CV1012 Fluid mechanics AY 16/17 Final Paper Answer sheet

(1a) Formula of hydrostatic force = $\gamma h_c A$.

Since the surface area, A of 3 plates are equal, γ (specific weight) are equal and h_c (distance of centroid to ~~free~~ free surface) are equal,

Hydrostatic force magnitudes of all 3 flat plates are the same.

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(1b) $\nu = \mu / \rho$

dimension of $\mu = [ML^{-1}T^{-1}]$

dimension of $\rho = [ML^{-3}]$

dimension of $\nu = \dfrac{[ML^{-1}T^{-1}]}{[ML^{-3}]}$

$= L^2T^{-1}$

$\therefore$ Dimension of ν is L^2/T (shown)

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(1c) $P_A + \cancel{a\rho g} - h(11\rho)g - \cancel{a\rho g} = P_B$

$P_A - P_B = 11h\rho g$

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When pipe is horizontal, $z_A = z_B$.

Using Bernoulli's equation,

$\frac{P_A}{\rho g} + \cancel{\frac{V_A^2}{2g}} + \cancel{z_A} = \frac{P_B}{\rho g} + \cancel{\frac{V_B^2}{2g}} + \cancel{z_B} + h_L$, From continuity equation, $V_A = V_B$.

$h_L = \dfrac{P_A - P_B}{\rho g}$

$= \dfrac{11h\cancel{\rho g}}{\cancel{\rho g}} = 11h$

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(1d) (i) h_c (distance from centroid to free surface) = $L\sin 45°$

Magnitude of Horizontal force of water acting on the gate : $\frac{1}{\sqrt{2}}L\rho g\left(\frac{2}{\sqrt{2}}L \times L\right)$

$= \cancel{1.71L^2\rho g}\ L^3\rho g$

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$= \frac{\frac{1}{12}L\left(\frac{2}{\sqrt{2}}L\right)^3}{\frac{1}{\sqrt{2}}L\left(\frac{2}{\sqrt{2}}L^2\right)} + \frac{1}{\sqrt{2}}L = 0.9428L \text{ (from the free surface.)}$$

(ii) Magnitude of vertical force $= \left(\frac{1}{2} \times \frac{1}{\sqrt{2}}L \times \frac{2}{\sqrt{2}}L\right) \times L \times \rho g$

$= \frac{1}{2}L^3\rho g$

By principle of moments,

$L^3\rho g \times (0.9428L - 0.7071L) = \frac{1}{2}L^3\rho g \times dx$

$0.2357L^4\rho g = \frac{1}{2}L^3\rho g \times dx$

$\therefore dx = 0.2357L$

(2a) At point B, velocity = 0.
Using Bernoulli's equation,

$$\frac{P_a}{\rho g} + \frac{V_a^2}{2g} + \cancel{z_a} = \frac{P_B}{\rho g} + \cancel{\frac{V_B^2}{2g}} + \cancel{z_B}.$$

$$\Rightarrow \frac{P_a}{\rho g} + 1 = \frac{P_B}{\rho g} \quad - (1)$$

$P_a + L\rho g + h(11\rho)g - (L+h)\rho g = P_b$

$P_a - P_b = -10h\rho g \quad - (2)$

From (1): $\frac{P_a}{\rho g} - \frac{P_B}{\rho g} = -1 \quad - (3)$

Substitute (2) into (3):

$\frac{-10h\rho g}{\rho g} = -1$

$h = 0.1\,m$

(2b) Apply momentum equation in the x direction,

$$F_{x\,net} = (\rho Q V)_{out} - (\rho Q V)_{in}$$

$$= \rho Q V \cos\alpha - \rho Q V$$

Applying momentum equation in the y direction,

$$F_{y\,net} = (\rho Q V)_{out} - (\rho Q V)_{in}$$

$$= -\rho Q V \sin\alpha - 0$$

Since $F_{x\,net} = F_{y\,net}$,

$$\Rightarrow \cancel{\rho Q} V \cos\alpha - \cancel{\rho Q} V = -\cancel{\rho Q} V \sin\alpha$$

$$\cos\alpha - 1 = -\sin\alpha$$

$$\cos\alpha + \sin\alpha = 1$$

$$(\cos\alpha + \sin\alpha)^2 = 1^2$$

$$\cos^2\alpha + \sin^2\alpha + 2\sin\alpha\cos\alpha = 1$$

Since $\cos^2\alpha + \sin^2\alpha = 1$, and $2\sin\alpha\cos\alpha = \sin 2\alpha$,

$$\Rightarrow \sin 2\alpha = 0$$

$$\alpha = 90^\circ, 180^\circ, 0^\circ, 270^\circ$$

$\therefore$ Values of α in which $F_x = F_y$: $0^\circ, 90^\circ, 180^\circ, 270^\circ$

\#

(3a) (i) Laminar flow is a type of flow where the friction factor is a function of Renold's number only & is independent of $\frac{\varepsilon}{D}$. Renold number is less than 2100.

(ii) Turbulent flow is characterised by irregular, random and chaotic nature. Renold number is usually greater than 4000.

(3b) (i) Applying Bernoulli eqn, where V_1, P_1 and P_2 are ~~0ft~~ nil.

$$10 = h_f + \frac{v_2^2}{2g} + 8.72$$

$$= f\frac{100}{0.12}\left(\frac{1^2}{2g}\right) + \frac{1}{2g} + 8.72$$

$$= 42.474f + 0.05097 + 8.72$$

$\Rightarrow 42.474f = 1.229$

$f = 0.02894$

$Re = \frac{VD}{\nu} = \frac{1 \times 0.12}{1 \times 10^{-6}} = 120{,}000$

Using Moody Diagram, $\frac{\varepsilon}{D} = 0.004$.

$\therefore$ Roughness height $\varepsilon = 0.004 \times 0.12$
$= 0.00048\,m$

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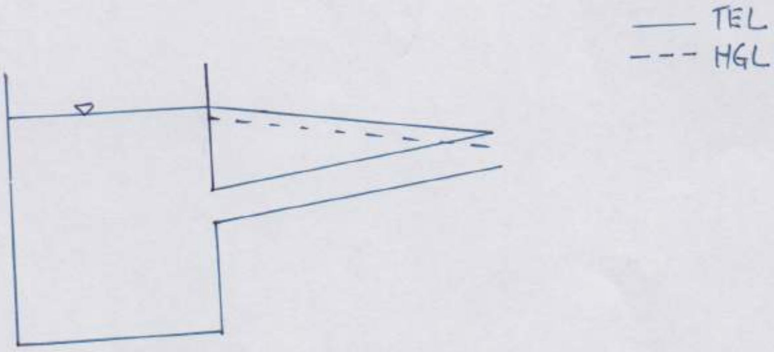
(ii)



(3c)(i) $\Delta P = f(H, D, \omega, Q, \rho, g)$

$\Delta P = [ML^2T^{-3}]$

$H = [L]$

$D = [L]$

$\omega = [T^{-1}]$

$Q = [L^3T^{-1}]$

$\rho = [ML^{-3}]$

$g = [LT^{-2}]$

Taking D, ω and ρ as repeating variables, and P, Q and gH as dependent variables.

Total no. of Pi terms = 3.

$\pi_1 = QD^a\omega^b\rho^c$ = dimensionless

$= (L^3T^{-1})(L^a)(T^{-b})(M^cL^{-3c}) = M^0L^0T^0$

$\Rightarrow M^cL^{3+a-3c}T^{-1-b} = M^0L^0T^0$

$\Rightarrow c = 0,\ b = -1,\ a = -3$

$\therefore \pi_1 = \frac{Q}{D^3\omega}$ #

$\pi_2 = gHD^a\omega^b\rho^c$ = dimensionless

$\Rightarrow (L^2T^{-2})(L^a)(T^{-b})(M^cL^{-3c}) = M^0L^0T^0$

$c = 0,\ b = -2,\ a = -2$

$\therefore \pi_2 = \frac{gH}{\omega^2D^2}$ #

$\pi_3 = PD^a\omega^b\rho^c$ = dimensionless

$\Rightarrow (ML^2T^{-3})(L^a)(T^{-b})(M^cL^{-3c}) = M^0L^0T^0$

$c = -1,\ b = -3,\ a = -5$

$\therefore \pi_3 = \frac{P}{\rho\omega^3D^5}$

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(3c)(ii) $\frac{Q_1}{\omega_1D_1^3} = \frac{Q_2}{\omega_2D_2^3}$

When ω is reduced to one third,

flow discharge (Q) will be reduced by one third.

$\frac{gH_1}{\omega_1^2D_1^2} = \frac{gH_2}{\omega_2^2D_2^2}$

When ω is reduced by one third, H reduces by $\frac{1}{9}$.

$\frac{P_1}{\rho_1\omega_1^3D_1^5} = \frac{P_2}{\rho_2\omega_2^3D_2^5}$

When ω is reduced by one third, P reduces by $\frac{1}{27}$.

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(4a) (i) velocity of flow $= \dfrac{Q}{A}$

$$= \frac{0.2}{\pi\left(\frac{0.3^2}{4}\right)} = 2.829\,m/s$$

$$Re = \frac{VD}{\upsilon} = \frac{2.829 \times 0.3}{1 \times 10^{-6}} = 848826$$

$$\frac{\varepsilon}{D} \text{ of pipe} = \frac{0.3 \times 10^{-3}}{0.3}$$

$$= 0.001$$

Using Moody's diagram, $f = 0.02$

$\therefore$ Head loss per metre length $= \dfrac{fV^2}{D2g}$

$$= \frac{0.02 \times 2.829^2}{2 \times 0.3 \times 9.81}$$

$$= 0.0272$$

(ii) For smooth pipe, $f = 0.012$

$$h_f = \frac{0.012 \times 2.829^2}{2 \times 0.3 \times 9.81}$$

$$= \frac{0.096039}{5.886}$$

$$= 0.0163$$

h_f is reduced by $\dfrac{0.0272 - 0.0163}{0.0272} \times 100\% = 40\%$

Thus, by selecting a hydraulically smooth pipe, head loss per metre is reduced by 40% $\Rightarrow$ more economical design as this reduces energy loss.

(4b) $h_f = \frac{8fLQ^2}{g\pi^2 D^5}$, $K = \frac{8fL}{g\pi^2 D^5}$

$K_1 = 3.3$
$K_2 = 7.9$
$K_3 = 95.1$

Trial 1 : Taking $H_J = 50m$

$Z_A - Z_J = K_1Q_1^2$

Pipe 1 : $60 - 50 = 3.3Q_1^2 \Rightarrow Q_1 = 1.74 m^3/s$

Pipe 2 : $Z_J - Z_B = 5 = 7.9Q_2^2 \Rightarrow Q_2 = 0.7956 m^3/s$

Pipe 3 : $Z_J - Z_C = 50 - 41.8 = 95.1Q_3^2 \Rightarrow Q_3 = 0.2936 m^3/s$

$Q_{IN} = 1.74 m^3/s$
$Q_{OUT} = 0.7956 + 0.2936 = 1.089 m^3/s$
Need to increase H_J. $Q_{diff} = 0.651 m^3/s$

Trial 2 : Taking $H_J = 52m$.

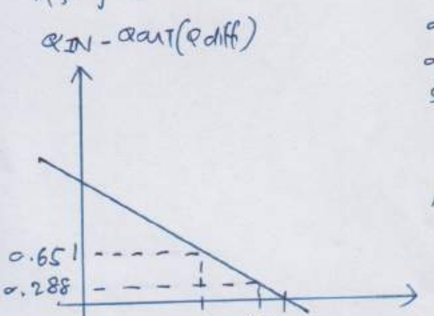
Pipe 1 : $Z_A - Z_J = 60 - 52 = K_1Q_1^2 \Rightarrow Q_1 = 1.557 m^3/s$

Pipe 2 : $Z_J - Z_B = 52 - 45 = K_2Q_2^2 \Rightarrow Q_2 = 0.9413 m^3/s$

Pipe 3 : $Z_J - Z_C = 52 - 41.8 = K_3Q_3^2 \Rightarrow Q_3 = 0.3275 m^3/s$

$Q_{IN} = 1.557 m^3/s$
$Q_{OUT} = Q_2 + Q_3 = 1.269 m^3/s$
Need to increase H_J. $Q_{diff} = 0.288 m^3/s$

Using linear interpolation,
$Q_{IN} - Q_{OUT} (Q_{diff})$



$y = mx + c$
$0.651 = m(50) + c$
$0.288 = m(52) + c$
solving the simultaneous equation,
$y = -0.1815x + 9.726$
At $Q_{diff} = 0$, $x = 53.59 m$

Trial 3 : Taking $H_J = 53.59m$,

Pipe 1 : $Z_A - Z_J = 60 - 53.59 = K_1Q_1^2 \Rightarrow Q_1 = 1.394 m^3/s$

Pipe 2 : $Z_J - Z_B = 53.59 - 45 = K_2Q_2^2 \Rightarrow Q_2 = 1.043 m^3/s$

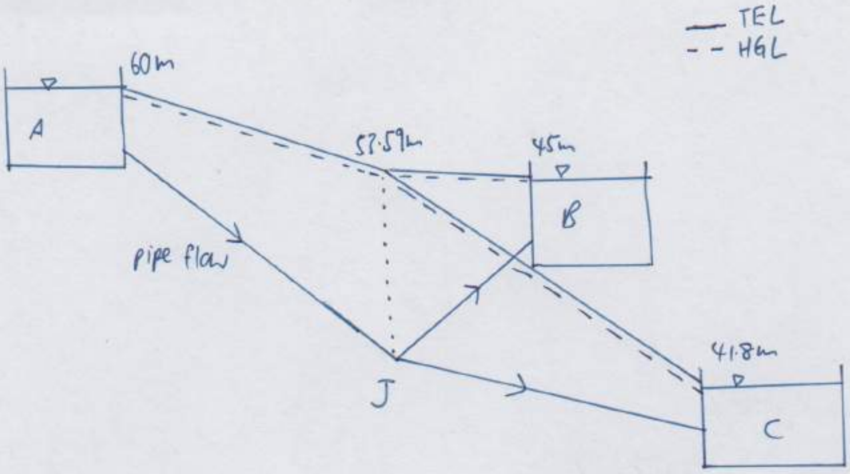
Pipe 3 : $Z_J - Z_C = 53.59 - 41.8 = K_3Q_3^2 \Rightarrow Q_3 = 0.352 m^3/s$

$Q_{IN} = 1.394 m^3/s$
$Q_{OUT} = Q_2 + Q_3 = 1.395 m^3/s$

Since $Q_{IN} \approx Q_{OUT}$, the elevation of J is accurate.

$\therefore$ $Q_1 = 1.394 m^3/s$
$Q_2 = 1.043 m^3/s$
$Q_3 = 0.352 m^3/s$ ※

(4b) (ii)



- All the best for your finals :)

Qi Long, 2017