CV2601

Sem I 2010-2011

1a $F = \tau \cdot A$

$= \mu \cdot \frac{du}{dy} \times A$

$= 1.5 \times 2 \times 2 \times \frac{du}{dy}$

$= 6 \frac{du}{dy}$

$= \frac{6}{y}$

At $y = 0.01_m$ $\longrightarrow$ $F = \frac{6}{0.01} = 600\text{ N}$

At $y = 0.025m$ $\longrightarrow$ $F = \frac{6}{0.025} = 240\text{ N}$

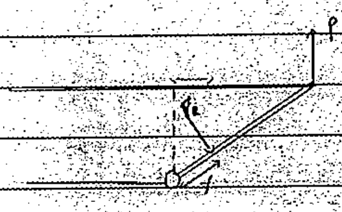
At $y = 0.04\ m$ ($y = 0.01_m$ from upper fixed plate) $\longrightarrow$ $F = \frac{6}{0.01} = 600\text{ N}$

Hence $y = 0.025_m$ to expect F to be minimum

b $F_h = \rho g A \cdot h$

$= 1000 \times 9.81 \times (3 \times 2) \times 1.5$

$= 88290\text{ N}$



$F_v = \rho g V$

$= 1000 \times 9.81 \times \frac{1}{2}(4 \times 3) \times 2$

$= 117{,}720\text{ N}$

$F_R = \sqrt{88290^2 + 117720^2}$

$= 147{,}150\text{ N}$

$y = \frac{1}{3} \times 5 = \frac{5}{3}\text{ m}$

$\circlearrowleft + \sum M = 0$

$4 \times P - F_R \times y = 0$

$P = 61{,}312.5\text{ N}$

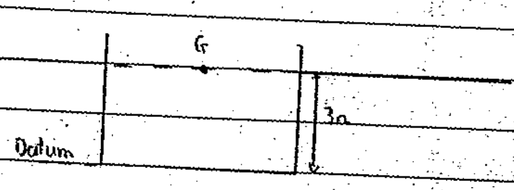
c $+\uparrow \Sigma F_y = 0$

$F_b = W$

$\rho_w \cdot g \cdot V_i = W$

$W = 1025 \times 9.81 \times 3 \times 5 \times 10$

$W = 1508.3\ kN$



Point $B = \frac{1}{2} \times 3 = 1.5m$

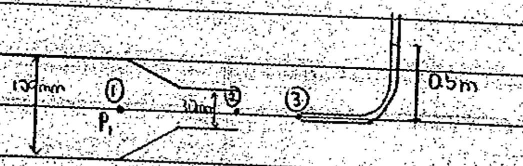
Point $G = 3m$

~~$BG = G - B = 1.5m$~~

$BM = \frac{I_{xx}}{V_i} = \frac{\frac{1}{12} \times 10 \times 5^3}{5 \times 10 \times 3} = \cancel{0.694}$

Since $BG > BM$, it will float in unstable equilibrium

2a



Apply Bernoulli's equation at point ② and ③

$P_1 + \rho g h_1 + \frac{1}{2}\rho V_2^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho V_1^2$

$0 + 0 + \frac{1}{2}(1000) V_2^2 = \rho g H + 0 + 0$

$V_2 = 3.13\ m/s$

Apply continuity equation at ① and ②:

$A_1 V_1 = A_2 V_2$

$\frac{1}{4}\pi (100)^2 V_1 = \frac{1}{4}\pi (30)^2 (3.13)$

$V_1 = 0.282\ m/s$

Apply Bernoulli's equation at (1) and (2)

$P_1 + \rho g h_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho V_2^2$

$P_1 + 0 + \frac{1}{2}(1000)(0.282)^2 = 0 + 0 + \frac{1}{2}(1000)(3.13)^2$

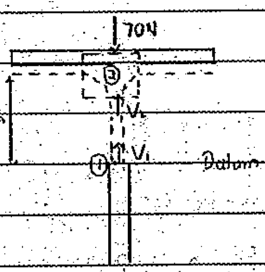
$P_1 = 4858.7\ N$

b | $F_n = \dot{M}_{out} - \dot{M}_{in}$

$-F = 0 - \rho Q V_2$

$70 = 1000 \times 0.1 V_2$

$V_2 = 0.7 \, m/s$



Apply Bernoulli's equation at (1) & (2):

$P_1 + \rho g h_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho V_2^2$

$0 + 0 + \frac{1}{2}(1000)V_1^2 = 0 + 1000 \times 9.81 \times 0.5 + \frac{1}{2} \times 1000 \times (0.7)^2$

$V_1 = 3.21 \, m/s$

$Q = A \cdot V$

$0.1 = \frac{1}{4}\pi D^2 \times 3.21$

$D = 0.199 \, m$

$\approx 0.2 m$

3(a)(i) $D = 150mm$ $\quad W_{shaft} = \dfrac{\rho g Q h_A}{\eta}$ $\quad \dfrac{1}{\sqrt{f}} = -2\log\left(\dfrac{\varepsilon/D}{3.7}\right)$

$\varepsilon = 0.38mm$

$l = 2000m$ $\quad \dfrac{1}{\sqrt{f}} = -2\log\left(\dfrac{\frac{0.38}{150}}{3.7}\right)$

$W_{shaft} = 95kW$

$f = 0.025$

$h_A = \Delta H = H_L + \Sigma$ Minor Loss

$\dfrac{\eta \times W_{shaft}}{\rho g Q} + 0 = 0.5\dfrac{V^2}{2g} + f\dfrac{L}{D} \times \dfrac{V^2}{2g} + \dfrac{V^2}{2g}$

Assume the efficiency of the pump is 100% which is not possible in real case

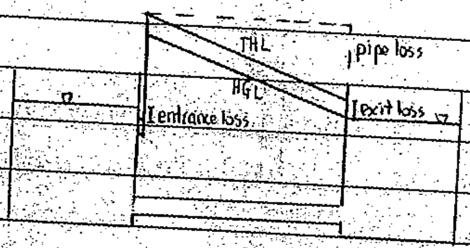
$\dfrac{95000}{1000 \times 9.81 \times Q} = 1.5 \times \dfrac{16Q^2}{2g \times \pi^2 \times D^4} + 0.025 \times \dfrac{2000}{0.15} \times \dfrac{16Q^2}{2g \times \pi^2 \times D^4}$

$\dfrac{9.68}{Q} = 244.8Q^2 + 54,405Q^2$

$9.68 = 54,649.3Q^3$

$Q = 0.0562 \, m^3/s$

(ii)



(iii) No because : - The assumption of the efficiency which is 100%
- The assumption that the roefficient of entrance loss for sharp-edge is 0.5

b $D = 0.3\,m$ $Re = \frac{V \cdot D}{\nu} = \frac{3.25 \times 0.3}{10^{-6}} = 975{,}000$

$\varepsilon = 0.046\,mm$

$H = 7\,m$

$V = 3.25\,m/s$ $\frac{\varepsilon}{D} = \frac{0.046}{300} = 1.53 \times 10^{-4}$

$L = 6000\,m$

From the diagram, $f = 0.0145$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + f\frac{L}{D}\frac{V^2}{2g}$$

$$\frac{P_1}{1000 \times 9.81} + 0 = 0 + 7 + 0.0145 \times \frac{6000}{0.3} \times \frac{3.25^2}{2 \times 9.81}$$

$P_1 = 1{,}600\,kPa$

4(a)(i) $h = f(d, g, \rho, \sigma, \theta)$

Repeating variables: d, g, ρ

Number of π: 2

π_1: $h d^a g^b \rho^c$

$[L][L]^a([L][T]^{-2})^b([M][L]^{-3})^c = [M]^0[L]^0[T]^0$

M: $c = 0$

T: $-2b = 0$

L: $1 + a + b - 3c = 0$

$a = -1$

$\therefore \pi_1 = \frac{h}{d}$

π_2: $\sigma d^a g^b \rho^c$

$[M][T]^{-2}[L]^a([L][T]^{-2})^b([M][L]^{-3})^c = [M]^0[L]^0[T]^0$

M: $1 + c = 0 \rightarrow c = -1$

T: $-2 - 2b = 0 \rightarrow b = -1$

L: $a + b - 3c = 0 \rightarrow a = -2$

$\therefore \pi_2 = \frac{\sigma}{\rho g d^2}$

Hence $\frac{h}{d} = \frac{\sigma}{\rho g d^2}$

(ii) $h_1 = 10\text{cm}$

$\frac{d_2}{d_1} = \frac{1}{2}$ $\quad \frac{\sigma_2}{\sigma_1} = \frac{1}{2}$ $\quad \frac{\rho_2}{\rho_1} = 2$

$h = \frac{\sigma}{\rho g d} \Rightarrow \frac{h_2}{h_1} = \left(\frac{\sigma_2}{\sigma_1}\right)\left(\frac{\rho_1}{\rho_2}\right)\left(\frac{\cancel{g_1}}{\cancel{g_2}}\right)\left(\frac{d_1}{d_2}\right)$

$\frac{h_2}{h_1} = \frac{1}{2} \times 2 \times 2$

$h_2 = 2 \times 10$

$h_2 = 20\text{cm}$

b(i) $\tau = \frac{r\Delta p}{2L}$

$$-\mu \frac{du}{dr} = \frac{r\Delta p}{2L}$$

$$\int du = -\frac{\Delta p}{2\mu L}\int r\,dr$$

$$u(r) = -\frac{\Delta p\, r^2}{4\mu L} + C$$

When $r = \frac{D}{2}$, $u(r) = 0$

$$C = \frac{\Delta p\, D^2}{16\mu L}$$

$$u(r) = -\frac{\Delta p\, r^2}{4\mu L} + \frac{\Delta p\, D^2}{16\mu L}$$

$$= \left(\frac{\Delta p}{16\mu L}\right) D^2\left[1 - \left(\frac{2r}{D}\right)^2\right]$$

(ii) $Q = \int_0^{\frac{D}{2}} u(r)\, dA$

$$= \left(\frac{\Delta p}{16\mu L}\right) D^2 \int_0^{\frac{D}{2}} \left[1 - \left(\frac{2r}{D}\right)^2\right] 2\pi r\, dr$$

$$= \frac{\pi\, \Delta p\, D^2}{8\mu L}\int_0^{\frac{D}{2}} \left(r - \frac{4r^3}{D^2}\right) dr$$

$$= \frac{\pi\, \Delta p\, D^2}{8\mu L}\left[\frac{1}{2}r^2 - \frac{r^4}{D^2}\Big|_0^{\frac{D}{2}}\right]$$

$$= \frac{\pi\, \Delta p\, D^4}{128\mu L}$$

(iii) We cannot assume that the flow is a fully developed steady flow. There is certain disturbance that affect it's velocity, such as minor loss. Hence, the flow rate will definitely decrease.