

CV1012 - FLUID MECHANICS. SEM 2 AY 11/12

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Q1a) P_1 and P_2 experience the same shear stress,

$$\Rightarrow \tau_1 = \tau_2$$

$$\Rightarrow \mu_1 \left(\frac{v_1}{y_1} \right) = \mu_2 \left(\frac{v_2}{y_2} \right)$$

$$(0.4) \left(\frac{v-3}{0.02} \right) = 0.2 \left(\frac{v}{0.02} \right)$$

$$20v - 60 = 10v$$

$$\therefore 10v = 60$$

$$v = 6 \text{ m/s}$$

} Relative velocity!

Q1b)

Pressure of air = $\rho g h$

$$= (13600)(9.81)(0.16)$$

$$= 21346.56 \text{ Pa}$$

$$\text{Pressure Gauge Measurement} = 21346.56 + (1000)(9.81)(4)$$

$$= 60586.56 \text{ Pa}$$

Q1c)

$$F_{v_1} = \rho g V = (1000)(9.81)(2 \times 2 \times 2) = 78480 \text{ N at } 1 \text{ m from hinge.}$$

$$F_{v_2} = \rho g V = (1000)(9.81) \left(\frac{1}{4} \pi \times 2 \times 2 \times 2 \right) = 61638.048 \text{ at } 1.15 \text{ from hinge.}$$

$$F_h = \rho g A h_c = (1000)(9.81)(2 \times 2)(1) = 39240 \text{ N acting at } 1.333 \text{ m from hinge.}$$

$$I_c = \frac{1}{12}(2^4) = 1.333 \quad (3)$$

$$\Delta y = \frac{I}{A y_c} = \frac{1.333}{2 \times 2 \times 1} = 0.333 \text{ m}$$

$$\Sigma M = 0$$

$$(78480)(1) + (61638.048)(1.15) + 39240(1.333) = F(2)$$

$$\Rightarrow F = 100841.8 \text{ N}$$

Q1d)

$$\text{Weight} = 7000 \text{ kN} = \rho g V$$

$$= (1020)(9.81)(30)(8h - 12)$$

$$h = 4.39 \text{ m}$$

$$\therefore \text{dist. from top} = 5 - 4.39$$

$$= 0.613 \text{ m}$$

$$I_{yy} = \frac{1}{12}(30)(8^3) = 1280$$

$$V_y = (1.39)(8)(30) + \left(\frac{1}{2} \right) (8)(3)(30) = 693.6$$

$$BM = 1.85 \text{ m}$$

\therefore For stable equilibrium $BM > BG$, $BG < 1.85 \text{ m}$



$$P_1 + \rho_{\text{water}}(g)(0.1) = P_2 + \rho_{\text{Hg}}(g)(0.1)$$



2a)

$$P_1 - P_2 = \rho g H = (13600)(9.81)(0.1)$$

$$= 13341.6 \text{ Pa}$$

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$$P_1 - P_2 = 12360.6 \text{ Pa}$$

$$V_1 = \frac{Q}{A} = \frac{Q}{0.00785} \text{ ms}^{-1} \quad \text{--- (A)}$$

$$V_2 = \frac{Q}{0.00196} \text{ ms}^{-1} \quad \text{--- (B)}$$

$$P_1 - P_2 = \rho g \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g} \right)$$

$$13341.6 = 1000(9.81) \left[\frac{Q^2}{2g(0.00196)^2} - \frac{Q^2}{2g(0.00785)^2} \right]$$

$$Q = 0.0105 \text{ m}^3/\text{s}$$

2b)

By continuity eqn, $Q_1 = Q_2$

$$V_1(24) = V_2(0.4)$$

$$V_2 = 60V_1 \quad \text{--- (1)}$$

By Bernoulli eqn, $\frac{V_1^2}{2g} + z = \frac{V_2^2}{2g} + z$

$$\frac{V_1^2}{2g} + 6 = \frac{(60V_1)^2}{2g} + 0.1$$

$$2579V_1^2 = 2.9$$

$$V_1 = 0.149 \text{ ms}^{-1}$$

$$V_2 = 10.76 \text{ ms}^{-1}$$

By momentum eqn, $F_{\text{net}} = M_{\text{out}} - M_{\text{in}}$

$$F_1 - F - F_2 = M_{\text{out}} - M_{\text{in}}$$

$$(1000)(9.81)(24)(3) - F - (1000)(9.81)(0.4)(0.05) = (1000)(9.81)(10.76) - (1000)(9.81)(0.149)$$

$$F = 60234.19 \text{ N}$$

$$= 60.2 \text{ kN} \quad (\text{reaction forces } \Rightarrow \text{equal in magnitude})$$



Q3.a)

$$\pi_1 = Q M^a g^b L_1^c$$

$$(L^3 T^{-1}) (M L^{-1} T^{-1})^a (L T^{-2})^b (L)^c = M^0 L^0 T^0$$

$$\Rightarrow a = 0 \quad \Rightarrow M = 0$$

$$\Rightarrow 3 - a + b + c = 0$$

$$\Rightarrow -1 - a - 2b = 0$$

$$b = -\frac{1}{2}$$

$$\Rightarrow c = -\frac{5}{2}$$

$$\therefore \pi_1 = \frac{Q}{g^{1/2} L_1^{5/2}}$$

$$\pi_2 = \rho M^a g^b L_1^c$$

$$(M L^{-3}) (M L^{-1} T^{-1})^a (L T^{-2})^b (L)^c = M^0 L^0 T^0$$

$$\Rightarrow 1 + a = 0 \quad \Rightarrow a = -1$$

$$\Rightarrow -3 - a + b + c = 0 \Rightarrow b + c = 2$$

$$\Rightarrow -a + 2b = 0 \Rightarrow b = \frac{1}{2}$$

$$\Rightarrow c = \frac{3}{2}$$

$$\pi_2 = \frac{\rho g^{1/2} L_1^{3/2}}{M}$$

$$\pi_3 = L_2 M^a g^b L_1^c$$

$$(L) (M L^{-1} T^{-1})^a (L T^{-2})^b (L)^c = M^0 L^0 T^0$$

$$\Rightarrow a = 0$$

$$\Rightarrow 1 - a + b + c = 0 \Rightarrow b + c = -1$$

$$\Rightarrow -a - 2b = 0 \Rightarrow b = 0$$

$$\Rightarrow c = -1$$

$$\pi_3 = \frac{L_2}{L_1}$$

$$\pi_1 = \phi(\pi_2, \pi_3)$$

$$\frac{Q}{g^{1/2} L_1^{5/2}} = \phi\left(\frac{\rho g^{1/2} L_1^{3/2}}{M}, \frac{L_2}{L_1}\right)$$



NOTES

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3 b)

$$\rho_{ave} = 1.4 \text{ m}^3/\text{s}$$

$$\rho_{ave} = 900 \text{ kg m}^{-3}$$

$$\mu_{ave} = 2600 \text{ kg/ms}$$

$$g_{ave} = 9.81 \text{ m/s}^2$$

$$L_{1ave} = 12.6 \text{ m}$$

$$L_{2ave} = 6.3 \text{ m}$$

$$\tau_1 = \frac{\rho}{g^{1/2} L^{5/2}} = 7.93 \times 10^{-4}$$

$$\tau_2 = \frac{\rho g^{1/2} L^{3/2}}{\mu} = 18.19$$

$$\tau_3 = \frac{L_2}{L_1} = 2$$

estimate using $\tau_1 \Rightarrow 7.93 \times 10^{-4} (g^{1/2} / L^{5/2}) = 0$
 $Q = 43.92 \text{ m}^3/\text{s}$

3c)

Geometric Similarity: Similarity in length ratios

Dynamic Similarity: Similarity in Reynold's, Froude, Weber Number etc.

Kinematic Similarity: Ratio of velocity and acceleration between model and prototype constant through the flow field.

$$\frac{g_{model}}{g_{earth}} = 0.1662$$

Using Dynamic Similarity: $Fr_e = Fr_m$

$$\frac{V_e}{\sqrt{g_e y_e}} = \frac{V_m}{\sqrt{g_m y_m}}$$

$$\frac{V_m}{V_e} = \frac{\sqrt{g_m y_m}}{\sqrt{g_e y_e}}$$

$$= \frac{1.63 \times 1}{\sqrt{9.81 \times 1}}$$

$$V_m = 0.4076 V_e$$



4a)
$$p_1 \pi r^2 - (p_1 - \Delta p) \pi r^2 - \tau 2\pi r l = 0$$

$$\Delta p \pi r^2 - 2\pi r l \tau = 0$$

$$\tau = \frac{\Delta p r}{2l} \quad (\text{shown})$$

4b)
$$\tau = \mu \left(\frac{du}{dy} \right)$$

$$\frac{\Delta p r}{2l} = \mu \left(\frac{du}{dy} \right)$$

$$du = \left(\frac{\Delta p r}{2\mu l} \right) dy$$

$$u = - \int \frac{\Delta p r}{2\mu l} dy$$

$$= \left(\frac{\Delta p r^2}{4\mu l} + c \right)$$

When $r = R, u = 0$ at wall, $c = - \frac{\Delta p R^2}{4\mu l}$

$$u = \left(\frac{\Delta p r^2}{4\mu l} - \frac{\Delta p R^2}{4\mu l} \right)$$

$$= \frac{\Delta p R^2}{4\mu l} \left(1 - \left(\frac{r}{R} \right)^2 \right) \quad (\text{shown})$$

4c) When $r = 0.04, v = 0.8$.

$$0.8 = \frac{\Delta p (0.05)^2}{4\mu l} \left(1 - \left(\frac{0.04}{0.05} \right)^2 \right)$$

$$\frac{\Delta p}{4\mu l} = 888.89$$

At centerline, $r = 0, u = \frac{\Delta p R^2}{4\mu l} = (888.89)(0.05^2)$
 $= 2.22 \text{ m s}^{-1}$

$v_{\text{max}} = \frac{v_c}{2} = 1.11 \text{ m s}^{-1}$

Not final

4d) Assuming uniform velocity,

$$\tau = \mu \left(\frac{du}{dy} \right) = \mu \left(\frac{u}{y} \right)$$

Assuming Q is constant.

$$Q_1 = Q_2 \Rightarrow A_1 V_1 = A_2 V_2$$

$$\pi (0.05^2) V_1 = \pi (0.04^2) V_2$$

$$V_2 = 1.5625 V_1$$

$$\frac{\tau_B}{\tau_C} = \frac{\mu \left(\frac{V}{y} \right)}{\mu \left(\frac{V}{y} \right)} = \frac{\left(\frac{V_1}{0.1} \right)}{\left(\frac{1.5625 V_1}{0.08} \right)} = 0.512$$