CV1012 Fluid Mechanics.

April / May 2016.

1. (a) specific weight $= \frac{W}{V} = \frac{110}{0.01} = 11000\ N/m^3$.

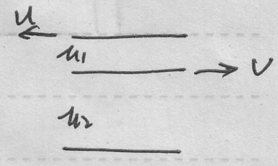
$W_{in\,water} = \rho_w g V$ $\qquad \frac{W}{V} = \rho g = 11000$

$\Rightarrow V = \frac{100}{1000 \times 10} = 0.01$ $\qquad \rho = 1100\ kg/m^3$

specific gravity $= \frac{\rho}{\rho_w} = \frac{1100}{1000} = 1.1$

(b). (i) $\tau = \mu \frac{du}{dy} = \mu \frac{u}{y} = 0.06 \times \frac{4}{0.006} = 40\ N/m$

The direction of shear stress that acts on the fixed plate is $+x$ direction #



(ii) $F = (\tau_1 + \tau_2) A$

$\overleftarrow{+}$

$= \left(\mu_1 \frac{u-(-V)}{y_1} + \mu_2 \frac{V}{y_2} \right) \cdot A.$

$= \left(0.01 \times \frac{2-(-4)}{0.02} + 0.06 \times \frac{4}{0.0006}\right) \times 1$

$= 70\ N.$ #

(c) $\begin{cases} P_A - P_B = 22000 \\ P_A + \rho g (h_2 - L\sin\theta) = P_B \end{cases}$

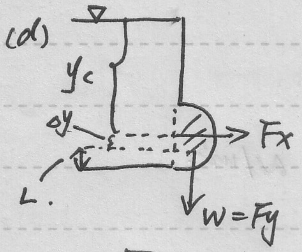
$\Rightarrow \rho g (h_2 - L\sin\theta) = -22000$

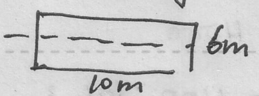
$900 \times 9.81 \times (5 - 15\sin\theta) = -22000$

$\sin\theta = 0.499$

$\Rightarrow \theta = 30^\circ$ #

(d)





$I_{xc} = \frac{1}{12} \times 10 \times 6^3$

$= 180\ m^4$

(i) $F_y = W = \rho g V$

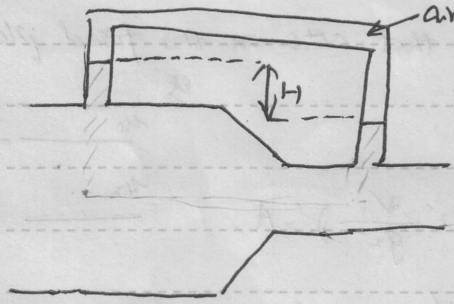
$= 1000 \times 9.81 \times (\frac{\pi}{2} \times 3^2) \times 10$

$= 1.39 \times 10^6\ N$

(ii) $\Delta y = \frac{I_{xc}}{A y_c} = \frac{180}{10 \times 6 \times (6+3)} \approx 0.333$

$L = r - \Delta y = 3 - 0.333 = 2.667m$ #

2. (a). $D_1 = 0.88\ m$ $D_2 = 0.6\ m$ $Q = 1 m^3/s$.



$P_2 = P_1 - \rho_w g H$.

$\frac{P_2 - P_1}{\rho g} = 1.1$

From Bernoulli equation:

$z_1 = z_2$

$\frac{V_1^2}{2g} + \frac{P_1}{\rho g} = \frac{V_2^2}{2g} + \frac{P_2}{\rho g}$

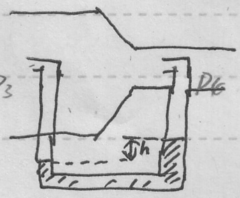
$H = \frac{P_2 - P_1}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$

$H = \frac{3.537^2 - 1.644^2}{2 \times 9.81} = 0.5 m$ #

$V_1 = \frac{Q}{A_1} = \frac{Q}{\pi(\frac{D_1^2}{4})} = \frac{1}{\pi(\frac{0.88^2}{4})}$

$V_1 = 1.644\ m/s$

$V_2 = \frac{1}{\pi(\frac{0.6^2}{4})} = 3.537\ m/s$.



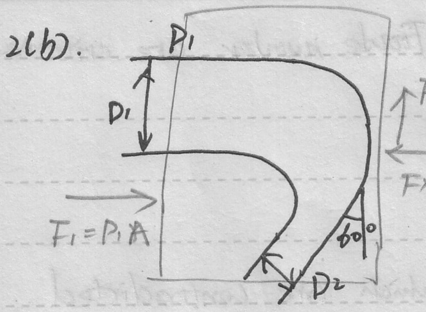
From Bernoulli equation

$z_3 = z_4$ $V_3 = V_4$

$\frac{P_3}{\rho_w g} = \frac{P_4}{\rho_w g} \Rightarrow P_3 = P_4$

$P_4 = P_3 - \rho_s g h \Rightarrow h = 0$ #.

2(b).



$D_1 = 0.15m \quad D_2 = 0.1m$

$P_1 = 70\times10^3 Pa \quad F_x = 5kN.$

① $\xrightarrow{+} \Sigma F_x = \Sigma(\rho Q V_x)_{out} - \Sigma(\rho Q V_x)_{in}$

$\cancel{P_1A_1 - 5\times10^3 = 1000\times}$

$P_1A_1 - F_x = -\rho Q V_2 \sin 60° - \rho Q V_1$

$70\times10^3\times\pi\left(\frac{0.15}{2}\right)^2 - 5000 = -1000\left(\frac{\sqrt{3}}{2}\cdot\frac{9}{4}+1\right)\frac{Q^2}{\pi\left(\frac{0.15}{2}\right)^2}$

$-3.763\times10^3 = -10^3\times2.949\times\frac{Q^2}{0.0176}$

$\Rightarrow Q = 0.15\ m^3/s$. #

$\Sigma Q_{in} = \Sigma Q_{out}$.

$Q_1 = Q_2 \qquad A_1V_1 = A_2V_2$

$\frac{V_1}{V_2} = \frac{A_2}{A_1} = \frac{D_2^2}{D_1^2} = \left(\frac{0.1}{0.15}\right)^2$

$V_2 = \frac{9}{4}V_1$

$V_1 = \frac{Q}{\pi(0.075)^2} \approx 8.5\ m/s$

$V_2 = \frac{Q}{\pi(0.05)^2} \approx 19.1\ m/s.$

② $+\uparrow\Sigma F_y = F_y = \Sigma(\rho Q V_y)_{out} - \Sigma(\rho Q V_y)_{in} = -\rho Q V\cdot\cos60° - 0.$

$= -1000\times0.15\times19.1\times\frac{1}{2}.$

$\Rightarrow F_y = -1.43\ kN$ #

(conservation of energy).

☆ Note: In this case, Beroulli equation can't apply because of external forces exerted by pipe. Instead, we should use $\Sigma F = \rho Q V_{out} - \rho Q V_{in}$, which is another transformation of conservation of moment.

3.(a) i. Several factors like fluid density, velocity, viscosity, etc can be analyzed together by dimensional analysis.

ii. Practical hydraulic problems can be simulated and analyzed by relative small laboratorial model using dimensional analysis.

3(b) Suppose both Reynolds number and Froude number are criteria for dynamic similarity.

$$Re = \frac{\rho V D}{\mu} \qquad \frac{V_1}{V_2} = \frac{L_2}{L_1}$$

$$Fr = \frac{V^2}{gL} \qquad \frac{V_1}{V_2} = \sqrt{\frac{L_1}{L_2}}$$ which are contradicted.

Therefore. either Re or Fr is used in hydraulic modelling.

(c) (i) In the laminar regime, $f = \frac{64}{Re} = \frac{64\mu}{\rho V D}$.

Because the head loss remains constant,

$$hf = \frac{64\mu}{\rho V D} \cdot \frac{1}{2} \frac{L}{g} \frac{V^2}{D}$$

$$= \frac{32\mu L}{\rho g} \frac{V}{D^2}$$

$$\frac{D_2}{D_1} = 2 \qquad \frac{V_2}{V_1} = \frac{D_2^2}{D_1^2} = 4.$$

$$\frac{Q_2}{Q_1} = \frac{A_2}{A_1} \cdot \frac{V_2}{V_1} = \left(\frac{D_2}{D_1}\right)^2 \cdot \left(\frac{V_2}{V_1}\right) = 16$$

It will be 1600% increase in flow rate.

(ii) In the wholly turbulent flow regime.

f is indepent of Re. $hf = \frac{8fLQ^2}{g\pi^2 D^5}$

$$\frac{D_2}{D_1} = 2. \qquad \frac{Q_2}{Q_1} = \sqrt{\left(\frac{D_2}{D_1}\right)^5} = \sqrt{2^5} = 5.66$$

\#

It will be 566% increase in flow rate.

(3) a. (i). $H = 25.37m$. $L_1 = 200m$ $D_1 = 0.3m$ $f_1 = f_2 = f_3 = 0.02$

$L_2 = 400m$ $D_2 = 0.4m$

$L_3 = 600m$ $D_3 = 0.3m$

$$H = \underbrace{\sum h_f}_{\text{major loss}} + \underbrace{\sum h_j}_{\text{minor loss}} \qquad h_f = \frac{8fLQ^2}{g\pi^2 D^5}$$

$$h_j = K \cdot \frac{V^2}{2g} = K \frac{8Q^2}{g\pi^2 D^4}.$$

$$25.37 = \frac{8f_1L_1Q^2}{g\pi^2 D_1^5} + \frac{8f_2L_2Q^2}{g\pi^2 D_2^5} + \frac{8f_3L_3Q^2}{g\pi^2 D_3^5}$$

$$+ 0.5 \times \frac{8Q^2}{g\pi^2 D_1^4} + \left(1 - \frac{D_1^2}{D_2^2}\right)^2 \cdot \frac{8Q^2}{g\pi^2 D_1^4}$$

$$+ 0.5 \cdot \frac{8Q^2}{g\pi^2 D_3^4} + 1 \times \frac{8Q^2}{g\pi^2 D_3^4}$$

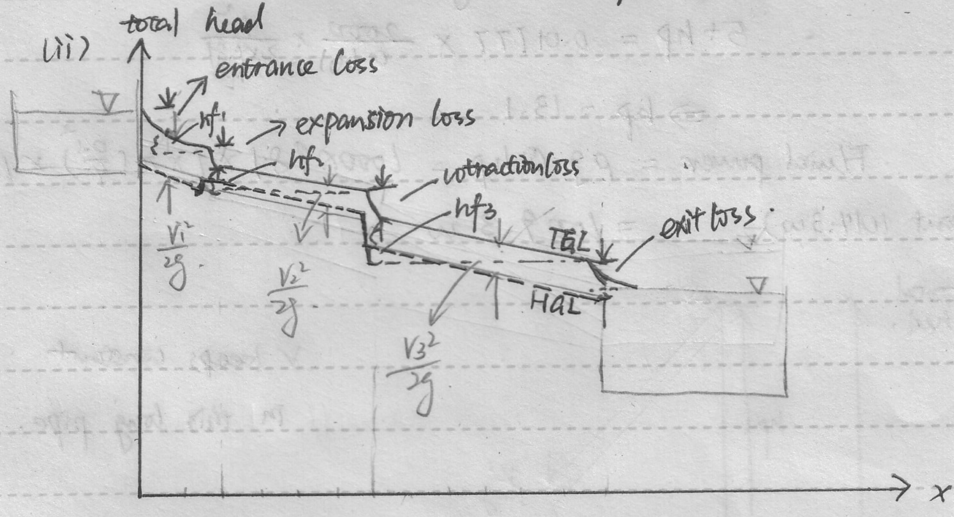
$$25.37 = (136Q^2 + 64Q^2 + 408Q^2)$$

$$+ (5.1Q^2 + 1.54Q^2 + 5.1Q^2 + 10.2Q^2)$$

$$630.54\,Q^2 = 25.37$$

$$\Rightarrow \quad Q \approx 0.2\ m^3/s \ \#$$

(ii)



(Note: $\frac{V_1^2}{2g} = \frac{V_3^2}{2g} > \frac{V_2^2}{2g}$)

4 (a) i In lamina flow regime, the friction factor linearly decreases with Reynolds number. ($f = \frac{64}{Re}$)

ii In turbulent flow regime, the friction factor non-linearly decreases with Reynolds number.

iii When relative pipe roughness increases, the friction factor also increase.

iv When Reynolds number is large enough, the friction factor is indepentent with Re, meaning almost horizontal lines as Re becomes larger.

(b) (i) smooth pipe. $f = \frac{0.316}{Re^{0.25}}$ $\quad Re = \frac{VD}{\nu} = \frac{1\times 0.1}{10^{-6}} = 10^5$

$$f = \frac{0.316}{\sqrt[4]{10^5}} = 0.01777$$

$$H + hp = hf$$

$$5 + hp = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g}$$

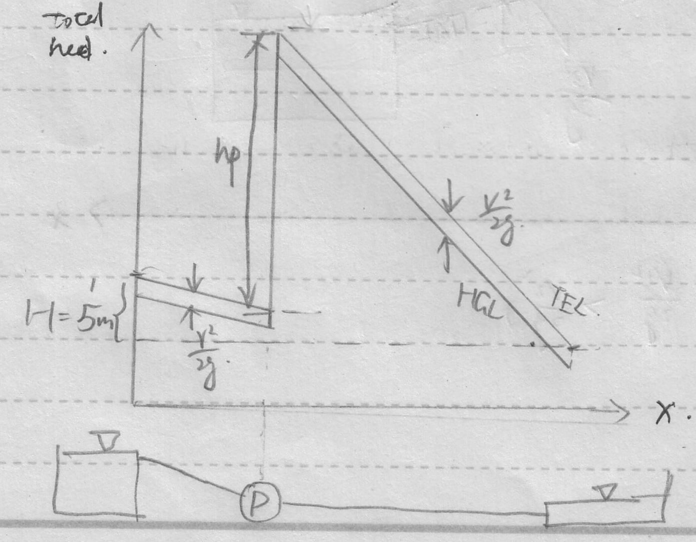
$$5 + hp = 0.01777 \times \frac{2000}{0.1} \times \frac{1^2}{2\times 9.81}$$

$$\Rightarrow hp = 13.1$$

Fluid power $= \rho g Q hp = 1000 \times 9.81 \times 1 \times \pi (\frac{0.1}{4})^2 \times 13.1$

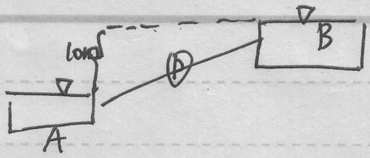
(roughly about 1014.3w) $= 1009.3\ w$

(ii)



V keeps constant in this long pipe.

(c) i.



$$-10 + h_p = f \cdot \frac{L}{D} \frac{V^2}{2g} = \frac{8fLQ^2}{g\pi^2 D^5}.$$

$$h_p = 10 + \frac{8 \times 0.03 \times 100}{9.81 \times \pi^2 \times (0.2)^5} Q^2$$

the pipe system characteristic curve: $\Rightarrow h_p = 774.6Q^2 + 10$ #

$$\begin{cases} h_p = 774.6Q^2 + 10 \\ h_a = 20 - 70Q^2 \end{cases}$$

$$774.6Q^2 + 10 = 20 - 70Q^2$$

$$\Rightarrow Q = 0.109 \text{ m}^3/\text{s} \quad \#$$

ii. Head rise coefficient $= \frac{gh_a}{\omega^2 D^2}$

$$\frac{h_{a2}}{h_{a1}} = \frac{\omega_2^2}{\omega_1^2} = 4 \qquad h_{a1} = \frac{1}{4} h_{a2} \quad ①$$

flow coefficient $= \frac{Q}{\omega D^3}$

$$\frac{Q_2}{Q_1} = \frac{\omega_2}{\omega_1} = 2. \qquad Q_1 = \frac{1}{2} Q_2 \quad ②$$

with ①②

$$h_{a1} = 20 - 70Q_1^2$$

$$\frac{1}{4} h_{a2} = 20 - 70\left(\frac{1}{2} Q_2\right)^2$$

$$\Rightarrow \begin{cases} h_{a2} = 80 - 70Q_2^2 \\ h_p = 774.6Q^2 + 10 \end{cases}$$

$$70 = 844.6Q^2$$

$$\Rightarrow Q = 0.288 \text{ m}^3/\text{s} \quad \#$$

All the best for your exam!

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