Fluid Mechanics. 11/12 Sem 1

CV 2601.

1. (a) $V = 5 m/s$, $y = 2 \times 10^{-3}$, $\mu = 0.01$

$$\tau = \mu \frac{\partial u}{\partial y}$$

$$= \mu \cdot \frac{5}{2 \times 10^{-3}}$$

$$= 0.01 \times 5 \times \frac{1}{2 \times 10^{-3}}$$

$$= 25 \, Pa.s$$

$$\therefore F = \tau \times A$$

$$= 25 \times 2 \times 1$$

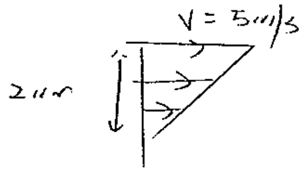
$$= 50 \, N.$$

$$\because \frac{dw}{dt} = F.V$$

$$P = F.V$$

$$= 50 \times 5$$

$$= 250 \, J/s.$$



Ans: The power against fluid shear stress is 250 J/s.

b) $\Delta \rho = 995.7 - 1000$

$$= -4.3 \, kg/m^3.$$

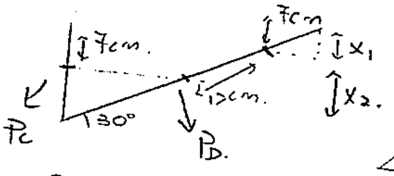
$$\therefore \Delta p = K \cdot \frac{\Delta \rho}{\rho_0}$$

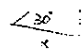
$$= 215 \times 10^7 \times \frac{4.3}{1000}$$

$$= 9245000 \, Pa.$$

Ans: The applied pressure is 9245000 Pa.

c)





$\tan 30° = \frac{7}{x}$

$\therefore x = \frac{7}{\tan 30°}$

$P_C = P_D$.

$P_A + \rho_{H_2O} g h = P_B + \rho_{oil}\, g\, x_1 + \rho_{mercury}\, g\, x_2$.

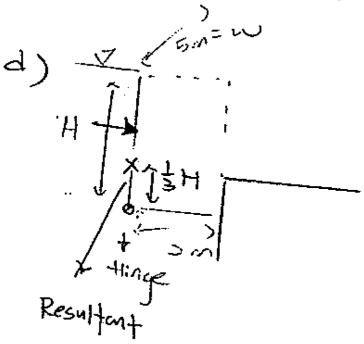
$5\times10^3 + 1\times10^3 \times 9.81 \times \frac{7}{100} = P_B + 0.87\times10^3 \times 9.81 \times \frac{7}{\tan 30°} \times \frac{1}{100} + 13.6\times10^3 \times 9.81 \times \frac{12}{\tan 30°} \times \frac{1}{100}$

$P_B = -23078.0722\ Pa$

$= -23.078\ kPa$.

Ans: The pressure at B is -23.078 kPa.

d)



F_H = weight of the volume prism

$F_H = \rho g H \times H \times 5 \times \frac{1}{2}$

$= \frac{5}{2} \rho g H^2 \ (\rightarrow)$

The F_H is acting at $\frac{1}{3}H$ from the hinge.

$F_V = \rho g H \times 2 \times 5$

$= 10 \rho g H \ (\uparrow)$.

The F_V will act on 1m left from hinge.

$\because$ The gate is in equilibrium;

$\therefore \Sigma M_{Hinge} = 0$

$\frac{1}{3}H \times \frac{5}{2} \rho g H^2 = 10 \rho g H \times 1$.

$\frac{5}{6} \rho g H^3 = 10 \cancel{\rho g} H$

$\frac{5}{6} H^3 - 10H = 0$

$H(\frac{5}{6}H^2 - 10) = 0$.

$\therefore H = \sqrt{12}$

$= 2\sqrt{3}$ m.

Ans: The height of H is $2\sqrt{3}$ m.

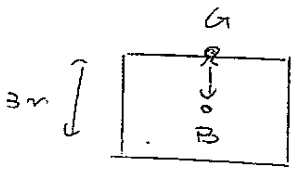
1 e)

$b = h = 8$

$I_{yy} = \frac{1}{12} bh^3$ $\quad V_i = 3 \times 8 \times 8$ $\quad$ ∘ $BG = 1.5m$.

$= \frac{1}{12} 8.8^3$ $\quad = 64 \times 3$

$= \frac{1024}{3} m^4$ $\quad = 192 m^3$



∘ $BM = \frac{I_{yy}}{V_i}$

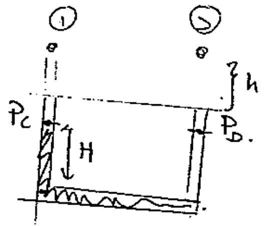
$= \frac{1024}{192} \times \frac{1}{3}$

$= \frac{16}{9} = 1.7778 m$.

$\because BM > BG$.

Hence the barge floating is in stable equilibrium.

2(a)



$$\therefore P_C + \rho_{Mercury} \times g \times H = P_D + \rho_{water} \times g \times H$$

$$\therefore P_C = P_D + 1000 \times 9.81 \times \frac{10}{100} - 13.6 \times 1000 \times 9.81 \times \frac{10}{100}$$

$$= P_D - 12360.6$$

$$\therefore P_1 + \rho g h = P_2 + \rho g h$$

$$P_C = P_{1D}$$

By Bernouli Equation. $\therefore P_1 = P_2 - 12360.6$.

$$\because \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \cancel{Z_2}$$

$\because Z_1 = Z_2$.

$V_2 = 0$.

$$\frac{V_1^2}{2g} + \frac{P_1}{\rho g} = \frac{P_2}{\rho g}$$

$$\therefore \frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g}$$

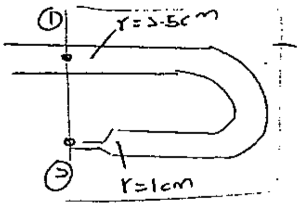
$$V_1^2 = \frac{2(P_2 - P_1)}{\rho}$$

$$= \frac{2 \times 12360.6}{1000}$$

$$V_1 = 4.972 \ m/s.$$

Ans: The velocity of the water is 4.972 m/s.

2(b)



control volume.

By Continuity eqn.

$A_2V_2 = A_1V_1$.

$$V_2 = \frac{A_1}{A_2} V_1$$

$$= \frac{\pi}{\pi} \cdot \frac{D_1^2}{D_2^2} \cdot V_1$$

$$\therefore V_2 = \frac{5^2}{2^2} \times 5$$

$$= \frac{5^3}{4} \text{ m/s.}$$

By using Bernouli eqn.

$E_① = E_2$ ($\because$ There is no head loss).

$$\frac{P_1}{\rho g} + \cancel{z_1} + \frac{V_1^2}{2g} = \cancel{\frac{P_2}{\rho g}} + \cancel{z_2} + \frac{V_2^2}{2g}$$

$z_1 = z_2$, $P_2 = 0$.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g}$$

$$\frac{P_1}{\rho} = \left[\frac{5^3}{4}\right]^2 \times \frac{1}{2} - \frac{5^2}{2}$$

$$P_1 = 475.78125 \times 1000$$

$$= 475781.25 \text{ Pa.}$$

Define $\rightarrow$ (Right) as positive

$$\therefore F_{Net} = \dot{M}_{out} - \dot{M}_{In}$$

$$= -\rho A_2 V_2^2 - \rho A_1 V_1^2.$$

$$= -\rho\left[\pi \times 1^2 \times \left(\frac{5^3}{4}\right)^2 + \pi \times 2.5^2 \times 5^2\right]$$

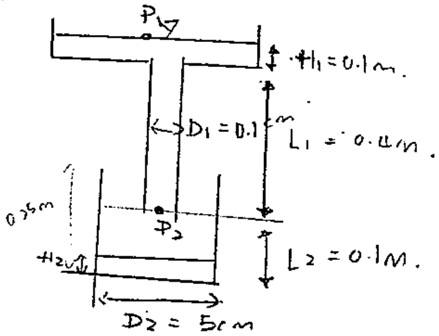
$$= -\rho\pi\left[945.3125\right]$$

$$= -2969.7869 \text{ kN } (\rightarrow)$$

- Force acting on fluid => Right $\therefore$ Force acting on pipe => left.

Ans: The hydrodynamic force acting on the horizontal bend is 2969.7869 kN to the left.

3.



$$\because \Sigma h_L = \text{entrance loss} + \text{friction loss}$$

$$= 0.5 \cdot \frac{V_2^2}{2g} + f \cdot \frac{l}{D} \cdot \frac{V_2^2}{2g}$$

$$= \left[\frac{0.2 \times 0.4}{0.1 \times 10^{-2}}\right] \frac{V_2^2}{2g}$$

$$= [80] \frac{V_2^2}{2g}$$

80.5

a). By using Bernouli eqn from pt ① to pt ②.

$$\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + Z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + Z_2 + \Sigma h_L.$$

$\because P_1 = 0,\ V_1 = 0,\ P_2 = 0,\ Z_2 = 0,\ Z_1 = 0.5$

$$0.5 = \frac{V_2^2}{2g} + \underset{80.5}{80}\frac{V_2^2}{2g}$$

$$= 81 \cdot \frac{V_2^2}{2g}.$$

$$\therefore V_2^2 = \frac{1 \times 9.81}{81}.$$

$$V_2 = 0.348\ m/s.$$

$\therefore$ The water flow out from the pipe will go into the ~~tank~~ container.

$$\therefore A_2 V_2 = A_3 V_3.$$

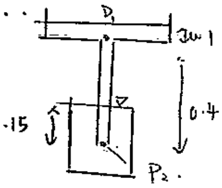
$$\frac{\cancel{\pi} \cdot D_1^2}{\cancel{4}} \times 0.348 = \frac{\cancel{\pi} \times D_2^2}{\cancel{4}} \times V_3$$

$$\therefore V_3 = \frac{0.1^2}{5^2} \times 0.348$$

$$= 1.392 \times 10^{-4}\ m/s.$$

Ans: The rate of the water rising within the container is 1.392×10^{-4} m/s.

b).



~~By using~~ Bernouli eqn:

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2} + h_L .$$

$Z_1 = 0.5,\ Z_2 = 0,\ P_1 = 0,\ P_2 = -\rho_w g \times 0.15.$

h_L = entrance + friction + exit

$$= 80\frac{V_2^2}{2g} + \frac{V_2^2}{2g}$$

$$= 81\frac{V_2^2}{2g}$$

$$0.5 = 0.15 + \frac{V_2^2}{2g} + 81\frac{V_2^2}{2g}.$$

$$0.35 = 82\frac{V_2^2}{2g}.$$

$$V_2^2 = \frac{0.35 \times 2}{82}$$

$$V_2 = 0.09239 \text{ m/s}.$$

$\because A_2V_2 = A_3V_3.$

$$V_3 = \frac{0.1^2}{5^2} \times 0.09239$$

$$= 3.6956 \times 10^{-5} \text{ m/s}.$$

Ans: The rate of rise of the water surface is 3.6956×10^{-5} m/s.

c).

It consists of two stage for the water to fill up the container.

I. water level < 0.1 m $\rightarrow$ Rate of h rising $= 1.392 \times 10^{-4}$ m/s.

II water level ≥ 0.1. $\rightarrow$ Rate of h rising $\Rightarrow$ keep on changing.

at $h = 0.25$ $\quad V = 3.6956 \times 10^{-5}$ m/s.

$\therefore$ For $\Delta h = 0.1$ m, $t_1 = \frac{0.1}{1.392 \times 10^{-4}} = 718.39$ s.

$0.1 < h < 0.25$, we assume the velocity change is linear, $V_i^2 = \frac{2 \times 9.81 \times 0.5}{82} \times \frac{0.1^2}{5^2}$

$V_i = 0.3480 \text{ m/s} \times \frac{0.1^2}{5^2}$

$= 1.392 \times 10^{-4}$ m/s.

$$\therefore \bar{V} = \frac{1.392 \times 10^{-4} + 3.6956 \times 10^{-5}}{2}$$

$$= 8.8078 \times 10^{-5} \text{ m/s}.$$

For $\Delta h = 0.15$ m, $t_2 = \frac{0.15}{8.8078 \times 10^{-5}} = 1703.03$ s.

$\therefore T_{total} = t_1 + t_2 = 718.39 + 1703.03$

$= 1881.42$ s.

Ans: The total time taken for the container to fill up is 1881.42s.

d)

No. It won't affect the filling time significantly as for the head losses = entrance + pipe + exit losses. The water pipe flow losses component contribute the most significant effect toward the rate of filling. Hence, the changing edge of the entrance will not affect the filling time significantly. as it only contribute a little of head loss in this condition.

4 (a).

∴ W increased to 3500 rpm.

$$\frac{Q_m}{\omega_m \cancel{D_m^3}} = \frac{Q_p}{\omega_p \cancel{D_p^3}} \quad \because D_m = D_p \qquad \frac{g h_m}{\omega_m^2 \cancel{D_m^2}} = \frac{g h_p}{\omega_p^2 \cancel{D_p^2}}$$

$$\therefore Q_p = \frac{3500}{1750} \times 0.1 = 2 \times 0.1 = 0.2\ m^3/s$$

$$\frac{h_m}{\omega_m^2} = \frac{h_p}{\omega_p^2}$$

$$\frac{h_p}{3500^2} = \frac{h_m}{1750^2}$$

$$\therefore h_p = 15m \times 4^2 = 60 \times 4 = 240m.$$

Ans: The flow rate is $0.2 m^3/s$ and the head is 240m.

b)

I) For the shaft:

$\because \Sigma F = ma$

$F_n - F_e = ma.$

We can explain this via Newton's Second law. Assuming only these 2 forces act on the shaft.

when $a \neq 0$.

$F_n = F_e + ma$

$\therefore F_n \neq F_e$

when $a = 0$, constant velocity. $\because a = r\omega^2$.

$V = const$

$W = const.$

$\therefore F_n - F_e = 0$

$F_n = F_e.$

• Ans: $F_n = F_e$ only when the shaft is rotating in constant velocity.

II) $F_n = f(L, D_1, D_2, \mu, w)$.

$\because$ Fe is a relationship between D_1, D_2, L and μ.

$F_n = \frac{kg.m}{s^2} \Rightarrow \frac{ML}{T^2}$ $\qquad$ $w = rads^{-1} \Rightarrow \frac{1}{T}$

$L = m \Rightarrow L$

$D_1, D_2 \Rightarrow L$.

$\mu \Rightarrow \frac{ML}{T}$

$\therefore k - r = 6 - 3$

$= 3\ \Pi$ group.

choosing L, μ, w as repeating units. (In order to construct a relationship between the rest of the unknown with D_1, D_2 and F_n).

$\downarrow \mu \quad \downarrow w$

$* \frac{ML}{T^2} = L^a \left(\frac{ML}{T}\right)^b \left(\frac{1}{T}\right)^c$.

For Π_1

$L: a + b = 1$ $\qquad \frac{1}{T}$ $\qquad \therefore \Pi_1 = \frac{F_n}{\mu w}$

$T: -b - c = -2$.

$M: b = 1$.

$\therefore a = 1 - 1$

$= 0$.

$\therefore -b - c = -2$.

$-1 + 2 = c$.

$\therefore c = 1$

For Π_2

$L = L^a \left(\frac{ML}{T}\right)^b \left(\frac{1}{T}\right)^c$ $\qquad \Pi_2 = \frac{D_1}{L}$

$\therefore b = 0, c = 0$

$\therefore a = 1$

For Π_3.

$L = L^a \left(\frac{ML}{T}\right)^b \left(\frac{1}{T}\right)^c$. $\qquad \Pi_3 = \frac{D_2}{L}$

$\therefore a = 1$.

Ans: The established relationship is shown as below.

$\therefore \frac{F_n}{\mu w} = \left(\frac{D_1}{L}, \frac{D_2}{L}\right)$.

III) Since the assumption of linear velocity of τ, $\therefore D_2 = 2D_1$.

$$\frac{D_2 - D_1}{2} - \frac{D_1}{2} \simeq 0.$$

$$\frac{F_n}{\mu w} = \phi\left(\frac{D_1}{L}, \frac{D_2}{L}\right)$$

$$\frac{D_2}{2} - \frac{D_1}{2} - \frac{D_1}{2} = 0.$$

$$\frac{D_2}{2} = D_1$$

$$\therefore D_2 = 2D_1.$$

$$\therefore \frac{F_n}{\mu w} = \phi\left(\frac{2D_1}{L}, \frac{D_1}{L}\right)$$ (As they can have a k to represent these 2 similar π group.

$$= \phi\left(\frac{D_1}{L}\right).$$

$$\therefore \frac{F_n}{\mu . w} = k . \frac{D_1}{L}.$$

$$\therefore F_n = \frac{k \mu w D_1}{L}.$$

Ans: $F_n = k . \frac{\mu w D_1}{L}$.

IV) If the assumption of linear velocity gradient is no longer valid. Then, we have to establish more relationship between these factor by assuming one of the factor is constant, then establish another relationship between these factor. By experimenting these during experiment, we can find out the relationship of F_n with these factors.