = 0.$

$\circlearrowleft \Sigma M_A = 0$

$-M - 20(6) - 20(6) = 0$

$M = -240\text{ kNm}$

$M = 240\text{ kNm}$ ↻

$\rightarrow \Sigma F_x = 0$

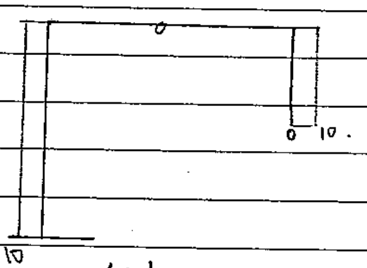
$20 - A_x = 0$

$A_x = 20\text{ kN}.$

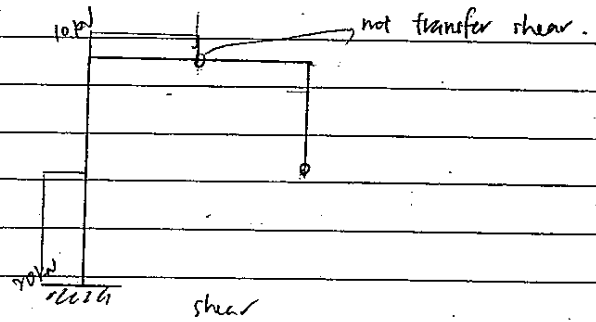
$\uparrow \Sigma F_y = 0$

$10 - 20 + A_y = 0$

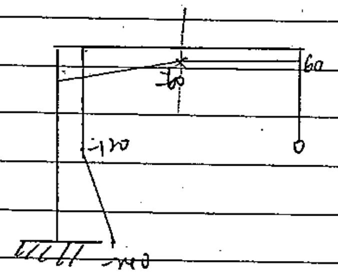
$A_y = 10\text{ kN}$



Axial



shear



moment.

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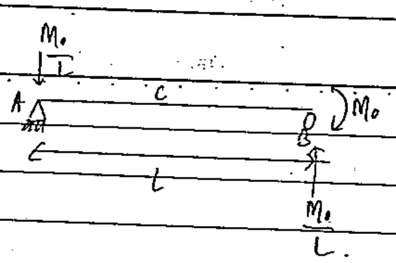
3.

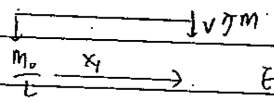
(a).



$\circlearrowleft \Sigma M_A = 0$

$B_y(L) - M_o = 0$

$B_y = \frac{M_o}{L} (\uparrow) \quad A_y = \frac{M_o}{L} \downarrow$



$M = -\frac{M_o}{L} x$

$EIv'' = -\frac{M_o}{L} x$

$EIv' = -\frac{M_o}{2L} x^2 + C_1$

$EIv = \frac{-M_o x^3}{6L} + C_1 x + C_2$

① $x = 0, \quad v = 0$

② $x = L \quad v = 0.$

① . $0 = -\frac{M_o}{6EIL}(0)^3 + C_1(0) + C_2$

$C_2 = 0.$

② . $0 = -\frac{M_o}{6EIL}(L)^3 + C_1(L).$

$\frac{M_o L^{\cancel{3}}}{6EI} = C_1 \cancel{L}$

$C_1 = \frac{M_o L}{6EI}$

$\therefore \quad v = -\frac{M_o x^3}{6EIL} + \frac{M_o L}{6EI} x$

v at C : $x = \frac{1}{2} L$

$v = -\frac{M_o}{6EIL}\left(\frac{1}{2}L\right)^3 + \frac{M_o L}{6EI}\left(\frac{1}{2}L\right)$

$= -\frac{M_o L^2}{48EI} + \frac{M_o L^2}{12EI}$

$= \frac{M_o L^2}{16EI}$

v' at B.

$v' = -\frac{M_o x^2}{2EIL} + \frac{M_o L}{6EI}$

$v'(L) = \frac{-M_o L^{\cancel{2}}}{2EI\cancel{L}} + \frac{M_o L}{6EI}$

$= -\frac{M_o L}{3EI}$

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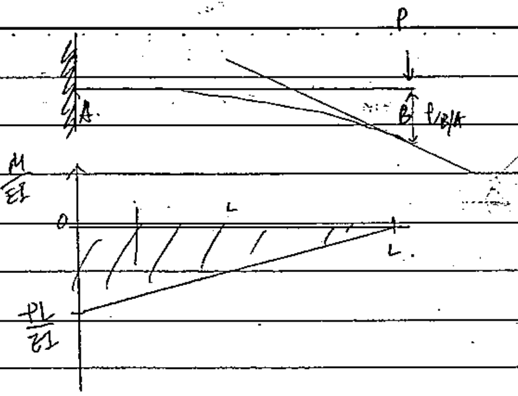
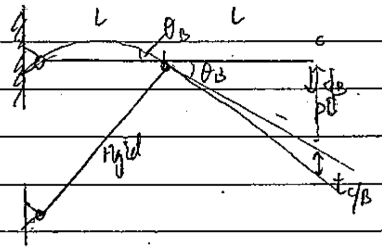
3(b)



$$\delta_B = t_{B/A} = \frac{1}{2}\left(-\frac{PL}{EI}\right)L\left(\frac{2L}{3}\right)$$

$$= -\frac{PL^3}{6EI}$$

(c)



from (a) $\theta_B = \frac{-M_o L}{3EI}$ let $M_o = PL$

$$\theta_B = \frac{-PL^2}{3EI}$$

$$\delta_c = \theta_B(L) + \left(\frac{-PL^3}{6EI}\right)$$

$$= -\frac{PL^3}{3EI} - \frac{PL^3}{6EI}$$

$$= \frac{PL^3}{-2EI}$$

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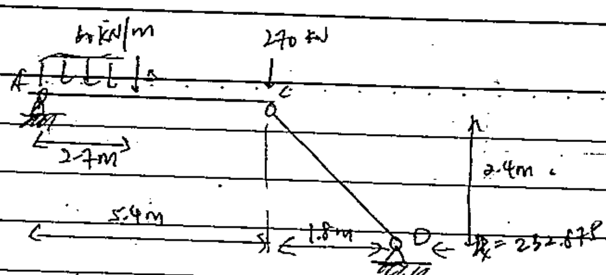
4.



$E = 200 \times 10^9$

$I = 6.656 \times 10^7 \, mm^4 = 6.656 \times 10^{-5} \, m^4$

A_{ABC} 2580 mm^2

$A_{CD} = 1935 \, mm^2 = 1935 \times 10^{-6}$

$\circlearrowleft \Sigma M_A = 0$

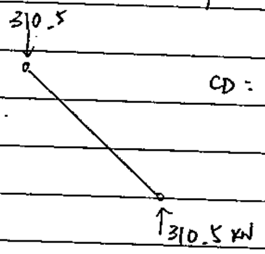
$-60(2.7)(1.35) - 270(5.4) + C_y(5.4) = 0$

$C_y = 310.5$ kN ↑

$+\uparrow \Sigma F_y = 0$

$310.5 - 270 - 60(2.7) + A_y = 0$

$A_y = 121.5$ kN ↑



CD : two force member.

∴ consider whole structure.

$\circlearrowleft \Sigma M_A = 0$

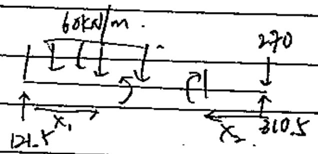
$-60(1.35)(2.70) - 270(5.4) + (7.2)(310.5) - D_x(2.4) = 0$

$D_x = 232.875$

$\xrightarrow{+} \Sigma F_x = 0$

$A_x = 232.875$ kN

For segment ABC



Internal force in CD : 388.125 kN

Internal force in ABC = 232.875.

$M_1 = -60(x_1)\left(\frac{x_1}{2}\right) + 121.5(x_1)$

$= 121.5x_1 - 30x_1^2$

$M_2 = -270(x_2) + 310.5(x_2)$

$= 40.5x_2$

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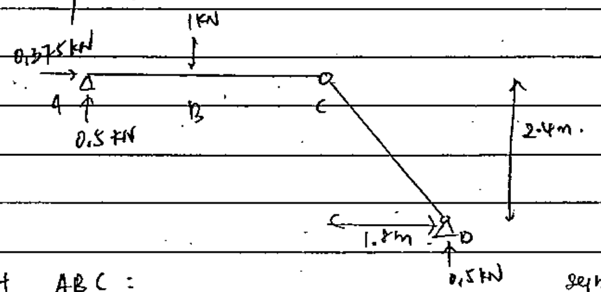
virtual system:



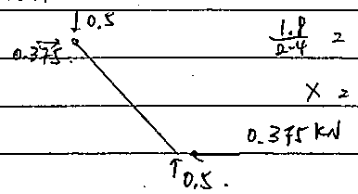
segment ABC :

C_y $\Sigma M_A = 0$

$-1(2.7) + C_y(5.4) = 0$

$C_y = 0.5\text{kN}$.

segment CD:



$\frac{1.8}{2.4} = \frac{x}{0.5}$

$x = 0.375\text{kN}$.

segment ABC

$M_1 = 0.5x_1$

$M_2 = 0.5x_2$

internal force in CD = 0.625 kN.

Internal force in CD = 0.375 kN.

deflection at B: $= \frac{1}{EI}\int_0^{2.7} (0.5x_1)(121.5x_1 - 30x_1^2)\, dx_1 + \frac{1}{EI}\int_0^{2.7} (0.5x_2)(40.5x_2)\, dx$

$+ \Sigma \frac{(0.625)(388.125)(3)}{AE} + \frac{(0.375)(232.875)(5.4)}{EA}$

$= \frac{199.3}{EI} + \frac{132.9}{EI} + \frac{727.7}{EA} + \frac{471.6}{EA}$

$= \frac{199.3}{(2\times10^8)(6.656\times10^{-5})} + \frac{132.9}{(2\times10^8)(6.656\times10^{-5})} + \frac{727.7}{(2\times10^8)(1935\times10^{-6})} + \frac{471.6}{(2\times10^8)(2580\times10^{-6})}$

$= 0.0277$ m

$= 27.7$ mm