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CV1012 / CV2601 AY12-13 SEM2.
Fluid Mechanics.

Date No.

1. (a). The pressure change:

$$\Delta P = -k\frac{\Delta V}{V_0} = -24750 \times (-1\%)\ MPa$$
$$= 24.75\ MPa$$

The pressure need to be increased 24.75 MPa.

(b). Suppose the gauge pressure is P_G

$$P_G + 0.7 \times 9.8 \times 1000 \times 0.5\ Pa + 1.4 \times 9.8 \times 1000\ Pa$$
$$= \cancel{45}\ 0.45 \times 13.6 \times 9.8 \times 1000\ Pa$$
$$\Rightarrow P_G = 42.8\ kPa$$

The absolute pressure:

$$P_A = P_G + 0.75 \times 13.6 \times 9.8 \times 1000 = 1.428 \times 10^5\ Pa$$

(C). The horizontal hydrostatic thrust.

$$F_h = \rho g A h_c = 1000 \times 9.8 \times 4 \times 1 \times 2\ N = 78400\ N$$

Vertical hydrostatic thrust:

$$(19600\pi\ N)$$
$$F_v = \rho g h = 1000 \times 9.8 \times \frac{1}{2}\pi \times 2^2 \times 1\ N = 61575.2\ N$$

As $\Delta y = I_c / A y_c = \frac{1}{12} \times 1 \times 4^3 / 4 \times 1 \times 2 = \frac{2}{3} m$.

hence the distance to the hinge:

$$d = y_c + \Delta y = 2m + \frac{2}{3}m = \frac{8}{3}m.$$

$$\Sigma M = 0 \quad F \times 4m = 78400\ N \times \frac{8}{3}m + 19600\pi\ N \times 4 \times 2m/3\pi$$
$$\Rightarrow F = 6.53 \times 10^4\ N$$

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Date No.

(d). Suppose the density is ρ_l

$$(0.15m)^2 \pi \times 0.9m \times \rho_l \times g = (0.15m)^2 \times \pi \times 0.6m \times \rho_w \times g.$$

$$\Rightarrow \quad \rho_l = 670 \ kg/m^3.$$

$$BM = I_{yy}/V_i = \frac{\pi D^4}{64} / \pi (\tfrac{D}{2})^2 \times 0.6 = 9.375 \times 10^{-3} m.$$

$$BG = 0.45m - 0.3m = 0.15m.$$

As $BM < BG$, it is not in stable equilibrium.

2. (a). $P_1 - P_2 = \rho g h = 13.6 \times 1000 \times 9.8 \times 0.05 \ Pa = 6664 \ Pa.$

By Bernoulli's equation:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2.$$

$$Z_1 = Z_2 \qquad Q_1 = Q_2 \Rightarrow \quad V_2 = 9 V_1$$

$$\frac{6664 Pa}{\rho g} = \frac{80 V_1^2}{2g} \Rightarrow V_1 = 0.29 \ m/s.$$

$$Q = 0.29 \times (0.15)^2 \pi \ m^3/s = 0.02 \ m^3/s$$

(b). $$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$P_1 = P_2 = 0 \qquad Z_1 = 0.1 m \quad Z_2 = 0.$

$V_1 = 12 m/s \Rightarrow \quad V_2 = 12.8 m/s.$

As $$F_{net} = \dot{M}_{out} - \dot{M}_{in} = \rho_1 Q_1 V_1 - \rho_2 Q_2 V_2.$$

$$= 1000 \times 0.03 \times (12 m/s - (-12.8 m/s))$$

$$= 662.4 N$$

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Date No.

3. (a). $\Delta P = N/m^2 = ML^{-1}T^{-2}$ $\rho = kg\,m^{-3} = ML^{-3}$

$\mu = kg\,m^{-1}s^{-1} = ML^{-1}T^{-1}$ $g = m/s^2 = LT^{-2}$

$L = m = L$

As $\Delta P \rho^a g^b L^c = M^0 T^0 L^0$

$$\begin{cases} 1 + a = 0 \\ -1 + (-3)a + b + c = 0 \\ -2 + (-2)b = 0. \end{cases} \quad \begin{cases} a = -1 \\ b = -1 \\ c = -1 \end{cases}$$

$$\pi_1 = \frac{\Delta P}{\rho g L}$$

As $\mu \rho^a g^b L^c = M^0 L^0 T^0$

$$\begin{cases} 1 + a = 0 \\ -1 + (-3)a + b + c = 0 \\ -1 + (-2)b = 0 \end{cases} \quad \begin{cases} a = -1 \\ b = -\frac{1}{2} \\ c = -\frac{3}{2} \end{cases}$$

$$\pi_2 = \frac{\mu}{\rho g^{\frac{1}{2}} L^{\frac{3}{2}}}$$

Therefore $\pi_1 = \phi\,\pi_2 \Rightarrow \frac{\Delta P}{\rho g L} = \phi\left(\frac{\mu}{\rho g^{\frac{1}{2}} L^{\frac{3}{2}}}\right)$

(b). Better go and consult your lecturer

Date No.

(c) It follows Froude number scaling. $Fr = \frac{V}{\sqrt{gL}}$

$$\frac{V_1}{\sqrt{gL_1}} = \frac{V_2}{\sqrt{gL_2}} \quad L_1 = 25L_2 \Rightarrow V_1 = 5V_2.$$

for drag force $F = \cancel{\frac{1}{2}C_D\frac{V^2}{2g}A} \cdot \frac{1}{2}\rho V^2 C_D A$

As $V_1 = 5V_2 \Rightarrow F_1 = 25F_2$

4. (a). $\varepsilon = 0.06\,mm$ $D = 150\,mm$ $\frac{\varepsilon}{D} = 0.0004$ $P_A = 0.85\,MPa$

$P_B = 0.337\,MPa$.

As $\frac{V_A^2}{2g} + \frac{P_A}{\rho g} + Z_1 + h_f = \frac{V_B^2}{2g} + \frac{P_B}{\rho g} + Z_2$ $V_A = V_B$.

$$\frac{(0.85-0.337)\times10^6 Pa}{854\times9.8} = f\cdot\frac{1220}{0.15}\cdot\frac{V^2}{2\cdot9.8}$$

by trial and error:

try $f=0.03$ $V=0.709$ $\Rightarrow Re = \frac{VD}{\nu} = 27760$ ✗

try $f=0.033$ $V=0.676$ $\Rightarrow Re = 26466$ ✓

(b). $Q = 0.0000157\,m^3/s$ $D = 0.1\,m$.

The average velocity $V = \frac{4Q}{\pi D^2} = 2\times10^{-3}\,m/s$

$V_c = 2V = 4\times10^{-3}\,m/s$.

※ Better to prove why $V_c = 2V$ in the exam.

(c). $h_f = f\frac{L}{D}\frac{V^2}{2g}$.

As $D_1 = 0.1\,m$ $D_2 = 0.05\,m$ $D_1 = 2D_2 \Rightarrow V_1 = \frac{1}{4}V_2$

$L_1 = 80\,m$ $L_2 = 100\,m$ $f_1 = f_2$

$h_{f1} = f\frac{V^2}{2g}\cdot\frac{80}{0.1}$ $h_{f2} = f\frac{16V^2}{2g}\frac{100}{0.05}$

$$\Rightarrow \frac{h_{f1}}{h_{f2}} = \frac{800}{2000\times16} = 0.025.$$